MOTION OF LUBRICATING DISC FOR THE ROTATORY THEORY OF LUBRICATION

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Abstract

The device of the lubrication of discs may be made kinematically equal to gears in the event that they have the identical radius at their touch line and rotate at identical angular velocities because the gears. For the device of discs, we can take the starting place on the floor of disc of radius R on the 2 discs. In the prevailing paper, there are a few new answers with the assist of geometrical figures, derivation for the expression for the weight ability, calculated tables and graphs for the weight ability within side the view of 2nd order rotatory principle of hydrodynamic lubrication. The evaluation of equation for load ability, tables and graphs suggested that the W isn't unbiased of viscosity and will increase barely with viscosity. Also the weight ability will increase with growing values of rotation range. The applicable tables and graphs verify those consequences within side the current paper.

Key words: Load Capacity, Reynolds equation, Rotation range, Taylor's range, Viscosity.

1. INTRODUCTION

The idea of dimensional classical theories of lubrication [4], [10] becomes given with the aid of using Osborne Reynolds [11]. In the wake of a test with the aid of using Beauchamp Tower [12], he had given a differential equation that becomes stated as Reynolds Equation [11]. The fundamental mechanism and formation of the fluid movie become found with the aid of using that test with the aid of using thinking about a few assumptions that the movie thickness of fluid is a great deal smaller than its axial and longitudinal dimensions and if lubricant layer is to provide strain among the bearing and the shaft then the layer will range the thickness of the fluid movie. After a few lengths Osborne Reynolds once more revised his personal differential equation that become the a great deal advanced model and become stated as: Generalized Reynolds Equation [7], [10]. The differential equation relies upon on viscosity, density, movie thickness, and transverse and floor velocities. The idea of the rotation [1] of the fluid movie approximately an axis, which lies throughout the fluid movie, offers a few notable answers within side the lubrication issues of the fluid mechanics. The starting place of rotation becomes found with the aid of using a few theorems of vorticity within side the rotating fluid dynamics. The rotation induces factor of vorticity within side the route of rotation of fluid movie and outcomes bobbing up from it are predominant, for huge Taylor's Number, it consequences in streamlines turning into restricted to the aircraft transverse to route of rotation of the fluid movie. The modern prolonged model of the Generalized Reynolds Equation [7], [10] is referred to as the Extended Generalized Reynolds Equation [1],[3] that takes under consideration of outcomes of uniform rotation approximately an axis, which lies throughout the fluid movie and relies upon on rotation range M [1], that's the rectangular root of the classical Taylor's Number. The generalization of the principle of hydrodynamic lubrication is stated because the Rotatory Theory of Hydrodynamic Lubrication [1], [3]. The idea of the Second Order Rotatory Theory of Hydrodynamic Lubrication [3], [8] become given with the aid of using keeping phrases containing as much as 2nd powers of M and with the aid of using neglecting better powers of M. The lubrication of discs may be made kinematic ally equal to gears in the event that they have the identical radius at their touch line and rotate on the identical angular velocities because the gears. For the device of discs, we can take the starting place on the floor of disc of radius R on the road of facilities of the 2 discs.

The film thickness 'h' is given by

$$h = h_0\left[1 + \frac{y^2}{2h_0}\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right)\right] \tag{1.1}$$

$$\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right) = \frac{1}{R} \tag{1.2}$$

$$tan\theta = \frac{y}{\sqrt{2Rh_0}} \tag{1.3}$$

$$h = h_0 sec^2\,\theta \tag{1.4}$$

Let us suppose that the disc is stationary at the lower surface transverse to the fluid film where sliding is absent and U=+U (constant).

Suppose the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction.

So the terms containing pressure gradient $\partial p/\partial x$ can be neglected in comparison to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence *P* may be taken as function of *y* alone.

2. GOVERNING EQUATIONS

The Extended Generalized Reynolds Equation [7] for the second order rotatory theory of hydrodynamic lubrication is given by equation (2.1).

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} - sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} - sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} + sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} + sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$= -\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} + sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} - sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} - cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\right] - \rho W^* \tag{2.1}$$

Where *x*, *y* and *z* are coordinates, μ is the viscosity, *U* is the sliding velocity, *P* is the pressure, ρ is the fluid density, and W^* is fluid velocity in *z*-direction.

The Extended Generalized Reynolds Equation for the second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* can be written as equation (2.2).

