Motion of Fluid Flow via Porous Bearings

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ABSTRACT

The second order rotatory theory of hydrodynamic lubrication was founded on the expression obtained by retaining the terms containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there are some new excellent fundamental solutions with the help of geometrical figures, expressions, calculated tables and graphs for the porous bearings in the second order rotator theory of hydrodynamic lubrication. The analysis of equations for pressure and load capacity, tables and graphs reveal that pressure and load capacity are not independent of viscosity and increases slightly with viscosity. Also the pressure and load capacity both increases with increasing values of rotation number. In the absence of rotation, the equation of pressure and load capacity gives the classical solutions of the classical theory of hydrodynamic lubrication. The relevant tables and graphs confirm these important investigations in the present paper.

Keywords: Lubrication theory, Pressure, Reynolds equation, Rotation,

I. INTRODUCTION

In the theory of hydrodynamic lubrication, two dimensional classical theory[4, 10] was first given by Osborne Reynolds[11]. In 1886, in the wake of a classical experiment by Beauchamp Tower[12], he formulated an important differential equation, which was known as: Reynolds Equation[11]. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some important assumptions given as:

[a] The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.

[b] If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation"[7, 10], which depends on density, viscosity, film thickness, surface and transverse velocities.

The rotation[1] of fluid film about an axis that lies across the film gives some new results in lubrication problems of fluid mechanics. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film.

The new extended version of "Generalized Reynolds Equation"[7, 10] is said to be "Extended Generalized Reynolds Equation"[1, 3], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M^1, i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication"[1, 3].

The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication"[3, 8] was given by retaining the terms containing up to first and second powers of M^1 respectively by neglecting higher powers of M^1.

The present paper analyzes about the pressure and load capacity in the porous bearings under the effect of second order rotation. These bearings are constructed by porous material and the lubricant flows out of the bearing surface with a definite velocity. These bearings are generally used in many useful devices, such as vacuum cleaners, extractor fans, motor car starters, hair dryers etc. These bearings are infinitely short. The geometry[4] of bearings is described in figure[1], of solid shaft in sintered metal bush and figure[2], of shaft and bush opened up.

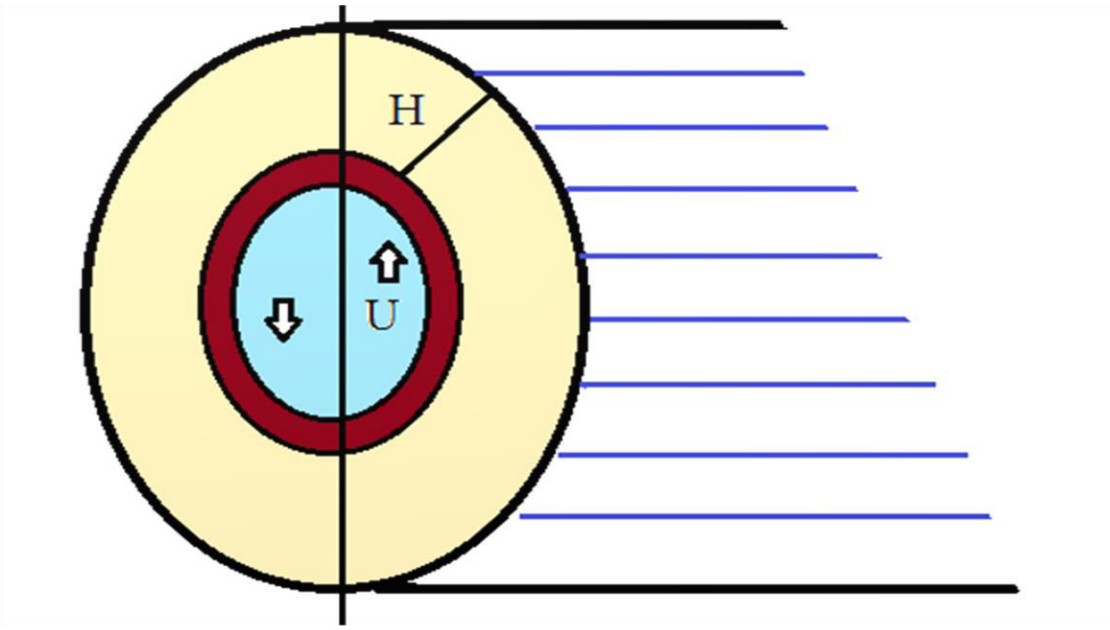

FIG.1. Solid shaft in sintered metal bush of thickness *H*.

