System of Rotating Discs in Second Order Rotation

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Abstract

The second order rotatory theory of hydrodynamic lubrication was founded on the expression obtained by retaining the terms containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there are some new excellent fundamental solutions with the help of geometrical figures, expressions, calculated tables and graphs for the infinitely long journal bearings in the second order rotatory theory of hydrodynamic lubrication. The analysis of equations for pressure and load capacity, tables and graphs reveal that pressure and load capacity are not independent of viscosity and increases slightly with viscosity. Also the pressure and load capacity both increases with increasing values of rotation number. In the absence of rotation, the equation of pressure and load capacity gives the classical solutions of the classical theory of hydrodynamic lubrication. The relevant tables and graphs confirm these important investigations in the present paper.

Key words: Continuity, Density, Film thickness, Reynolds equation, Rotation number, Taylor's number, Viscosity.

1. Introduction

1.1 Journal bearing:

In general the bearings [10, 11] can be divided in to four categories:
(1) Rolling element bearings for example; ball, cylindrical, spherical or tapered roller and needle etc.
(2) Dry bearings for example; plastic bushings, coated metal bushings etc.
(3) Semi-lubricated bearings for example; oil-impregnated bronze bushings etc.
(4) Fluid film bearings for example; crankshaft bearings etc.

Except from some radial-configuration aircraft engines, almost all piston engines use fluid film bearings [7]. This is true for the crankshaft and sometimes in the camshaft, although often the later runs directly in the engine structure. Here we have to discuss the working of the fluid film working and to demonstrate how engine designers are reducing friction losses through bearing technology [10]. The fluid film bearings operate by generating, as a by-product of the relative motion between the shaft and the bearing, a very thin film of lubricant at a sufficiently high pressure to match the applied load, as long as that load is within the bearing capacity [7]. Fluid film bearings represent a form of scientific process, by virtue of providing very large load carrying capabilities in a compact, lightweight implementation, and unlike the other classes, in most cases can be designed for infinite life. The fluid film bearings operate in any of the three modes:
(a) Fully-hydrodynamic
(b) Boundary
(c) Mixed.

In fully hydrodynamic or "full-film" [7, 11] lubrication, the moving surface of the journal is completely separated from the bearing surface by a very thin film of lubricant. The applied load causes the centerline of the journal to be displaced from the centerline of the bearing. This eccentricity creates a circular "wedge" in the clearance space.

The lubricant, by virtue of its viscosity, clings to the surface of the rotating journal, and is drawn into the wedge, creating a very high pressure, which acts to separate the journal from the bearing to support the applied load.

The bearing eccentricity is expressed as the centerline displacement divided by the radial clearance. The bearing eccentricity increases with applied load and decreases with greater journal speed and viscosity. The hydrodynamic pressure has no relationship at all to the engine oil pressure, except that if there is insufficient engine oil pressure to deliver the required copious volume of oil into the bearing, the hydrodynamic pressure mechanism will fail and the bearing and journal will be

destroyed. The pressure distribution in the hydrodynamic region of a fluid film bearing increases from quite low in the large clearance zone to its maximum at the point of minimum film thickness for the incompressible fluid like oil is pulled into the converging "wedge" [4, 5] zone of the bearing. However, this radial profile does not exist homogeneously across the axial length of the bearing. If the bearing has sufficient width, the profile will have a nearly flat shape across the high-pressure region.
The second mode of bearing operation is boundary lubrication. In boundary lubrication, the "peaks" of the sliding surfaces i.e., journal and bearing, are touching each other, but there is also an extremely thin film of the lubricant only a few molecules thick which is located in the surface "valleys". That thin film tends to reduce the friction from what it would be if the surfaces were completely dry.
The mixed mode is a region of transition between boundary and full-film lubrication. The surface peaks on the journal and bearing surfaces partially penetrate the fluid film and some surface contact occurs, but the hydrodynamic pressure is starting to increase.
When motion starts, the journal tries to climb on the wall of the bearing due to the metal-to-metal friction between the two surfaces.

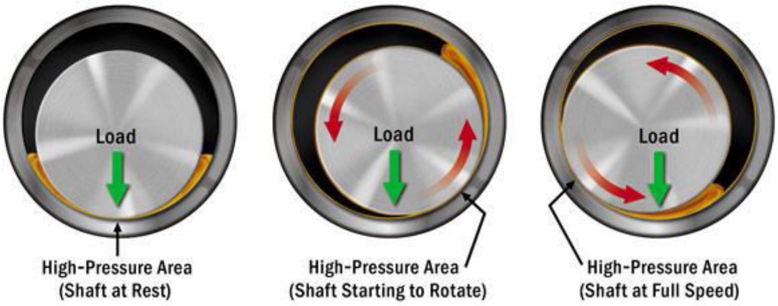

Figure 1(Journal Bearing Rotation [16])

