Analysis of System of Rotating Discs with MR-Fluid

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Abstract

The analysis of rotating discs is observed in areas of various engineering fields like as flywheels, turbine engines, gears and others. This paper analyses the subject of a plain rotating disc for the fluid flow, thereby providing an in-depth development of understanding of the flow physics and modeling approach for the second order rotation with MR-fluid. The solution of differential equation for the motion of system of discs with MR-fluid gives the load carrying capacity which numerically gives various solutions. In this analysis with MR-fluid the magnetic field is applying on the system that results the change in the behavior of fluid with variation of intensity of magnetic field. The load capacity varies positively on increasing values of Hartmann number and also with the intensity of magnetic field.

Key words: Hartmann number, Load capacity, Magnetic field, Reynolds equation, Taylor's number, Viscosity.

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1. INTRODUCTION

The idea of 2-dimensional classical theories of lubrication [4], [10] changed into given through the equation of Osborne Reynolds [13]. In the wake of an analysis as the results of Beauchamp Tower experiment [14], he had given a differential equation that stated as Reynolds Equation [13]. The simple mechanism and formation of the fluid film changed into discovered via means of that experiment with few assumptions that the fluid film thickness is much smaller than its axial and longitudinal dimensions and if lubricant layer is to supply stress among the bearing and the shaft then the layer will range the thickness of the fluid film. After sometimes, Osborne Reynolds once more revised his differential equation that changed into advanced model and stated as: Generalized Reynolds Equation [7], [10]. This differential equation relies upon on viscosity, density, film thickness, transverse and longitudinal velocities. The idea of the rotation [1] of the fluid film approximately an axis, which lies throughout the fluid film, offers a few amazing solutions within the lubrication issues of the fluid mechanics. The starting place of rotation changed via means of a few theorems of vorticity within the rotating fluid dynamics. The rotation induces aspect of vorticity within the course of rotation of fluid film and outcomes springing up from it are predominant, for big Taylor's Number, it effects in streamlines turning into limited to transverse to course of rotation of the fluid film. The today's prolonged model of the Generalized Reynolds Equation [7], [10] is referred to as the Extended Generalized Reynolds Equation [1], [3] that takes into consideration of outcomes of uniform rotation approximately an axis, which lies throughout the fluid film and relies upon on rotation quantity M [1], that is the root of the classical Taylor's Number. The generalization of the concept of hydrodynamic lubrication is stated because the Rotatory Theory of Hydrodynamic Lubrication [1], [3]. The idea of the Second Order Rotatory Theory of Hydrodynamic Lubrication [3], [8] changed into given via means of keeping expressions containing as much as 2^{nd} powers of M and neglecting big powers of M. The lubrication of discs may be made mechanically equal to gears in the event that they have the identical radius at their touch line and rotate on the identical angular velocities that of the gears. For the machine of discs, we are able to take the origin on the floor of disc of radius R on the lines of centers of the discs. The geometry of the machine of discs is given by the figure (1.1) and figure (1.2).

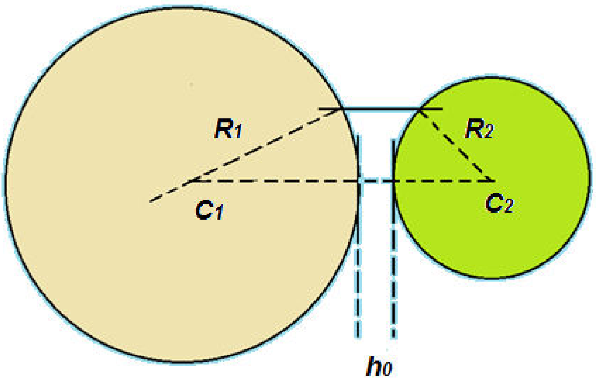

Figure- (1.1) (Geometry of system of discs)

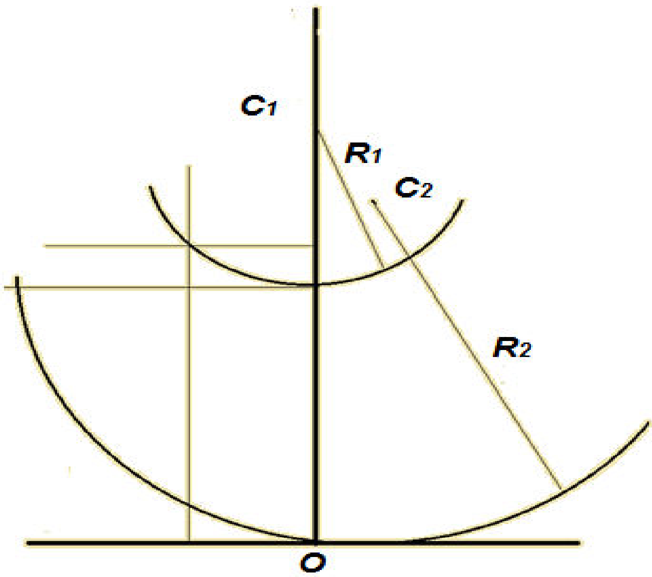

Figure- (1.2) (Film thickness of system of discs)