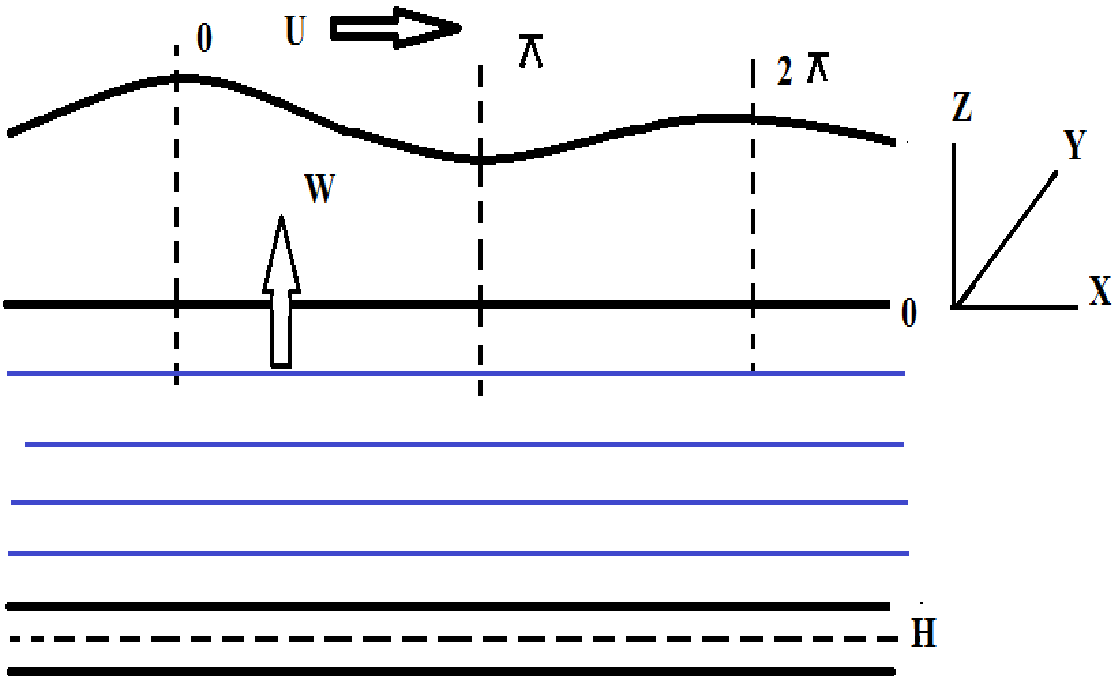

FIG.2. Geometry of shaft and sintered bush opened up.

Where *x, y* and *z* are coordinates, *H* is the wall thickness of porous bearing, *U* is the sliding velocity and *W* is the load capacity of the bearing.

If the bearing is infinitely short[2], then the pressure gradient $\frac{\partial P}{\partial x}$ in *x*-direction is much smaller than the pressure gradient $\frac{\partial P}{\partial y}$ in *y*-direction. In *y*-direction the gradient $\frac{\partial P}{\partial y}$ is of the order of $\left(\frac{P}{L}\right)$ and in the *x*-direction, and is of order of $\left(\frac{P}{B}\right)$; where *P* is the pressure, *B* is the breadth of bearing parallel to the direction of motion and *L* is the bearing length normal to the direction of motion. If $L << B$ then

$$\frac{P}{L} >> \frac{P}{B}, \text{ so } \frac{\partial P}{\partial x} << \frac{\partial P}{\partial y} \tag{1}$$

Then the terms containing $\frac{\partial P}{\partial x}$ can be neglected as compared to the terms $\frac{\partial P}{\partial y}$ containing in the expanded form of Generalized Reynolds Equation.

II. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation"[7] is given as:

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}-\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]-\rho W^{*} \quad (2)$$

Where ρ is the fluid density, μ is the viscosity, h is the fluid film thickness and W^* is fluid velocity in z-direction.

The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M [1] and by retaining the terms containing up to second powers of M and neglecting higher powers of M, can be written as:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right]-\rho W^{*}$$

(3)

On taking

h=$h(x)$, U=-U, P=$P(y)$ and W^* = $-\frac{\partial P}{\partial z}\Big|_{z=0}\frac{\phi}{\mu}$ (4)

Where $\frac{\partial P}{\partial z}$ the pressure gradient at the bearing surface and ϕ is the property called permeability, which varies with porosity and size of pores. From the requirements of continuity, we have for the porous matrix

$$\frac{\phi}{\mu}\nabla W^* = \nabla^2 P=0 \text{ i.e., } \nabla^2 P=0 \tag{5}$$

The problem then is to solve the governing equation (2) for the pressures in oil film simultaneously with that of Laplace for the porous matrix with a common pressure gradient $\frac{\partial P}{\partial z}$ at the boundary, we have

$$\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+\frac{\partial^2 P}{\partial z^2}=0 \tag{6}$$

We have two assumptions to solving the equations (2) and (6) as follows:

(A)The bearing is infinitely short.

(B) $\frac{\partial P}{\partial z}$ is linear across the matrix and is zero at the outer surface of the porous bearing shell.