If there is an adequate supply of lubricant, the motion of the journal starts to drag the lubricant into the wedge area and hydrodynamic lubrication begins to occur along with the boundary lubrication.
If we assume that the load and viscosity remain relatively constant during this startup period, then as revolution per minute increases, the hydrodynamic operation strengthens until it is fully developed and it moves the journal into its steady state orientation. The direction of the eccentricity and the minimum film thickness, do not occur in line with the load vector and are angularly displaced from the load.
There are also some other form of fluid-film lubrication, which includes the squeeze-film lubrication [10] i.e., the piston engine etc. Figure-1 (Hydrodynamic journal bearing)

Squeeze-film action is based on the fact that a given amount of time is required to squeeze the lubricant out of a bearing axially, thereby adding to the hydrodynamic pressure, and therefore to the load capacity. Since there is little or no significant rotating action in the wrist-pin bores, squeeze-film hydrodynamic lubrication is the prevailing mechanism which separates wrist pins from their bores in the rods and pistons.
The figure-1 shows a hydrodynamic journal bearing and a journal, are rotating in the clockwise direction. The rotation of the journal causes pumping of the lubricant that flows around the bearing in the direction of rotation. If there is no force applied to the journal then its position remains unaltered and concentric to the bearing position. The loaded journal moves from the concentric position and forms converging gap between the journal surfaces and bearing. Now the movement of journal forced the lubricant to squeeze through the gap generating the pressure. The pressure falls to the cavitations pressure [10] i.e., closer to the atmospheric pressure in the gap in which the cavitations forms.

In general the two types of cavitations are form in the journal bearing.

(a) **Gaseous cavitations:** This is associated with air and gases mixed with lubricant. If the pressure of lubricant falls below the atmospheric pressure then the gases come out to form the cavitations.
(b) **Vapors cavitations:** This is formed when the load applied to the bearing fluctuates at the high frequency. The pressure of fluid falls rapidly and causes the cavitations due to fast evaporation.

Now the fluid pressure creates the supporting force which separates the journal from the surface of the bearing. The hydrodynamic force of friction and force of fluid pressure counterbalance the external load. So the position of journal can be determined by these forces. In the hydrodynamic regime, the journal climbs in the rotational direction. If the working of journal is in the boundary and mixed lubrication then the hydrodynamic pressure ends and the journal climbs in the opposite to the rotational direction.

1.2 Second Order Rotatory Theory of Hydrodynamic Lubrication:

In the theory of hydrodynamic lubrication, two dimensional classical theories [4, 10] were first given by Osborne Reynolds [11]. In 1886, in the wake of a classical experiment by Beauchamp Tower [12], he formulated an important differential equation, which was known as: Reynolds Equation [11]. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some important assumptions given as:

(1.1)The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.

(1.2)If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation" [7, 10], which depends on density, viscosity, film thickness, surface and transverse velocities. The rotation [1] of fluid film about an axis that lies across the film gives some new results in lubrication problems of fluid mechanics. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film.

The new extended version of "Generalized Reynolds Equation" [7, 10] is said to be "Extended Generalized Reynolds Equation"[1,3], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M [1], i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication" [1, 3]. The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication" [3, 8] was given by retaining the terms containing up to first and second powers of M respectively by neglecting higher powers of M.

The lubrication of discs can be made kinematically equivalent to gears if they have the same radius at their contact line and rotate at the same angular velocities as the gears. For the system of discs, we will take the origin at the surface of disc of radius R on the line of centers of the two discs. The geometry of the system of discs is given by the figure () and figure ().

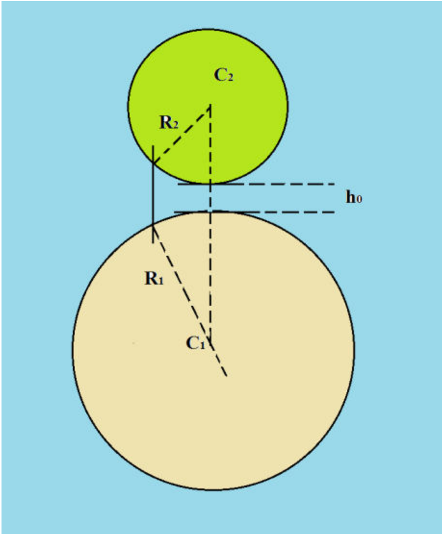

Figure (Geometry of system of discs)

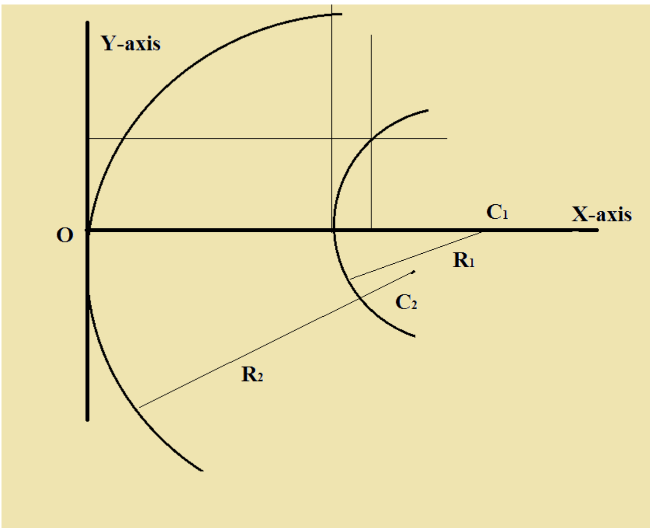

Figure (Film thickness of system of discs)