For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+$$
$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$

$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right]$$

(2.2)

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right]$$

(2.3)

Taking *h=h(y), U=+U, P=P(y)* (2.4)

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{dP}{dy}\right]=$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right]$$

(2.5)

For the determination of pressure distribution in the positive regions, the boundary conditions are as follows:

A. P=0 at $h=h_0$ or P=0 at y=0 or P=0 at θ=0

B. (ii) P=dP/dθ=0 at $y=y_1$ or θ=γ (say) (2.6)

Where is determined by putting $\theta=\gamma$ and P=0 in the equation of pressure.

3. PRESSUR EQUATION:

The solution of the differential equation under the boundary conditions imposed is given by

$$P=-\sqrt{\frac{Rh_0}{2}}\,M\rho U\left[\frac{17\,M^2\rho^2{h_0}^4}{1680\mu^2}F(\theta)-\tan\theta\,F(\gamma)\right] \quad (3.1)$$

Where $F(\theta)$ is given by

$$F(\theta)=\tan\theta\left[\frac{1}{9}\,sec^8\theta+\frac{8}{63}\,sec^6\theta+\frac{48}{315}\,sec^4\theta+\frac{192}{945}\,sec^2\theta+\frac{384}{945}\right] \quad (3.2)$$

4. LOAD CAPACITY

The load capacity is given by

$$W=\int_\gamma^0 P\,dy=\int_\gamma^0 P\,sec^2\theta\,\sqrt{2Rh_0}\,d\theta \quad (4.1)$$

$$=\sqrt{2Rh_0}\int_\gamma^0 P\,sec^2\theta\;d\theta$$

$$W=Rh_0M\rho U\left[\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}\int_\gamma^0 F(\theta)sec^2\theta\;d\theta-\int_\gamma^0\tan\theta\;sec^2\theta\;d\theta\right] \quad (4.2)$$

$$W = -\frac{17M^3\rho^3 {h_0}^3 UR}{1680\mu^2}\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^{8}\gamma+\frac{8}{315}sec^{6}\gamma+\frac{16}{315}sec^{4}\gamma+\frac{192}{945}tan^{2}\gamma\right) - \frac{Rh_0M\rho U}{2}\,tan^2\gamma \qquad (4.3)$$

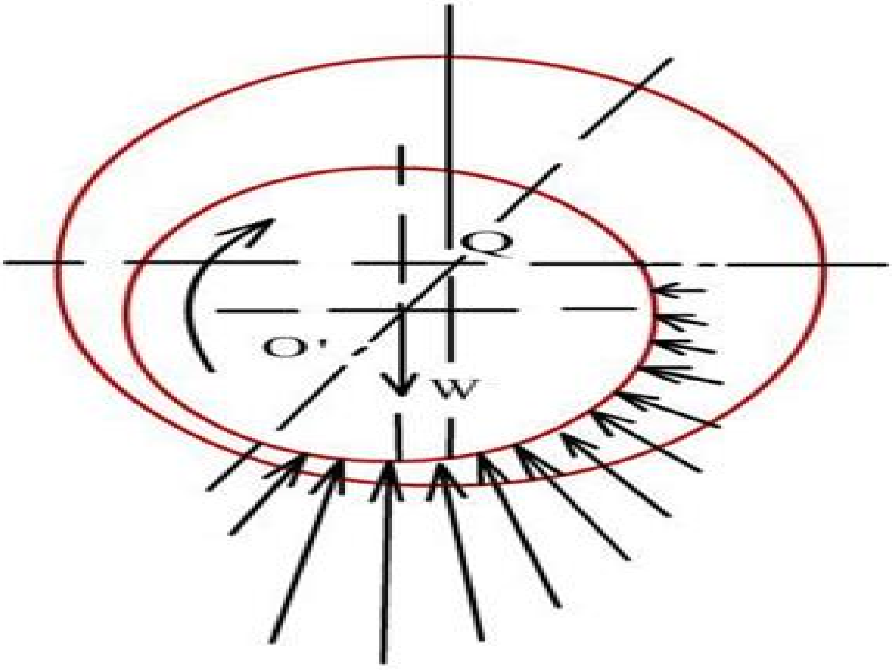

Figure- (4.1) (Load Capacity [17])