From (A), (B), we have

$$\frac{\partial^2 P}{\partial x^2}=0,\ \frac{\partial^2 P}{\partial z^2}=K \text{ (constant)},\ \frac{\partial^2 P}{\partial y^2}=-K \tag{7}$$

From (4), we have

$$\frac{\partial P}{\partial z}\Big|_{Z=0}=KH=\frac{\partial^2 P}{\partial y^2}\Big|_{Z=0}H \tag{8}$$

Now the equation (2) becomes

$$\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\right]\frac{d^2P}{dy^2}+\left[\frac{M\rho^2}{120\mu^2}\frac{d}{dx}\left(h^5-\frac{31M^2\rho^2h^9}{3024\mu^2}\right)\right]\frac{dP}{dy}$$
$$=\frac{d}{dx}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]-\rho\left(-\frac{dP}{dz}\Big|_{z=0}\frac{\phi}{\mu}\right) \tag{9}$$

The film thickness 'h' and 'x' can be taken as:

$$h=C\,(1+e\cos\theta),\ x=R\,\theta \tag{10}$$

Where C is the radial clearance, e is the eccentricity ratio, R is the radius of bearing and θ is the angular coordinate being measured from x-direction.

For the determination of pressure the boundary conditions are as follows:

$P=0,\ y = \pm\frac{L}{2}$ (11)

III. DETERMINATION OF PRESSURE

The solution of the differential equation (9) under the boundary condition (11) gives the pressure for porous bearing as follows:

$$P = \frac{(3\mu CUe\sin\theta + 12KH\phi R)(L^2-4y^2)}{4(1+e\cos\theta)^3 R} + \frac{\rho Ce\sin\theta(U\mu + 4KH\phi)(L^2y-4y^3)}{8\mu R(1+e\cos\theta)^2}M + \frac{(53U\mu\rho^2 Ce\sin\theta(1+e\cos\theta) - 68RKH\phi\rho^2(1+e\cos\theta)(L^2-4y^2)}{2240\mu^2 R}M^2 \quad (12)$$

Taking the terms of pressure equation (12) as:

$$A = \frac{(3\mu CUe\sin\theta + 12KH\phi R)(L^2-4y^2)}{4(1+e\cos\theta)^3 R} \quad (13)$$

$$B = \frac{\rho Ce\sin\theta(U\mu + 4KH\phi)(L^2y-4y^3)}{8\mu R(1+e\cos\theta)^2} \quad (14)$$

$$C = \frac{(53U\mu\rho^2 Ce\sin\theta(1+e\cos\theta) - 68RKH\phi\rho^2(1+e\cos\theta)(L^2-4y^2)}{2240\mu^2 R} \quad (15)$$

Hence the equation for pressure takes the form: $P = A + BM + CM^2$. (16)

(IV) DETERMINATION OF LOAD CAPACITY

The load capacity W for porous bearing is given by

$W = \sqrt{W_x^2 + W_y^2}$ (17)

where W_x and W_y are the components of the load capacity in x-direction and y-direction respectively.

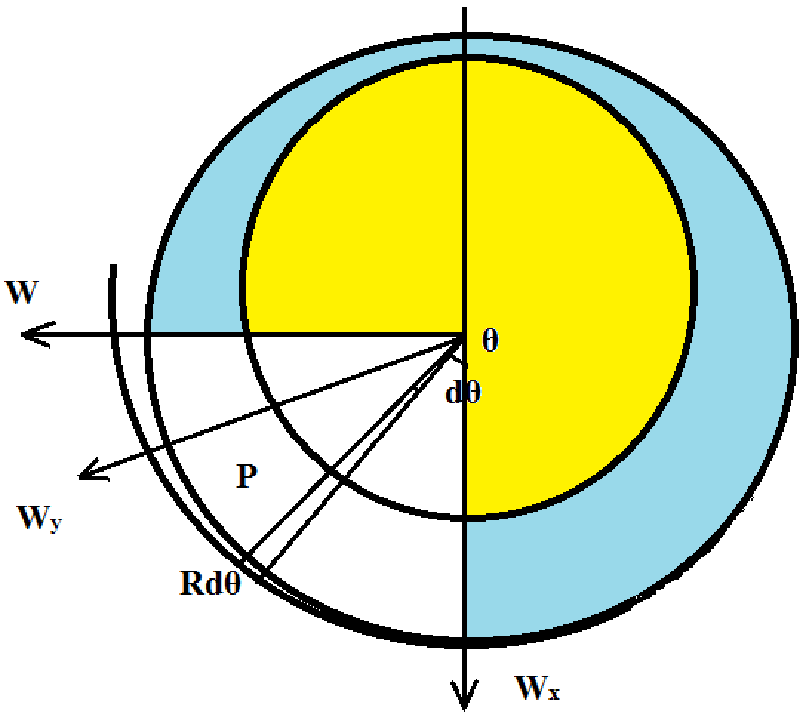

FIG.3. Geometry for the Components of Load Capacity.

