Mathematical Formulation on the Multi-phase Flow through Composite Stenosis

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Abstract

The current article provides an overview of the mathematical model describing blood flow through the Composite Stenosis Artery. Stenosis or sclerosis is a rare and unusual condition that blocks the flow of blood to the semi lunar valve leading to very serious consequences. The issue of resilience is growing at an alarming rate in both developing and developing countries. In the middle of this paper there is an analysis of mathematical and statistical principles of blood flow with a combination of stenosis of the arteries and a few important results. The result shows that the blood pressure varies accordingly with the radius of the artery. Blood pressure varies directly with the scale.

Keywords: Structure, multi-stage flow, stenosis.

1. Introduction:

Due to the growing problems in the human body and the animal such as gravity, etc., a brand new branch of science called biomechanics has been established. In order to talk about blood flow at the solidity of a compound in a related vein, we often remember about the characteristics of blood such as its composition, blood consistency and so on. about Biomechanics 1st. (Caro, C. G. et al., 1978), (Charm, S. E. and Kurland, G. S., 1965).

The word Biomechanics is derived from a combination of the two Greek words bios (meaning-life) and mechanics (meaning - mechanics). Biomechanics could be a government department of science that is more closely associated with engineering than biology. Within this branch the scientific methods used help research Biological systems. By Biological systems we usually mean man, animal, plants, fungi and cells. So during this science we often study the function and structure of man, animals, plants etc. as a result of this integration of the equipment used in the field of biology i.e., in biomechanics, the local unit of ancient engineering techniques used to dismantle and illustrate biological-related issues. The field of science is based on the assumption that the field of man-made systems is much simpler than biological systems, so a simple numerical method will usually study each biological challenge. Thus mathematical modeling, process simulation and test scales will tend to solve problems related to humans and animal bodies. Several branches of machinery play a vital role in the study of biological systems. Most of the time most used by mechanical branches are method analysis, structural analysis, Kinematics, Dynamics etc. In recent years the scope and application of biomechanics has greatly increased (Ahmed, P. S. and Giddens, D. P., 1983), (Bandyopadhyay, S and Layek, G. C., 2012), (Ku, D. N., 1997).

Credit for the Biomechanics event goes to a philosopher who was a key figure who created the connection between physics and living science. The fifteenth century saw the general development of biomechanics. One of the most influential figures in the field was William Harvey (1578-1657) who argued that blood should flow freely within the vascular system, although at that time there was no formation of blood vessels even without a magnifier. not a lie. An astronomer proposes his view that when blood flows, it is directed in only one direction. Mathematically he proved that the strength of the guts was 2 ounces per rhythm. Italian star Giovanni Alfonso Borelli (1608-1679) described muscle action using mechanical ideas (Huckaba, C. E. and Hahu A. W., 1968), (Cokelet, G. R., 1972), (Deshpande, M. D. et al., 1976).

2. Problem Development

Blood flow through capillaries of blood is often thought of as a multi-phase flow due to its composition i.e., half the fluid, plasma and cell components, WBCs, RBCs and platelets. At present it represents the issue of stenosis in a type of matrix that usually considers blood flow with stenosis in the artery of the circular cross section.

$$\frac{R(z)}{R_0} = 1 - 2\frac{2\delta}{R_0 L_0}(Z - d) \quad ; \quad d \le d + L_0/2\,, \tag{1}$$

$$= 1 - \frac{\delta}{2R_0}\left[1 + cos\frac{2\pi}{L_0}\left(z - d - L_0/2\right]; \; d + L_0/2 \le z \le d + L_0\right. \tag{2}$$

$$= 1 \quad ; \text{otherwise}, \tag{3}$$

In the above equation, the radius of artery with stenosis is given by R≅R(z) and without stenosis is given by R_0, the length and location of stenosis are represented by L_0 and d respectively, also δ represents the maximum projection in lumen located at $z=d+L_0/2$.