5. CALCULATION TABLE AND GRAPHS:

By taking the values of different mathematical terms in *C.G.S.* system the calculated tables and graphical representations are as follows:

5.1 Table

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = 60^0$

S.NO.	*M*	*W*
1.	0.1	6.2844159
2.	0.2	46.2472872
3.	0.3	153.5670693
4.	0.4	361.9222176
5.	0.5	704.9911875
6.	0.6	1216.452434
7.	0.7	1929.984414
8.	0.8	2879.265581
9.	0.9	4097.974391
10.	1.0	5619.7893

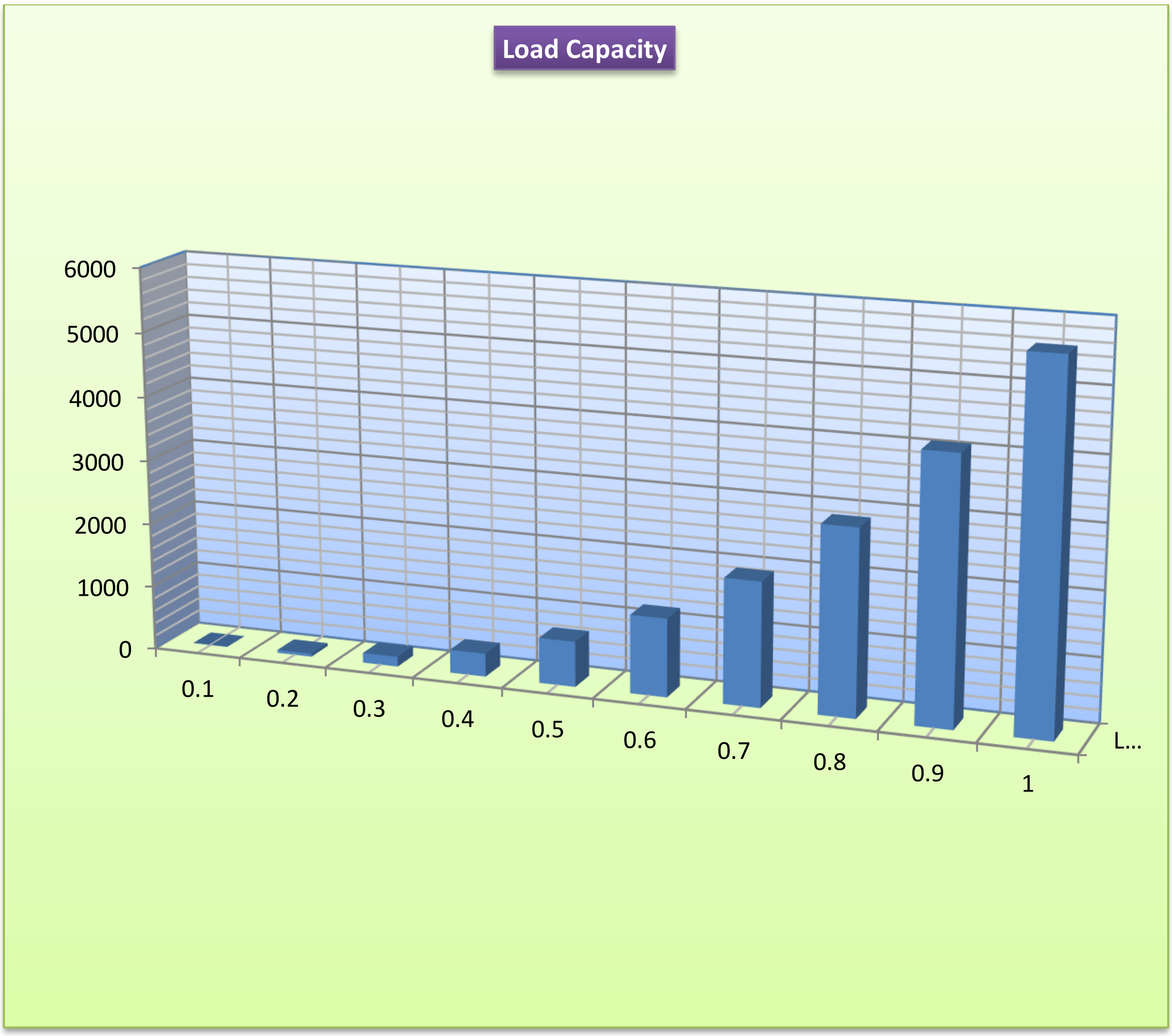

Figure-5.1 (Variation of load capacity with respect to rotation number M with exponential trendline)