The fluid film thickness 'h' will be:

$$h = h_0\left[1 + \frac{y^2}{2h_0}\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right)\right] \tag{1.1}$$

$$\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right) = \frac{1}{R} \tag{1.2}$$

$$tan\theta = \frac{y}{\sqrt{2Rh_0}} \tag{1.3}$$

$$h = h_0 sec^2\theta \tag{1.4}$$

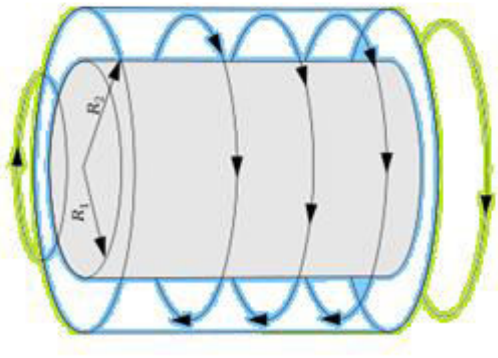

Figure- (1.3) (System of rotation of discs [20], [21])

2. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS:

The Extended Generalized Reynolds Equation of the second order rotation, in increasing powers of *M* can expressed as equation (2.1).

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right] \tag{2.1}$$

In the above equation; *x, y* and *z* belongs to the coordinate system, μ is the dynamic viscosity of the fluid, *U* is the sliding velocity, *P* belongs to pressure, ρ is the density of fluid.

Let us consider that the disc is stationary at the lower surface transverse of the fluid film at the place of zero sliding and *U*=+*U* (constant). Also considering that the pressure variation in *x*-direction is too low in comparison to the variation in its perpendicular direction. So the expressions having terms of pressure gradient $\partial p/\partial x$ can be omitted in comparison to the terms having $\partial p/\partial y$ in the final differential equation, so that *P* can be taken as function of only *y*. Taking *h=h(y), U=+U, P=P(y)*;

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2.2}$$

We have

$$M^2 = T_a = \frac{4\Omega^2 L^2}{\mu^2} \tag{2.3}$$

$$H_a = L\mathrm{B}\sqrt{\frac{\sigma}{\mu}} \tag{2.4}$$

Where,

T_a =Taylor's number

H_a=Hartmann number

Ω =Characteristic angular velocity

L =Characteristic length scale perpendicular to the direction of rotation

B=Magnetic field intensity

σ =Electric cunductivity

Hence the differential equation for the motion of system in the fluid can be expressed as:

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17\,\Omega^2 H_a^4 \rho^2 h^4}{420\mu^2 \mathrm{B}^4\sigma^2}\right)\rho\frac{\partial P}{\partial y}\right] = -\frac{d}{dy}\left[\frac{\Omega \mathrm{B}^2\rho^2 U}{\sigma H_a^2}L^3\left\{-\frac{h^3}{12\mu}\left(1-\frac{17\,\Omega^2 H_a^4\rho^2 h^4}{420\mu^2\mathrm{B}^4\sigma^2}\right)\right\}\right] \tag{2.5}$$

The imposed boundary conditions are as follows:

(i) *P=0 at h=h_0 or P=0 at y=0 or P=0 at θ=0*

(ii) *P=dP/dθ=0 at y=y_1 or θ=γ* (say) (2.6)

3. DETERMINATION OF LOAD CAPACITY

The solution of the differential equation under the boundary conditions imposed is given by

$$P = -\sqrt{\frac{Rh_0}{2}}\,M\rho U\left[\frac{17\,\Omega^2\rho^2\mathrm{B}^2 L^4\,{h_0}^4}{420\mu^2}\,F(\theta) - \tan\theta\,F(\gamma)\right] \tag{3.1}$$

Where $F(\theta)$ is given by

$$F(\theta) = \tan\theta\left[\frac{1}{9}\,sec^8\theta + \frac{8}{63}\,sec^6\theta + \frac{48}{315}\,sec^4\theta + \frac{192}{945}\,sec^2\theta + \frac{384}{945}\right] \tag{3.2}$$