The film thickness 'h' is given by

$$h = h_0\left[1 + \frac{y^2}{2h_0}\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right)\right] \quad (\ )$$

$$\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right) = \frac{1}{R} \quad (\ )$$

$$tan\theta = \frac{y}{\sqrt{2Rh_0}} \quad (\ )$$

$$h = h_0 sec^2\,\theta \quad (\ )$$

Let us assume that the disc is stationary at the lower surface transverse to the fluid film where sliding is absent and U=+U (constant).

Suppose the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction. So the terms containing pressure gradient $\partial p/\partial x$ can be neglected as compared to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence *P* may be taken as function of *y* alone.

2. Governing Equations and Boundary Conditions:

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation" [7] is given by equation (1). Let us consider the mathematical terms as follows:

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}-\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]-\rho W^{*} \tag{1}$$

Where *x, y* and *z* are coordinates, *U* is the sliding velocity, *P* is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in *z*-direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* and by retaining the terms containing up to second powers of *M* and neglecting higher powers of *M*, can be written as equation (2). For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2}$$

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right]$$

(3)

Taking *h=h(y), U=+U, P=P(y)* (4)

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{dP}{dy}\right]=-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right]$$

(5)

For the determination of pressure distribution in the positive regions, the boundary conditions are as follows:

(i) $P=0$ at $h=h_0$ or $P=0$ at $y=0$ or $P=0$ at $\theta=0$

(ii) $P=dP/d\theta=0$ at $y=y_1$ or $\theta=\gamma$ (say) (6)

Where is determined by putting $\theta=\gamma$ and P=0 in the equation of pressure.

Pressure

The solution of the differential equation under the boundary conditions imposed is given by

$$P=-\sqrt{\frac{Rh_0}{2}}\,M\rho U\left[\frac{17\,M^2\rho^2{h_0}^4}{1680\mu^2}\,F(\theta)-\tan\theta\,F(\gamma)\right]\;(\quad)$$

Where $F(\theta)$ is given by

$$F(\theta)=\tan\theta\left[\frac{1}{9}\,sec^8\theta+\frac{8}{63}\,sec^6\theta+\frac{48}{315}\,sec^4\theta+\frac{192}{945}\,sec^2\theta+\frac{384}{945}\right]\;(\quad)$$

5. Calculation tables and graphs

By taking the values of different mathematical terms in C.G.S. system the calculated tables and graphical representations are as follows:

$U=80,\ \rho=1.0,\ R=3.35,\ h_o=0.0167,\ \mu=0.0002,\ \theta=30^0,\ \gamma=\ 60^0$

S.NO.	*M*	*P*
1.	0.1	93.22554478
2.	0.2	186.4498789
3.	0.3	279.6717919
4.	0.4	6145.652604
5.	0.5	466.1035115
6.	0.6	559.310897
7.	0.7	652.5110187
8.	0.8	745.7026661