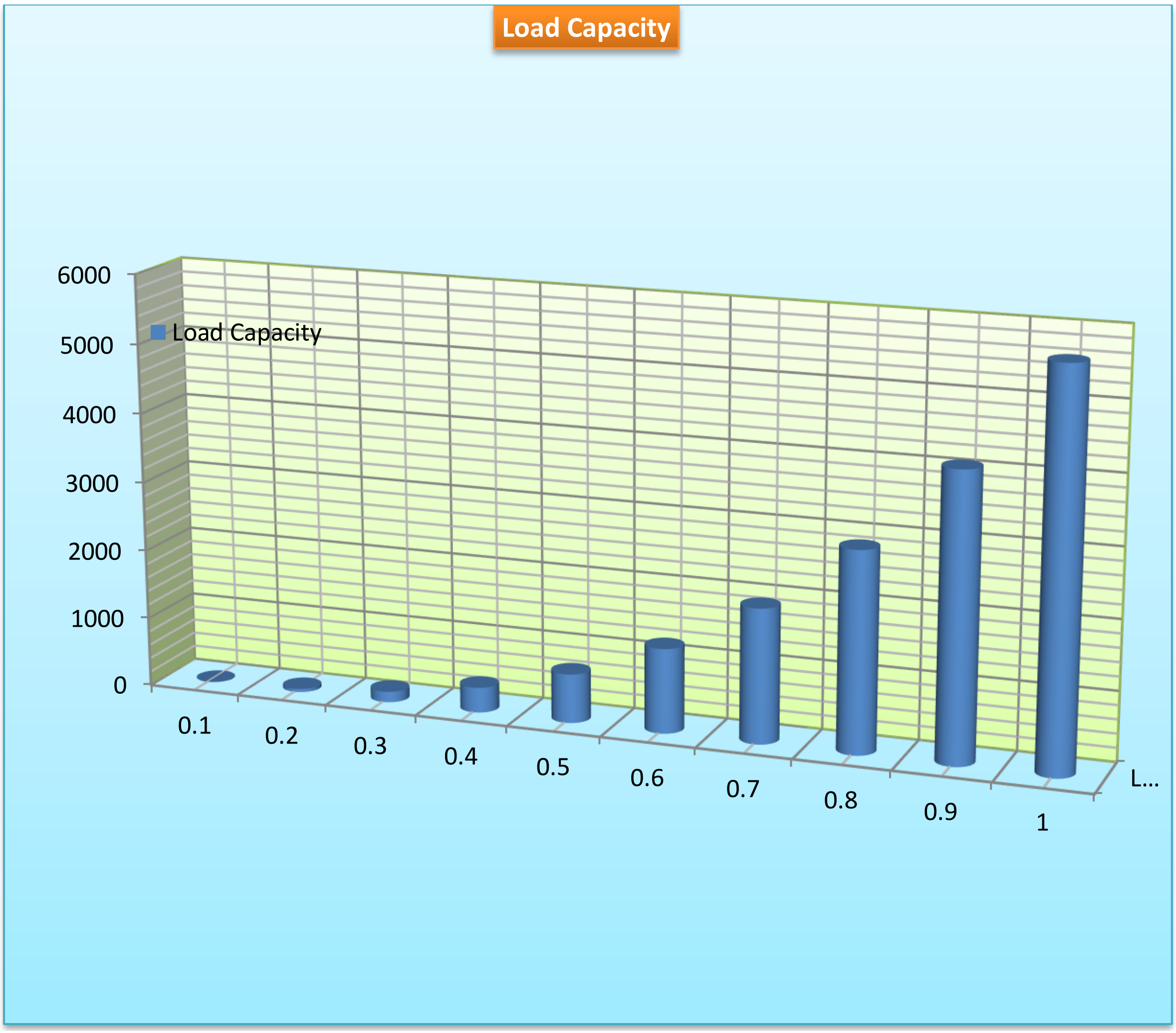

Figure-5.2 (Variation of load capacity with respect to rotation number M with logarithmic trendline)

