Motion of Fluid through Fitted Bearing

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Abstract

The second order rotatory theory of hydrodynamic lubrication was founded by the expression obtained by retaining the factors containing first and second powers of rotation number in the extended generalized Reynolds equation. In the present paper, there are some new excellent fundamental solutions with the help of geometrical figures, expressions, calculated table and graphs for the fitted bearings in the second order rotator theory of hydrodynamic lubrication. The analysis of equations for pressure and load capacity, table and graphs analyses that pressure and load capacity both increases with increasing values of rotation number. In the absence of rotation, the equation of pressure and load capacity gives the classical solutions of the classical theory of hydrodynamic lubrication. The relevant table and graphs confirm these important investigations in the present paper.

Keywords: Load capacity, Pressure, Reynolds equation, Taylor's number, Viscosity.

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1. Introduction

In the theory of hydrodynamic lubrication, two dimensional classical theories [4, 10] were first given by Osborne Reynolds [11]. In 1886, in the wake of a classical experiment by Beauchamp Tower [12], he formulated an important differential equation, which was known as: Reynolds Equation [11]. The formation and basic mechanism of fluid film was analyzed by that experiment on taking some important assumptions given as:

[a] The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.

[b] If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation" [7, 10], which depends on density, viscosity, film thickness, surface and transverse velocities.

The rotation [1] of fluid film about an axis that lies across the film gives some new results in lubrication problems of fluid mechanics. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film.

The new extended version of "Generalized Reynolds Equation" [7, 10] is said to be "Extended Generalized Reynolds Equation" [1, 3], which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number *M* [1], i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication" [1, 3].

The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication"[3, 8] was given by retaining the terms containing up to first and second powers of *M* [1] respectively by neglecting higher powers of *M* [1].

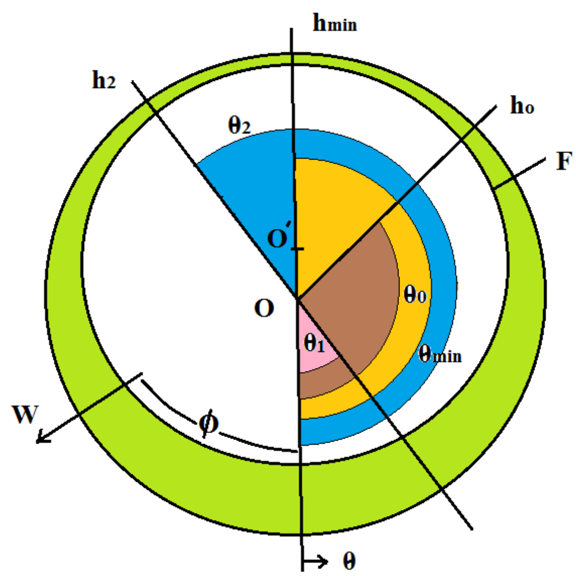

Figure 1. Geometry of Journal Bearing

The bearings having its diameter equal to the journal are known as fitted bearings or non clearance bearings. In these bearings the radial clearance is zero. We have the two cases for considering the approximations and corresponding boundary conditions. The geometries of journal bearing and fitted bearing are given by figure (1) and figure (2) respectively.