$$W_x = -2\int_0^{\pi}\int_0^{L/2} P\, Cos\theta\, R\, d\theta\, dy \tag{18}$$

$$W_y = 2\int_0^{\pi}\int_0^{L/2} P\, Sin\theta\, R\, d\theta\, dy \tag{19}$$

The W_x and W_y in the increasing values of M are given by

$$W_x = -\left\{\frac{\mu U e^2}{C^2(1-e^2)^2} + \frac{KH\phi R\pi e}{(1-e^2)^{\frac{5}{2}}}\right\}L^3 + \frac{\rho C(\mu U + 4KH\phi)}{64\mu}\left\{\frac{1}{e}log\frac{1+e}{1-e} - \frac{2}{1-e^2}\right\}L^4\, M + \left\{\frac{106\mu U e^2\rho^2 C^2 - 204RCKH\phi\rho^2\pi e}{13440\mu^2}\right\}L^3 M^2 \tag{20}$$

$$W_y = \left\{\frac{\mu U\pi e}{4C^2(1-e^2)^{3/2}} + \frac{4KH\phi R}{(1-e^2)^2}\right\}L^3 + \frac{\pi e\rho C(\mu U + 4KH\phi)}{128\mu(1-e^2)^{3/2}}L^4\, M + \left\{\frac{53\mu U\rho^2 C^2\pi e - 272RCKH\phi\rho^2}{4480\mu^2}\right\}L^3 M^2 \tag{21}$$

V. CALCULATION TABLES

By taking the values of different mathematical terms in C.G.S. system as follows: θ=30°, μ=0.0002, C=0.0067, ρ=0.9, U=10^2, h=0.00786, y=1, H=0.05, ϕ=0.0025, R=3.35, D=2R; the calculated values of pressure and load capacity with respect to M by taking μ= constant = 0.0002, are given by Table 1.

Table 1.

e↓	L/D↓	M→	0.2	0.4	0.6	0.8	1.0
0.2	0.5	*P*	844.7764864	897.5040214	950.1928974	10002.844914	1055.458273
		W	318359.2200	318360.4200	318362.3400	318364.9997	318368.3960
0.2	1.0	*P*	4782.680583	5081.196179	5379.496305	5677.580961	5975.450148
		W	2546874.560	2546884.912	2546901.120	2546923.118	2546951.093
0.9	0.5	*P*	1185.885315	1423.933488	1662.326572	1901.064569	2140.147477
		W	402079.4716	402082.259	402085.7945	402090.0741	402095.0935
0.9	1.0	*P*	6713.859539	8061.563214	9411.219599	10762.82869	12116.3905
		W	3216635.899	3216658.291	3216686.69	3216721.014	3216761.365

Also by taking the values of different mathematical terms in C.G.S. system as follows: $\theta=30^o$, e=0.9, $C=0.0067$, $\rho=0.9$, $U=10^2$, $h=0.00786$, $H=0.05$, $\phi=0.0025$, $R=3.35$, $L/D=1$; the calculated values of pressure and load capacity with respect to μ by taking M=constant=1.0, are given by Table 2.

Table 2.

μ	0.0002	0.0003	0.0004	0.0005	0.0006
P	12091.54659	12653.37738	13218.21638	13784.39107	14351.27039
W	3217061.181	4825306.167	6433586.076	8041213.813	9650176.837