9.	0.9	838.8846285
10.	1.0	932.0556954

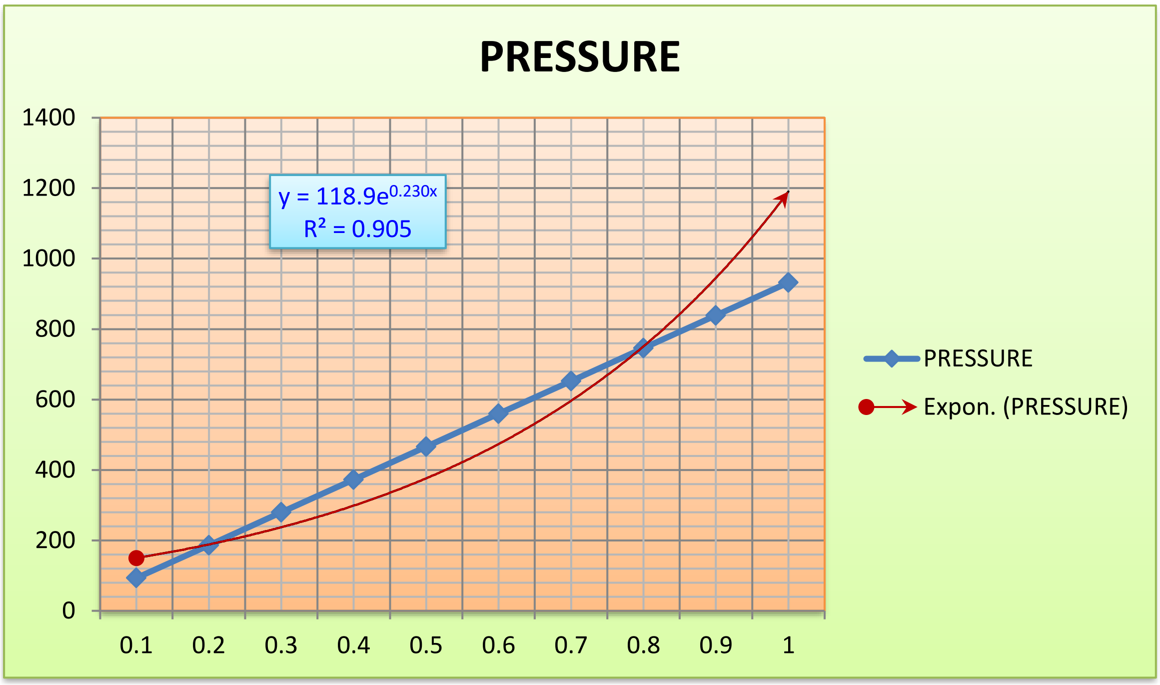

Figure (Variation of pressure with respect to rotation number M with exponential trendline)

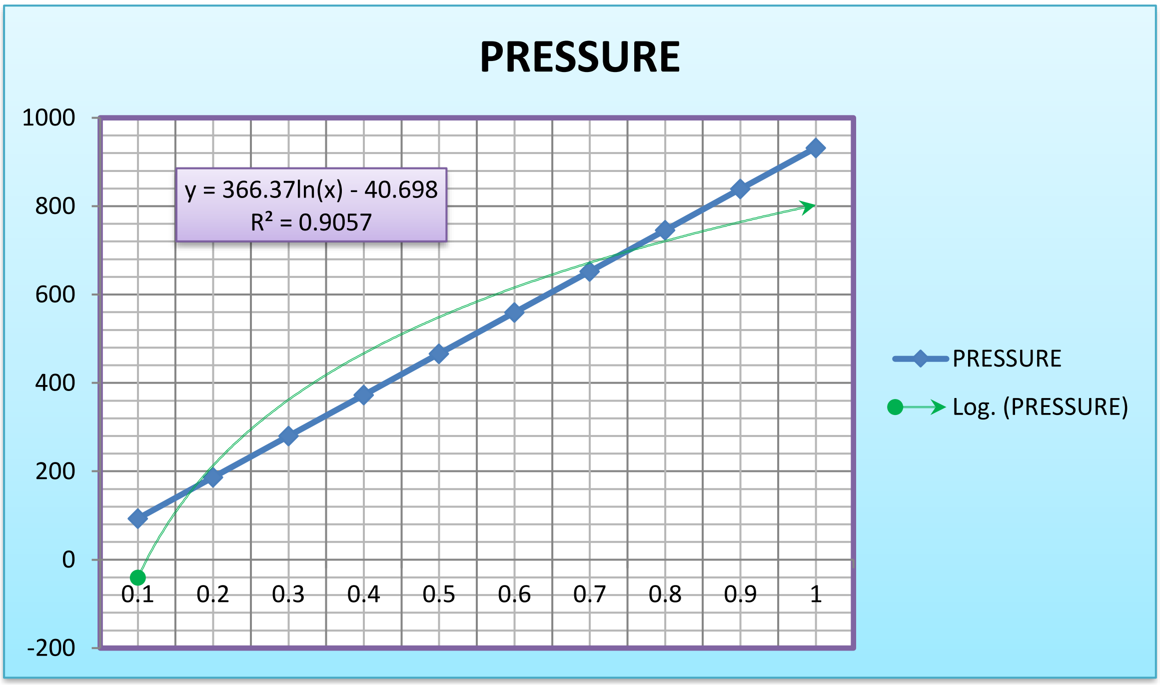

Figure (Variation of pressure with respect to rotation number M with logarithmic trendline)