The load capacity for the system of discs can be written as:

$$W = \int_\gamma^0 P\,dy = \int_\gamma^0 P\,sec^2\theta\,\sqrt{2Rh_0}\,d\theta \tag{3.3}$$

$$W = -\frac{17M\rho^3{h_0}^3 UR\Omega^2}{420\mu^2}\mathrm{B}^2L^4\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^8\gamma+\frac{8}{315}sec^6\gamma+\frac{16}{315}sec^4\gamma + \frac{192}{945}tan^2\gamma\right) - \frac{Rh_0 M\rho U}{2}\,tan^2\gamma \tag{3.4}$$

$$W = -\frac{17M\rho^3{h_0}^3 UR\Omega^2}{420\mathrm{B}^2\sigma^2}H_a^4\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^8\gamma+\frac{8}{315}sec^6\gamma+\frac{16}{315}sec^4\gamma+\frac{192}{945}tan^2\gamma\right) - \frac{Rh_0 M\rho U}{2}\,tan^2\gamma \tag{3.5}$$

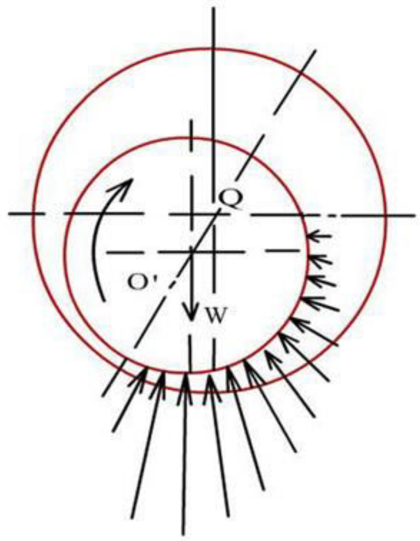

Figure- (4.1) (Load Capacity [19])

4. CALCULATION TABLE AND GRAPHS:

The numerical data for variations in the load carrying capacity with respect to magnetic field and Hartmann number can be calculated that is given by the table-4.1 and table-4.2 respectively.

4.1 Table: 4.1

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = \ 60^0,\ \Omega = 23.88059,\ M = 0.1,\ \sigma = 1, L = 1$

S. No.	*B*	*W*
1.	0.5	334740.702
2.	1.0	1338960.794
3.	1.5	3012660.948
4.	2.0	5355841.163
5.	2.5	8368501.439

4.2 Table 4.2

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = \ 60^0,\ \Omega = 23.88059,\ M = 0.1,\ \sigma = 1, B = 5$

S. No.	H_a	*W*
1.	0	0.67134
2.	40	4.099119
3.	60	8.383843
4.	80	14.382456
5.	100	22.094959

The graphical representations of the variation of pressure with respect to the magnetic field *B* and Hartmann number H_a are shown by the figure 4.1 and figure 4.2 respectively.

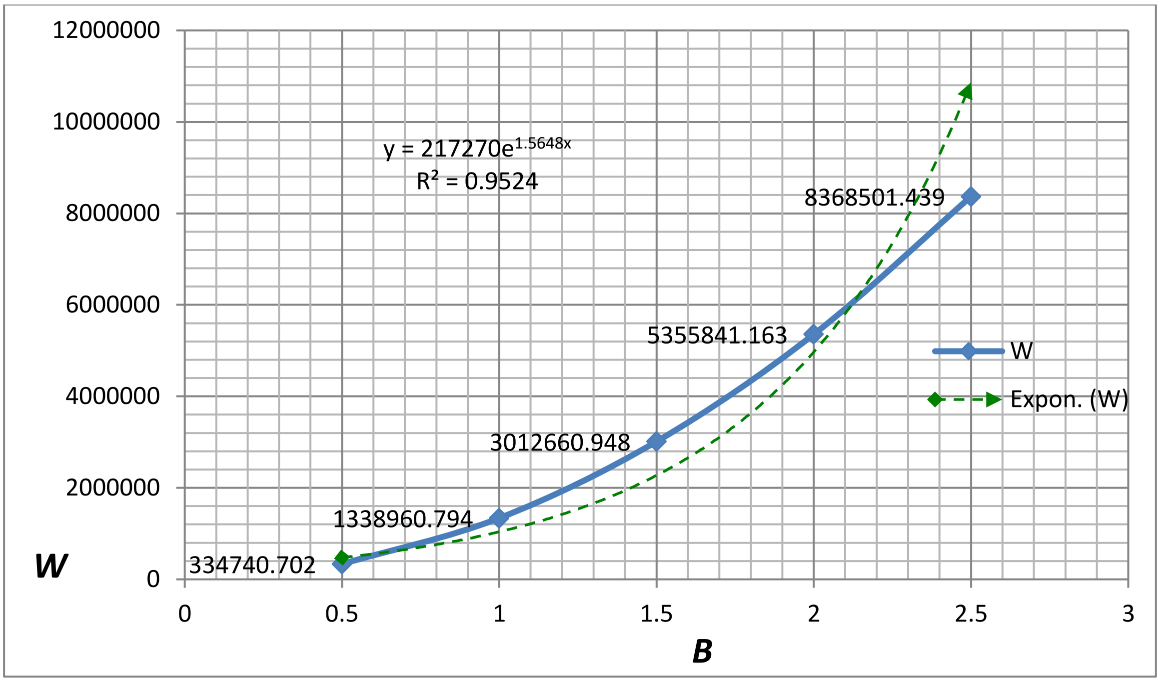

Figure-4.1 (Variation of ***W*** with respect to Magnetic field ***B*** with exponential trend line)