VI. GRAPHS

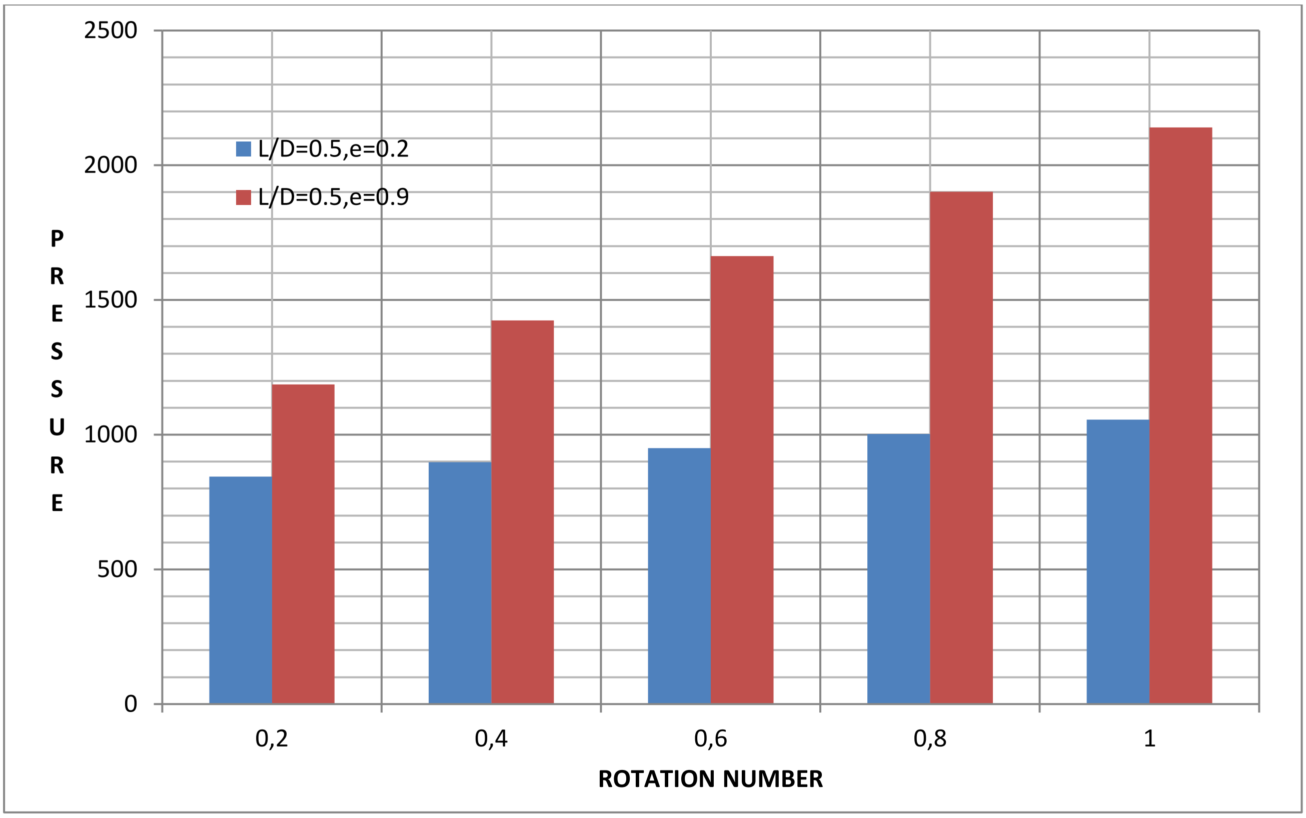

FIG.4. Variation of pressure with respect to *M* for e=0.2, 0.9; L/D=0.5.

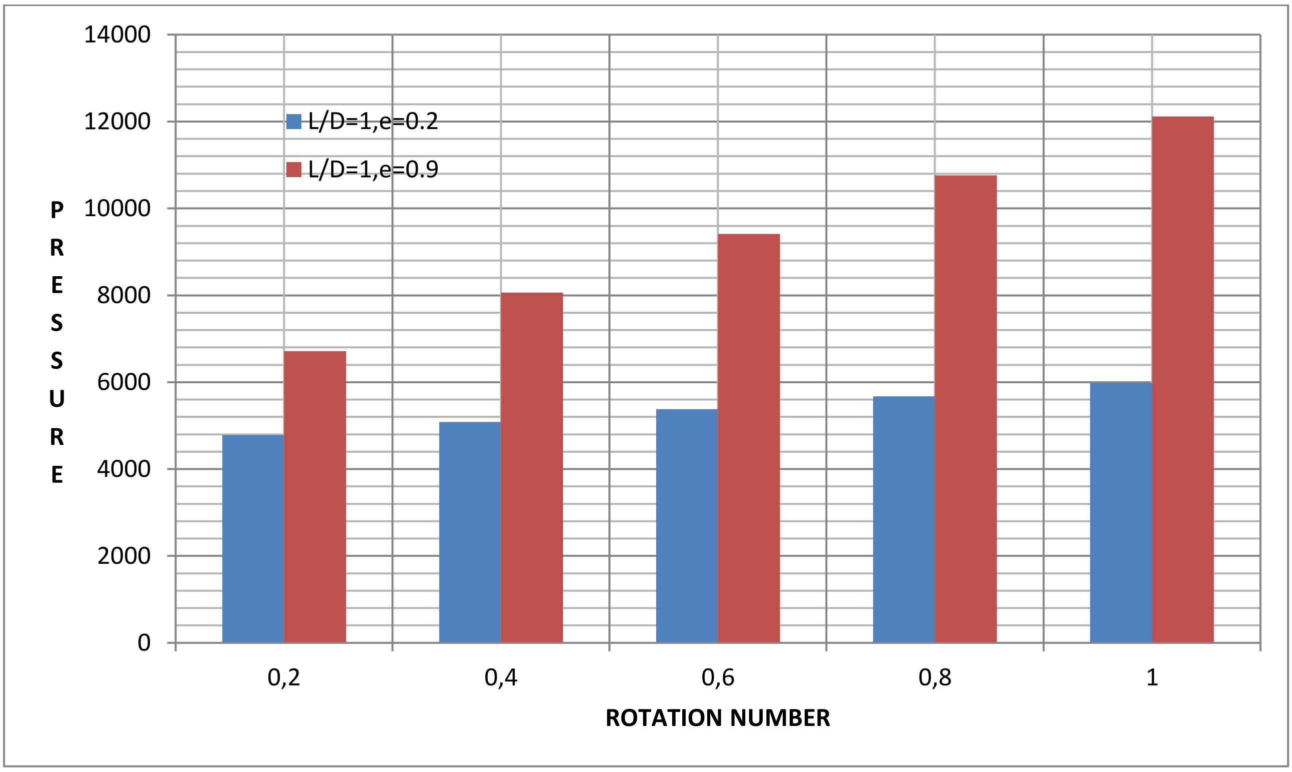

FIG.5. Variation of pressure with respect to *M* for e=0.2, 0.9; L/D=1.0.