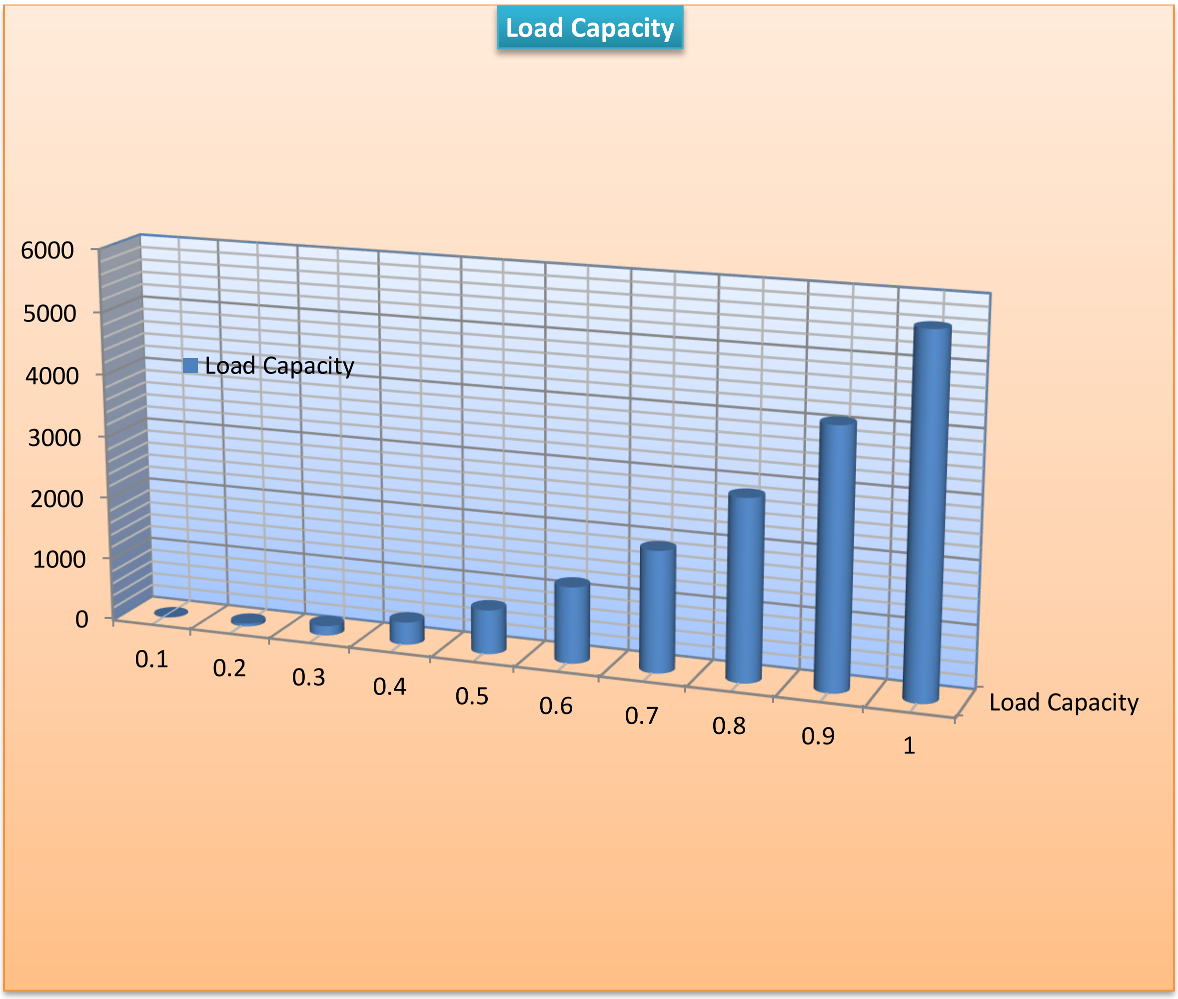

Figure-5.3 (Variation of load capacity with respect to rotation number M with third degree polynomial trendline)