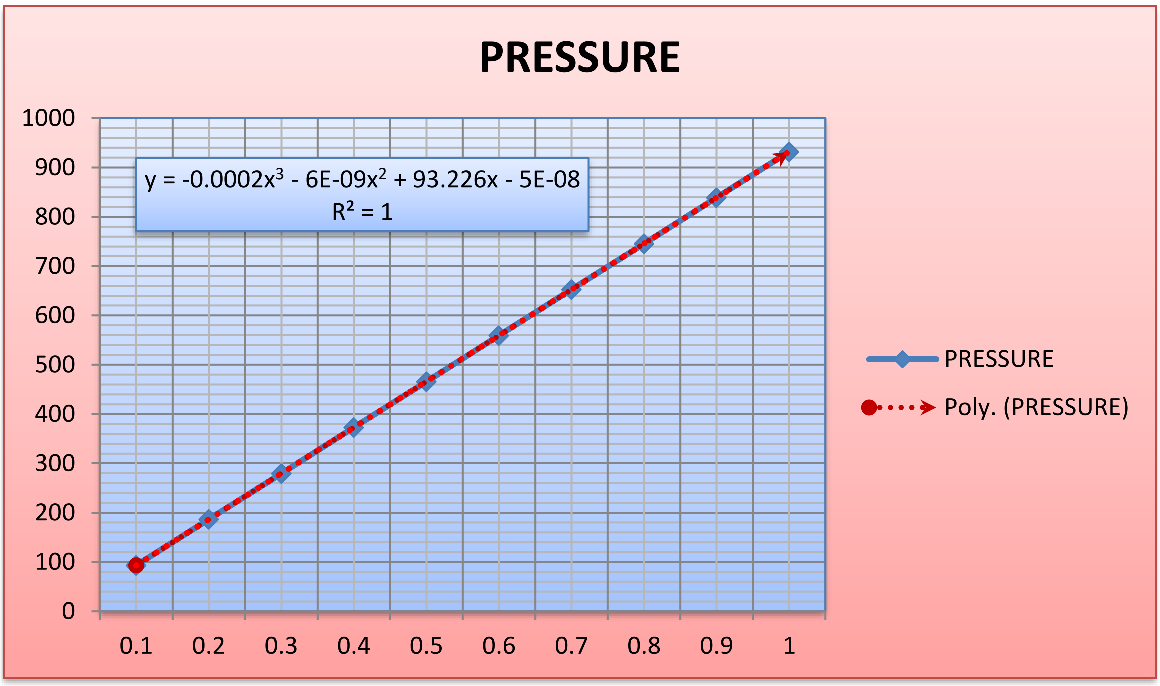

Figure (Variation of pressure with respect to rotation number M with third degree polynomial trendline)

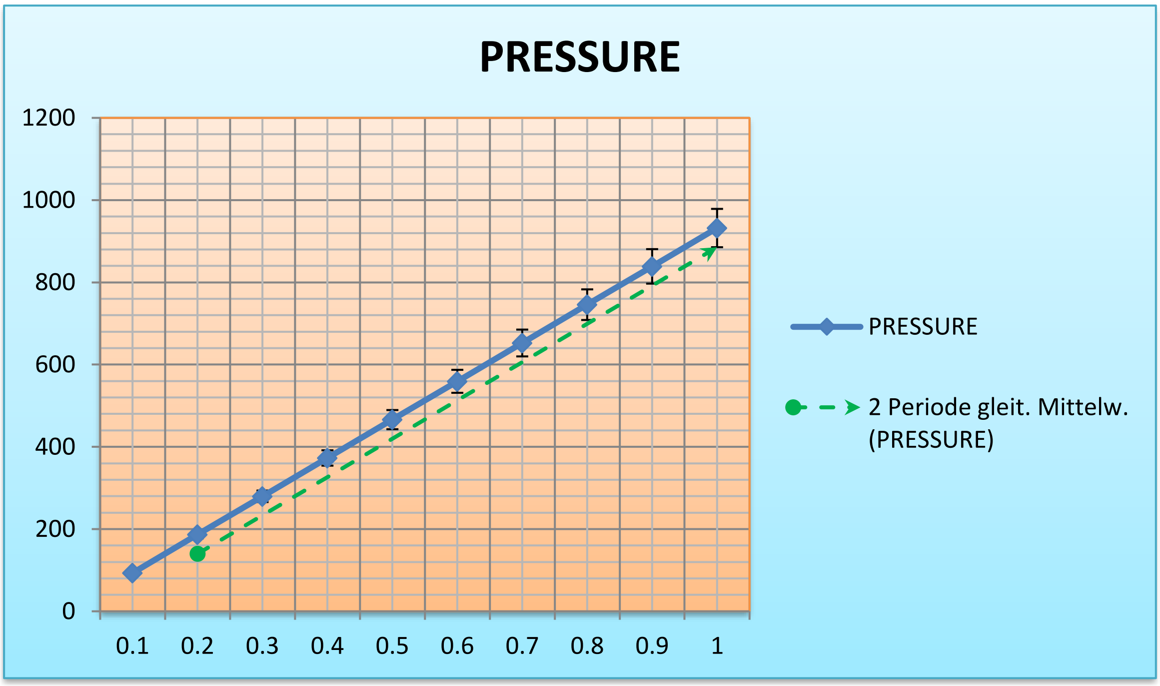

Figure (Variation of pressure with respect to rotation number M with moving average with 200% error bars)