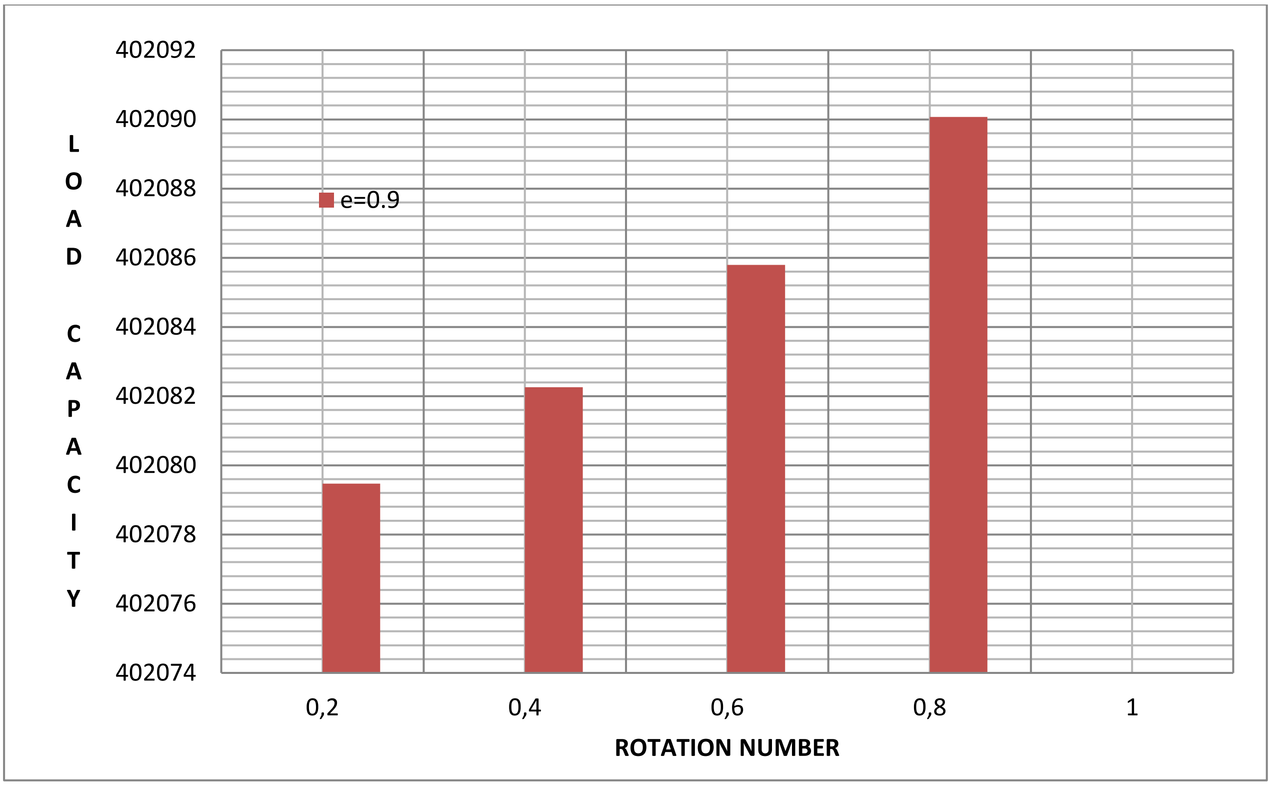

FIG.6. Variation of load capacity with respect to *M* for e=0.9; L/D=0.5.