3. Mathematical Solution

As the blood is the mixture of erythrocytes (red cells) and plasma therefore the blood flow through a stenosis in an artery is supposed to be a two–phase. The equation that describes the two phase flow of model of blood is given by (Srivastava et al., 2007, 09, 12).

$$(1 - C)\rho_f\left\{u_f\frac{\partial U_f}{\partial z} + v_r\frac{\partial u_f}{\partial r}\right\} = -(1 - C)\frac{\partial p}{\partial z} + (1 - C)\mu_s(C)(\nabla^2 - \frac{1}{r^2}v_f + CS\left(v_p - v_f\right) \tag{4}$$

$$\frac{\partial}{\partial r}\left[(1 - C)v_f\right] + (1 - C)\frac{v_f}{r} + \frac{\partial}{\partial z}\left[(1 - C)u_f\right] = 0 \tag{5}$$

$$\frac{\partial}{\partial r}\left[Cv_p\right] + \frac{Cv_p}{r}\frac{\partial\left[Cu_p\right]}{\partial z} = 0 \tag{6}$$

Here “r” denotes the radial coordinate directed perpendicular to axis of the tube, (u_f, v_f) and (u_p, v_p) denotes the axial and radial components of the fluid and particle velocities respectively, C denotes the volume fraction density of the particles, p denotes the pressure, $\mu_s(C) \approx \mu_s$ is the mixture viscosity , (Ku, D. N., 1997), (Mishra, S. and Siddiqui, S. U., 2012); S denotes the drag coefficient of interaction for the force exerted by one phase on the other, ρ_r and ρ_p show the actual density of the material constituting the fluid, i.e. the plasma and the particle i.e., erythrocyte phases respectively, fluid phase is given by (1-C)ρ_f and the particle phase densities is given by Cρ_p. Also here $\nabla^2 = \partial^2/\partial^2 + (1/r)$ $(\partial/\partial r) + \partial^2/\partial z^2$

is a two-dimensional Laplacian operator. In the above analysis the subscripts *f* represents the quantities associated with plasma and *p* represents the quantities associated with erythrocyte phases (Medhavi, A. et al., 2012), (Mekheimer et al., 2011). The expressions for the drag coefficient of interaction, S and the viscosity of the suspension, μ_s is given as

$$S = \frac{9}{2}\frac{\mu_0}{a_0^2}\frac{4 + 3[8C - 3C^2]^{\frac{1}{2}} + 3C}{(2 - 3C)^2}, \mu_s(C) = \frac{\mu_0}{1 - mC} \tag{7}$$

$$m = 0.070\, e^{[2.49C + (1107/T)\exp(-1.69C)]} \tag{8}$$

Here μ_0 is constant plasma viscosity and a_0 is the radius of a red cell. In this discussion Temp T is measured in Kelvin scale.

For a mild stenosis (i.e. stenosis for which $\partial/R_0 << 1$) the equations that govern the laminar, steady, one-dimensional flow of blood in and artery are as follows (Srivastava, V. P. et al., 2007, 12).

$$(1 - C)\frac{dp}{dz} = (1 - C)\frac{\mu_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\mu_f + CS\left(u_p - u_f\right) \tag{9}$$

$$C\frac{dp}{dz} = CS\left(u_p - u_f\right) \tag{10}$$

The boundary conditions for the stenosis problem are as follows

$$\frac{\partial \mu_f}{\partial r} = 0\ at\ r = 0;\ \mu_f = 0\ at\quad r = R(z) \tag{11}$$

The solution of the differential equation under the boundary conditions is given as:

$$u_f = -\frac{R_0^2}{4(1 - C)\mu_s}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2\right\} \tag{12}$$

$$u_p = -\frac{R_0^2}{4(1 - C)\mu_s}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2 + \frac{4(1 - C)\mu_s}{SR_0^2}\right\} \tag{13}$$

The volumetric flow rate Q is given by

$$Q = -\ \frac{\pi R_0^4}{8(1 - C)\mu_s}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^4 - \beta\left(\frac{R}{R_0}\right)^2\right\} \tag{14}$$

$$\frac{dp}{dz} = -\frac{8(1 - C)\mu_s Q}{\mu R_0^4}\ \emptyset(z) \tag{15}$$

With $\beta = 8C(1-C)\mu_0/SR_0^2$, a non-dimensional suspension parameter, and

$$\emptyset(z) = 1/F(z)F(z) = (R/R_0)^4 + \beta(R/R_0)^2$$

The pressure drop, $\Delta p(= p \ at \ z = -L, -p \ at \ z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_{-L}^{L}\left(-\frac{dp}{dz}\right)dz = \frac{8(1-C)\mu_s Q}{\pi R_0^4}\Psi \tag{16}$$

Where

$$\Psi = \int_0^4 [\emptyset(z)]_{\frac{R}{R_0}=1}dz + \int_d^{d+L_0/2} [\emptyset(z)]_{\frac{R}{R_0}from(1)}dz + \int_{d+L_0/2}^{L_0} [\emptyset(z)]_{\frac{R}{R_0}from(2)}dz + \int_{L_0}^{L} [\emptyset(z)]_{\frac{R}{R_0}=1}dz$$