Load Capacity

The load capacity is given by

$$W = \int_{\gamma}^{0} P \, dy \; (\quad)$$

$$=\int_{\gamma}^{0} P\, sec^2\theta\, \sqrt{2Rh_0}\, d\theta$$

$$=\sqrt{2Rh_0}\int_{\gamma}^{0} P\, sec^2\theta\, d\theta$$

$$W = Rh_0 M\rho U\left[\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}\int_{\gamma}^{0} F(\theta) sec^2\theta\, d\theta - \int_{\gamma}^{0}\tan\theta\, sec^2\theta\, d\theta\right] ()$$

$$W = -\frac{17M^3\rho^3{h_0}^3UR}{1680\mu^2}\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^8\gamma+\frac{8}{315}sec^6\gamma+\frac{16}{315}sec^4\gamma+\frac{192}{945}tan^2\gamma\right) - \frac{Rh_0M\rho U}{2}\, tan^2\gamma\ ()$$

5. Calculation tables and graphs

By taking the values of different mathematical terms in C.G.S. system the calculated tables and graphical representations are as follows:

5.1 Table

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = 60^0$

S.NO.	*M*	*W*
1.	0.1	6.2844159
2.	0.2	46.2472872
3.	0.3	153.5670693
4.	0.4	361.9222176
5.	0.5	704.9911875
6.	0.6	1216.452434
7.	0.7	1929.984414
8.	0.8	2879.265581
9.	0.9	4097.974391
10.	1.0	5619.7893

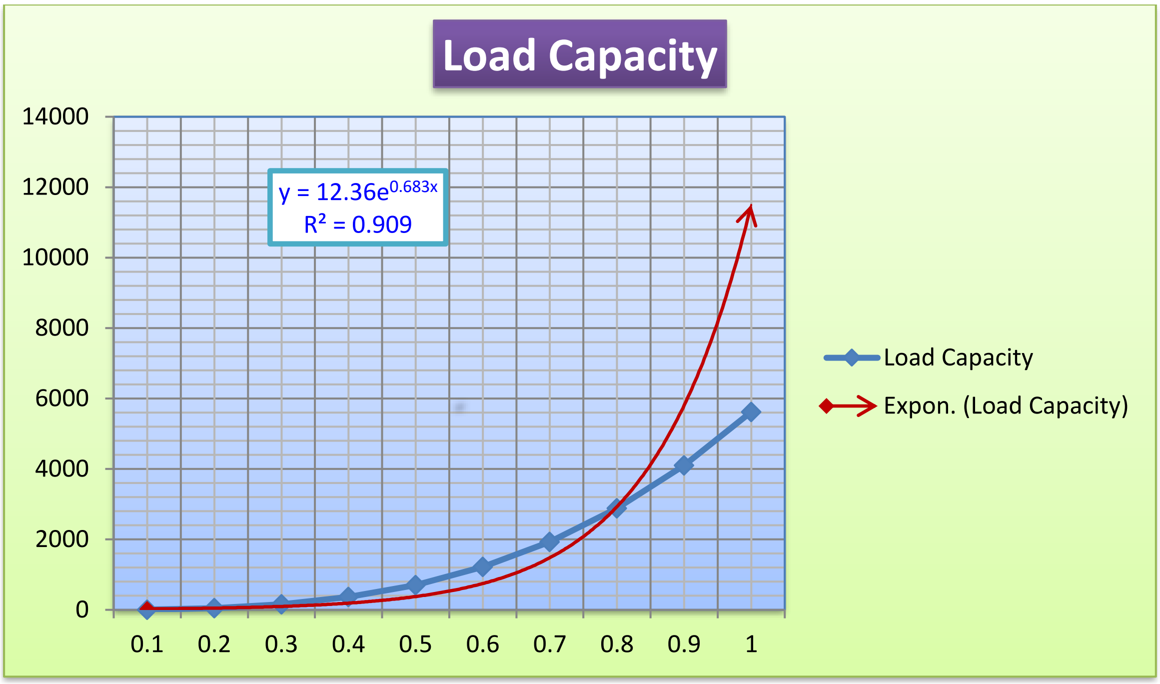

Figure (Variation of load capacity with respect to rotation number M with exponential trendline)