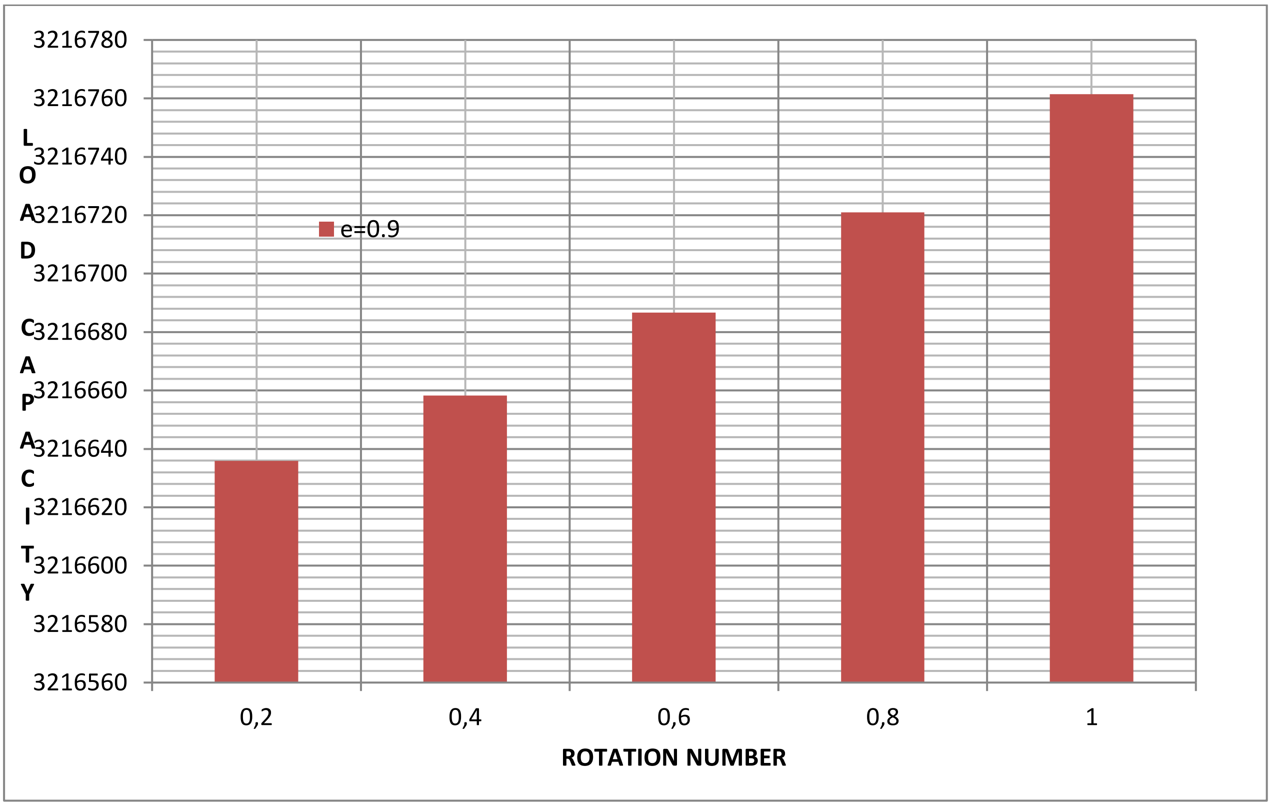

FIG.7. Variation of load capacity with respect to *M* for e=0.9; L/D=1.0.

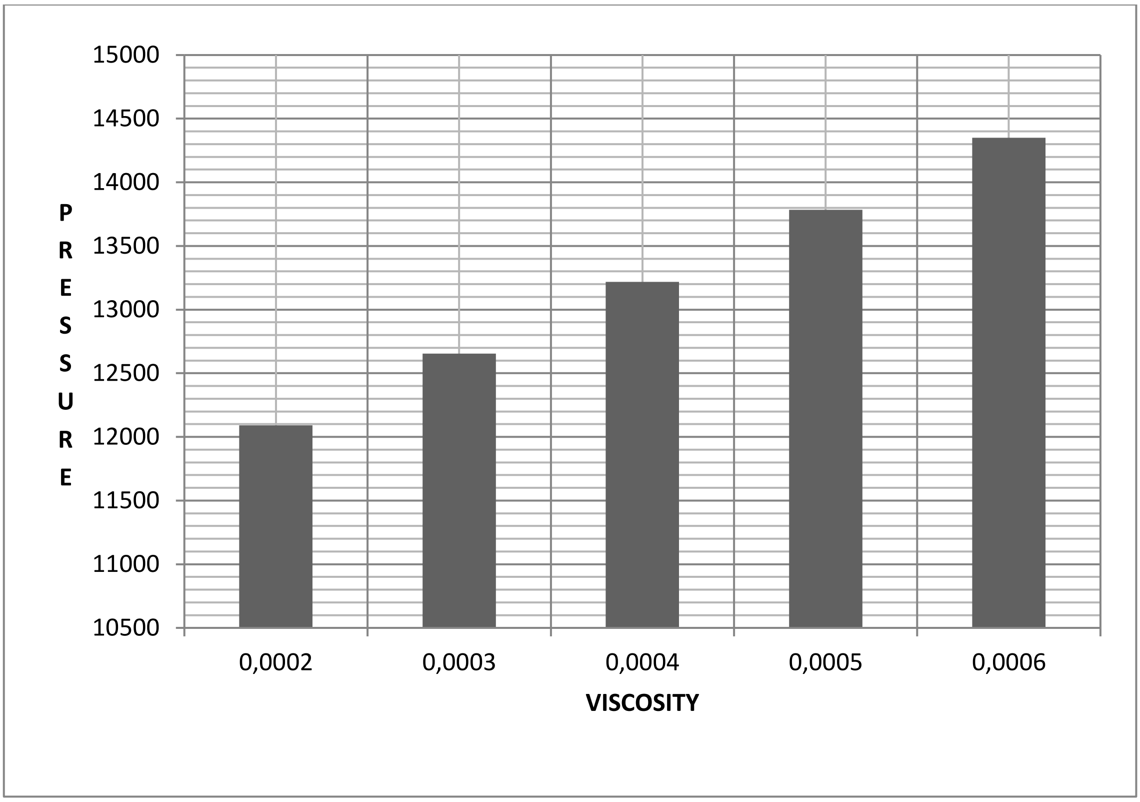

FIG.8. Variation of pressure with respect to viscosity for e=0.9; L/D=1.0.