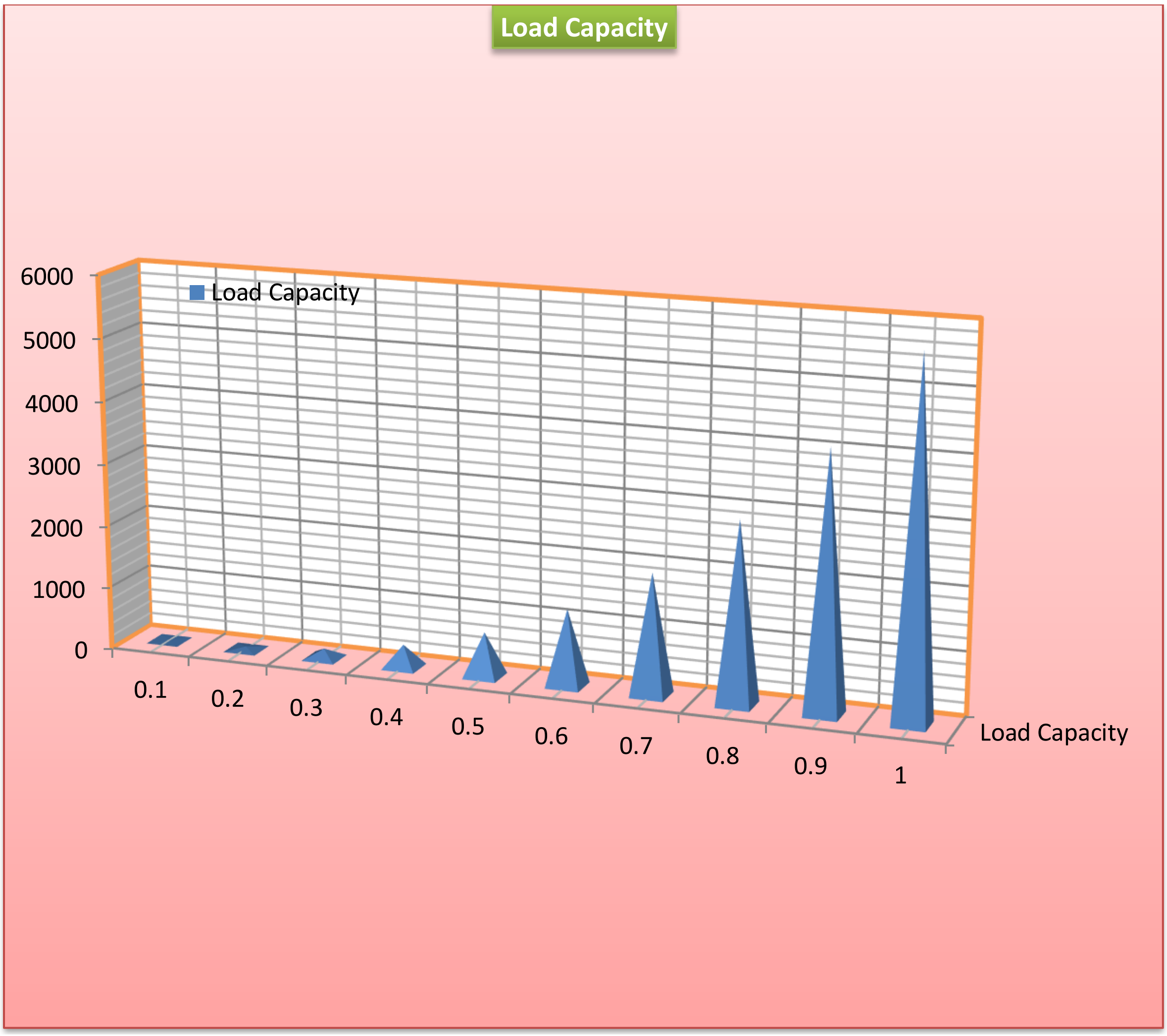

Figure-5.4 (Variation of load capacity with respect to rotation number M with 2-period moving average trendline with 5% error bars)