Now the analytical evaluation of the first and fourth integrals of the above expression for Ψ is easy but the evaluation of the second and the third integrals a much difficult job so that it would be better to evaluate these quantities numerically. As discussed by (Young, 1979), (Srivastava, 2007), the expression for the impedance, i.e., flow resistance λ, the wall shear stress in the stenotic region τ_w and the shearing stress at the stenosis throat τ_s is formulated as (Ku, D. N., 1997), (Medhavi, A, 2011).

$$\lambda = (1-C)\mu\left[\begin{array}{l}\dfrac{1-\frac{L_0}{L}}{1+\beta} - \dfrac{R_0L_0}{2\beta\delta L}\left\{1 - \dfrac{1}{1-\frac{\delta}{R_0}} + \dfrac{1}{\beta}tan^{-1}\dfrac{\frac{\delta\sqrt{\beta}}{R_0}}{1+\beta-\frac{\delta}{R_0}}\right\} \\ \quad + \dfrac{L_0}{2\pi L}\displaystyle\int_0^{\pi}\dfrac{d\theta}{(a+bcos\theta)^2[(a+bcos\theta)^2+\beta]}\end{array}\right] \tag{17}$$

$$\tau_w = \frac{(1-C)\mu}{(R/R_0)^3 + \beta(R/R_0)} \tag{18}$$

$$\tau_s = \frac{(1-C)\mu}{(1-\delta/R_0)^3 + \beta(1-\delta/R_0)} \tag{19}$$

$$where\ \lambda = \frac{\bar{\lambda}}{\lambda_0}, \quad (\tau_w, \tau_s) = \frac{(\overline{\tau_w}, \overline{\tau_s})}{\tau_0}, \quad \bar{\lambda} = \frac{\Delta p}{Q}, \quad \overline{\tau_w} = -\frac{R}{2}\left(\frac{dp}{dz}\right),$$

$$\overline{\tau_s} = \left[-\frac{R}{2}\frac{dp}{dz}\right]_{R/R_0=(1-\delta/R_0)}, \quad \mu = \frac{\mu_s}{\mu_o}, \quad \lambda_0 = \frac{8\mu_0 L}{\pi R_0^3}, a = 1 - \frac{\delta}{2R_0}, \quad b = \frac{\delta}{2R_0},$$

$$\theta = \pi - \frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)$$

In above expressions λ_0 and τ_0 are the flow resistance and wall shear stress for the normal artery i.e., artery having no stenosis in the absence of the particle phase i.e., C=0, Newtonian fluid (Nadeem, S. et al., 2011), (Ponalagusamy, R. et al., 2011).

By the discussed equations; we can conclude that, in the absence of the particles, i.e., C=0, the results for a Newtonian fluid are as follows:

$$\lambda_N = 1 - \frac{L_0}{6\delta}\left[1 - \frac{1}{\left(1 - \frac{\delta}{R_0}\right)^3}\right] + \frac{L_0}{2\pi L}\int_0^{\pi} \frac{d\theta}{(a + b\cos\theta)^4} \tag{20}$$

$$\tau_{wN} = \frac{1}{(R/R_0)^3}\ ;\ \tau_{sN} = \frac{1}{(1-\delta/R_0)^3} \tag{21}$$

4. Numerical Results and Discussion

Hence for observing the quantitatively effects of hematocrit and other parameters for the blood flow Computational analysis of the results obtained by various equations for the tube of radius 0.01 cm at temperature of 37^0C is done. The values of different parameters are as follows:

d(cm) = 0; L0 (cm) = 1; L(cm) = 1, 2, 5 ; C = 0, 0.2, 0.4, 0.6; δ/R_0 =0, 0.05, 0.10, 0.15, 0.20.

It will be noted that the present study corresponds to a case of the Newtonian fluid and no stenosis for the parametric values C=0 and δ/R = 0 respectively.

5. Conclusions

A large two-phase model of blood has been wont to observe the consequences of hematocrit on blood flow characteristics owing to the presence of a light pathology. The varied properties of the flow of blood like, the flow resistance, the wall shear stress within the stenosed region and also the shear stress at the pathology throat increase with the hematocrit also like the pathology size. The shear stress at the pathology throat has similar properties to it of the resistivity with reference to the other parameter.

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