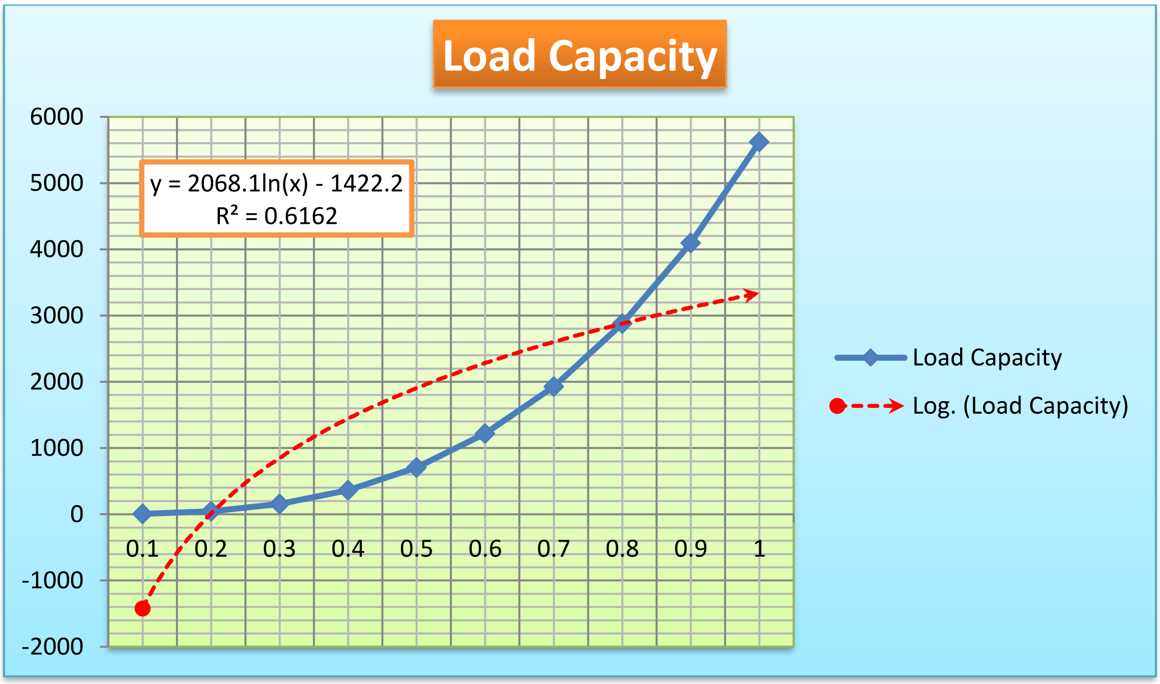

Figure (Variation of load capacity with respect to rotation number M with logarithmic trendline)

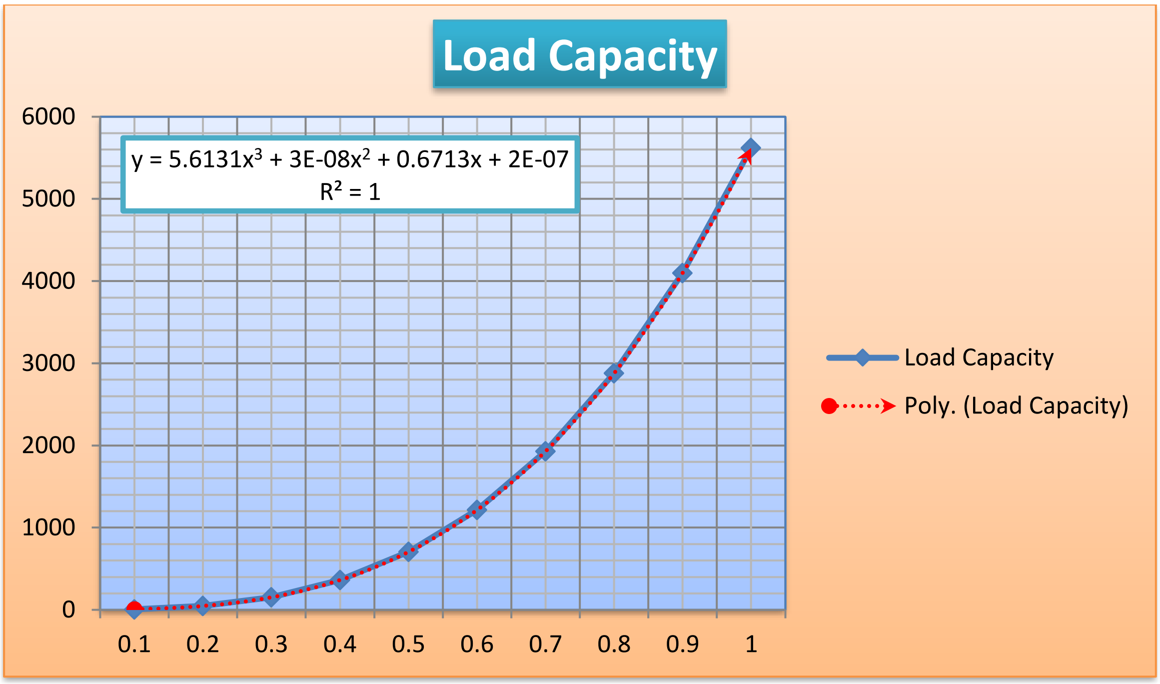

Figure (Variation of load capacity with respect to rotation number M with third degree polynomial trendline)