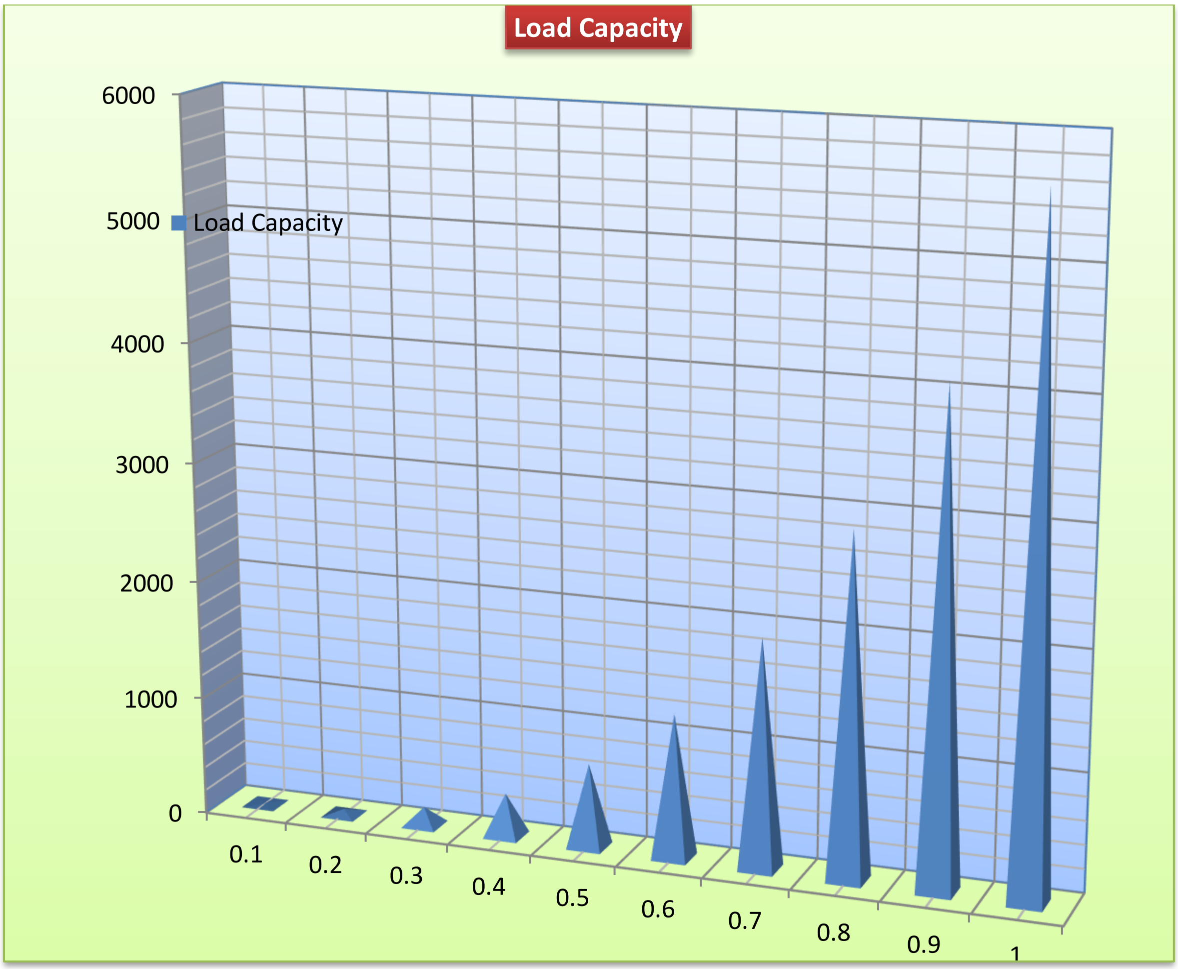

Figure-5.5 (Variation of load capacity with respect to rotation number M with linear trendline with 5% error bars)

6. Results and Discussion:

The version of load ability with admire to rotation range M is proven through the desk and graphs. The figure suggests the exponential trend line through y=12.36 e $^{0.683x}$ with R^2=0.909. The figure also suggests the logarithmic trend line through y=2068 log x-1422 with R^2=0.616. The figure suggests the polynomial trend line through $y=5.613x^3+3E\text{-}08X^2+0.671x+2E\text{-}07$ with R^2=1. The figure suggests the version of load ability with admire to rotation range M with transferring common with 2-duration transferring common trend line with 5% error bars. The figure suggests the version of load ability with admire to rotation range M with transferring common with linear trend line with 5% errors bars.

7. CONCLUSION:

The derived equation of load ability is given through equation. The calculated values of the W towards rotation range M is proven within side the desk and graphical illustration for the version of load capacity is likewise proven through figure-5.1 to figure-5.5. The comparisons of the W had been executed with the assist of geometrical figures, expressions, calculated tables and graphs for the lubricating discs within side the 2nd order rotatory concept of hydrodynamic lubrication. The evaluation of equation for load ability, desk and graphs display that load ability isn't impartial of viscosity and growth with growing values of rotation range.

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