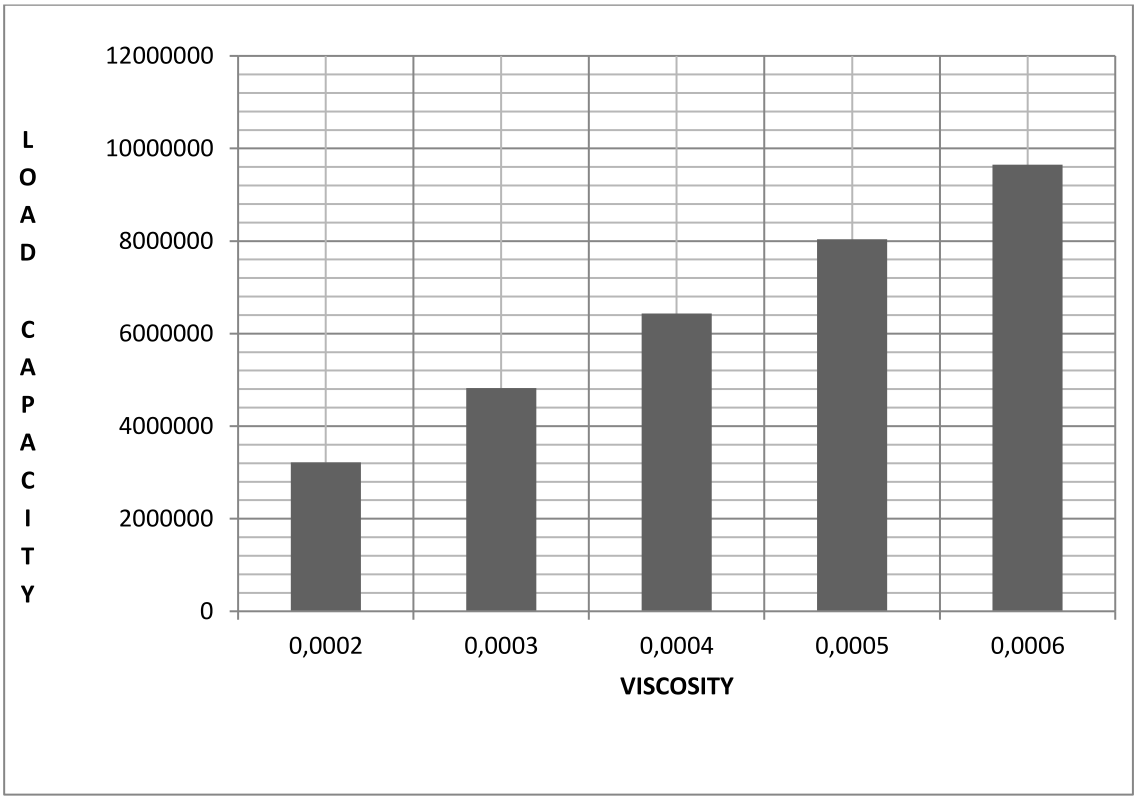

FIG.9. Variation of load capacity with respect to viscosity for e=0.9; L/D=1.0.

VII. DISCUSSION

The classical pressure equation for porous bearing was

$$P = \frac{3U\mu e\, sin\theta}{RC^2[\,(1+e\, cos\theta)^3+12\psi]}\left(\frac{L^2}{4}-y^2\right),\ \psi = \frac{H\Phi}{C^3} \tag{22}$$

This equation does not give infinite pressure at $\theta=\pi$ if e=1 due to the presence of the term (12ψ), whereas the pressure equation of second order rotatory theory given by (12) shows that it gives infinite pressure at $\theta=\pi$ if e=1.The equation (22) shows that pressure *increases linearly* with μ, whereas the equation (12), table-2, fig.8 and fig.9 shows that in second order rotatory theory, the pressure does not changes linearly with μ, it *slightly increases* with μ due to presence of the permeability factor in the numerator of term A, presence of μ in both numerator and denominator of term B and very small effect due to μ^2 in the denominator and μ in numerator of term C. We also get the same effect on load capacity shown by the equation (20), (21) and table-2.

The equations of pressure and load capacity shows that both increases with rotation number M. The term B of first order rotation gives much effect on P and W as compare to term C of second order rotation. These results are shown with the help of table-1 and fig.4-7.

VIII. CONCLUSIONS

The variation of pressure and load capacity with respect to viscosity ,when M is constant and with respect to rotation number M, when viscosity is constant; are shown by tables and graphs. Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure and load capacity are not independent of viscosity μ and slightly increases with μ, when M is constant; also the pressure and load capacity both increases with increasing values of $M,$ when viscosity is taken as constant. On taking (M=0) in the expression of pressure and load capacity, we get the classical solutions.

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