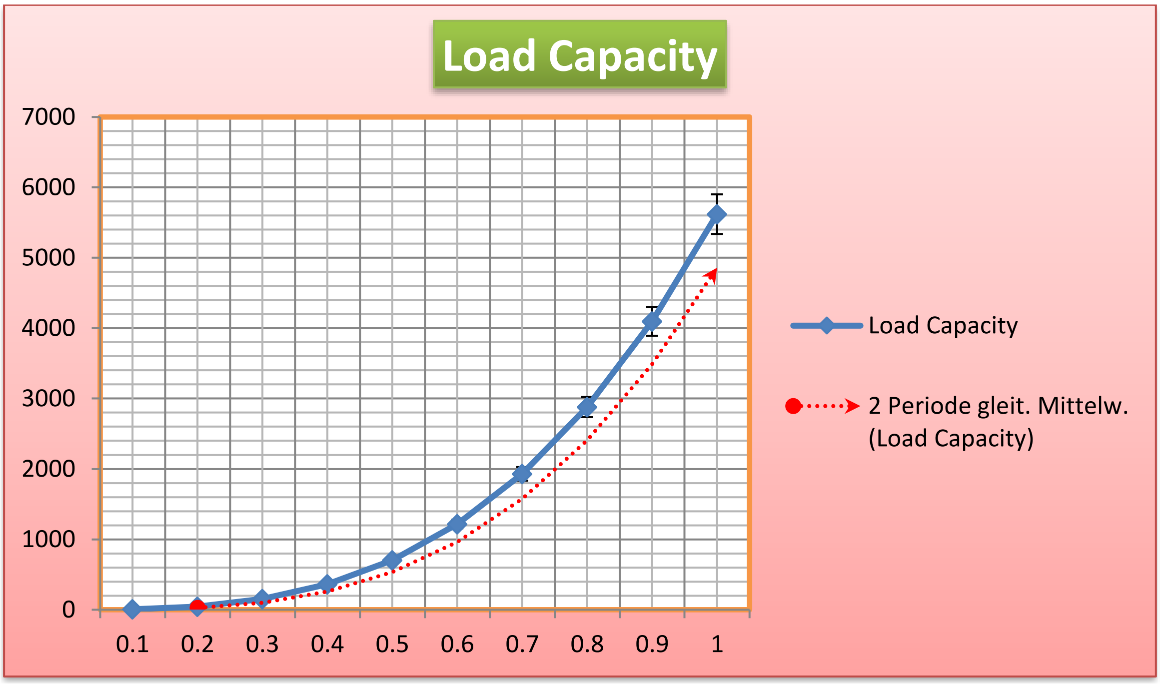

Figure (Variation of load capacity with respect to rotation number M with 2-period moving average trendline with 5% error bars)

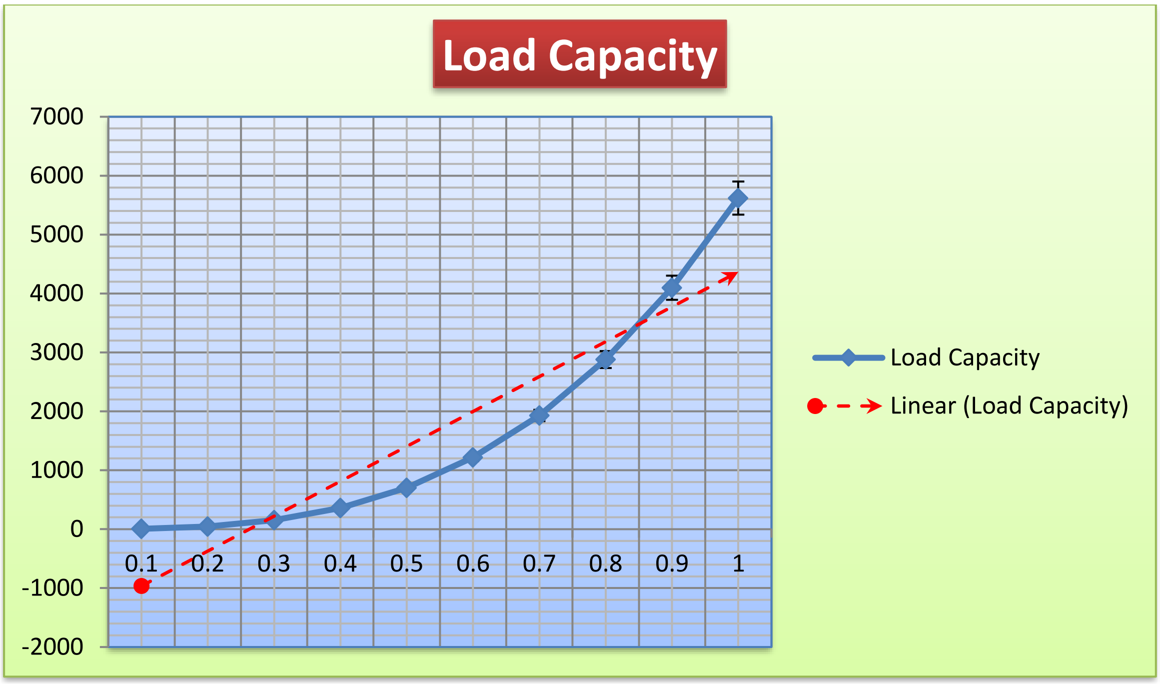

Figure (Variation of load capacity with respect to rotation number M with linear trendline with 5% error bars)

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