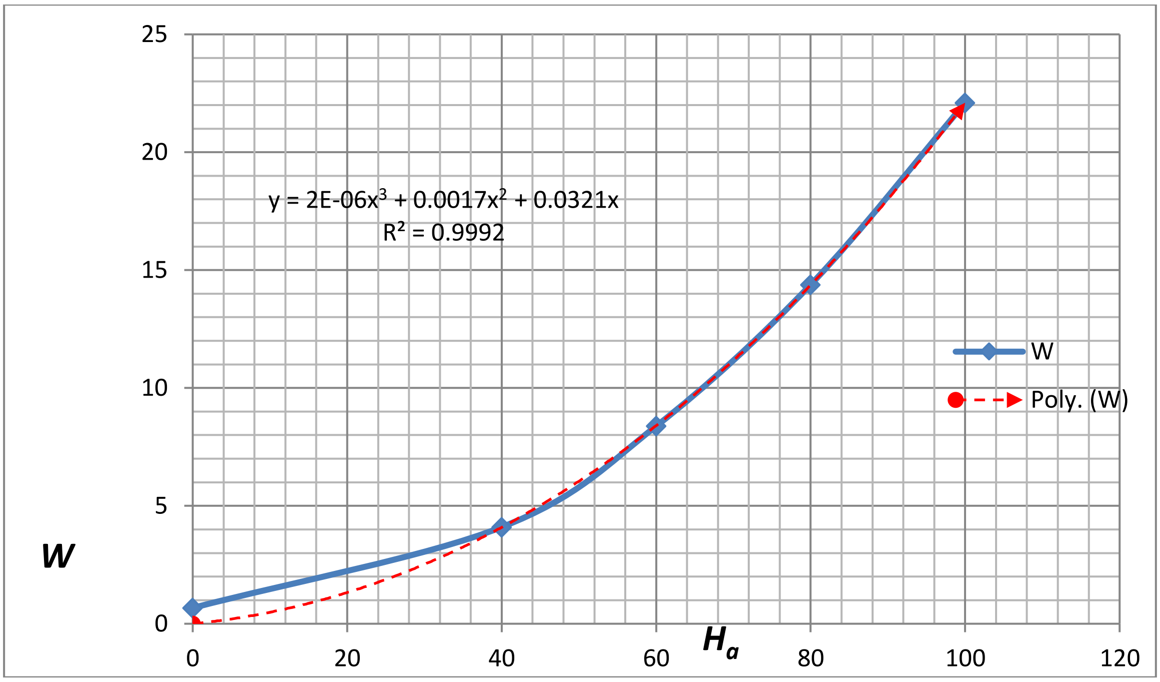

Figure-4.2 (Variation of ***W*** with respect to Hartmann number $\boldsymbol{H_a}$ with polynomial trend line)

5. RESULTS AND DISCUSSION:

The variation of load capacity W with respect to magnetic field B is shown by the table (4.1) and graph (4.1). The figure-4.1 shows the exponential trend line by $y=21727\ e^{1.564x}$ with $R^2=0.947$. The variation of load capacity W with respect to Hartmann number H_a by the table (4.2) and graph (4.2) with polynomial trend line $y=0.001x^2 +0.032x; R^2=0.998$. The figure-4.1, 4.2 indicate that the load capacity *W,* vary with magnetic field and Hartmann number.

6. CONCLUSION:

The derived equation of Load capacity is given by equation (3.4) and (3.5). The calculated values of the Load capacity against magnetic field ***B*** is shown in the table and graphical representation for the variation of Load capacity is also shown by figure-4.1. The comparisons of the Load capacity have been done with the help of geometrical figures, expressions, calculated tables and graphs for the lubricating discs in the second order rotatory theory of hydrodynamic lubrication. The analysis of equation for Load capacity, table and graphs show that pressure is not independent of dynamic viscosity μ and increase with increasing values of magnetic field intensity B, Hartmann number H_a, rotation number M, density of used fluid ρ, velocity of fluid U, characteristic length of the bearing L and film thickness h_0.

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8. CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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