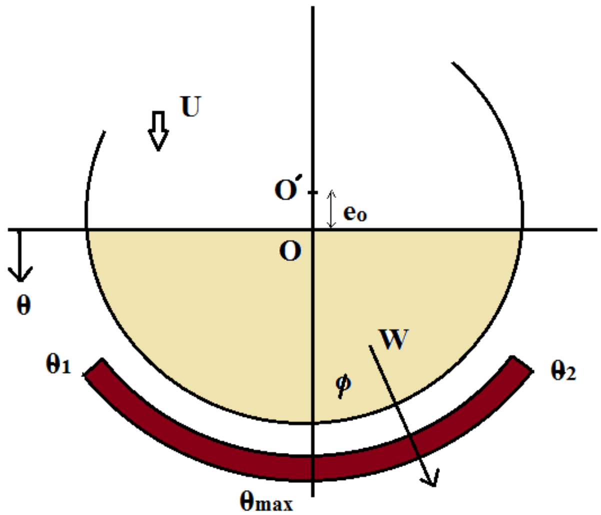

Figure2. Geometry of Fitted Bearing

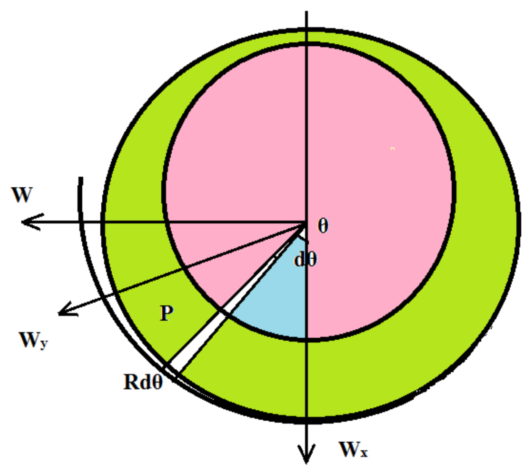

Figure3. Geometry for Components of Load Capacity

Where *x, y* and *z* are coordinates, *U* is the sliding velocity, *h* is the film thickness, θ is the angular coordinate, *P* is the pressure, ϕ is the permeability, *R* is the radius of bearing, e_0 is the eccentricity, *F* is the outward force and *W* is the load capacity of the bearing.

2. Governing Equations and Boundary Conditions

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation" [7] is given as:

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}-\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]-\rho W^{*} \tag{1}$$

Where ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in z-direction.

The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M [1] and by retaining the terms containing up to second powers of M and neglecting higher powers of M, can be written as:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right]-\rho W^* \tag{2}$$

Assumption I

Let we assume the bearing to be infinitely long in *y*-direction, which implies that the variation of pressure in *x*-direction is very small as compared to the variation of pressure in y-direction i.e., $\frac{\partial P}{\partial x} \ll \frac{\partial P}{\partial y}$, then the equation (2) will be

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{3}$$

Taking *h, U, P* has given

$$h = h(y),\ U = U,\ P = P(y) \tag{4}$$

The film thickness in angular coordinates is given as

$$h = e_0 \cos\theta \tag{5}$$

By rotating the angular coordinate 90º, in the direction of motion, we have

$$h= e_0 \sin\theta,\ y = R\theta \tag{6}$$

The equation (3) in view of above conditions can be written as:

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{dP}{dy}\right]=-\frac{d}{dy}\left[\frac{1}{2}\,M\,\rho^2\,U\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{7}$$

For the determination of pressure distribution excluding negative regions, the boundary conditions are given by

$$P = K\frac{dP}{d\theta} = 0 \text{ at } \theta = \theta_2, K = \text{constant.} \quad (8)$$

The values θ_1 and θ_2 are related by the condition

$$P = 0 \text{ at } \theta = \theta_1. \quad (9)$$

By neglecting the containing more than second powers of M, the equation for pressure is given by

$$P = \frac{1}{2} M \rho U R (\theta_2 - \theta) \quad (10)$$

The load capacity is given as

$$W = \sqrt{W_x{}^2 + W_y{}^2} \quad (11)$$

Where W_x and W_y are components of load capacities in x-directions and y-directions respectively.

$$W_x = \int_{\theta_1}^{\theta_2} LRPsin\theta \, d\theta \quad (12)$$

$$W_y = \int_{\theta_1}^{\theta_2} LRPcos\theta \, d\theta \quad (13)$$

Where L is the bearing length normal to the direction of motion.

With the help of above equations, we have

$$W_x = \frac{L\rho UR^2M}{2}[\theta_1 (cos\theta_1 - cos\theta_2) + (sin\theta_1 - sin\theta_2)] \quad (14)$$

$$W_y = \frac{L\rho UR^2M}{2} [\theta_1 (sin\theta_2 - sin\theta_1) + (cos\theta_2 - cos\theta_1)] \quad (15)$$

Assumption II

Let we assume the bearing to be infinitely long in x-direction, which implies that the variation of pressure in y-direction is very small as compared to the variation of pressure in x-direction i.e., $\frac{\partial P}{\partial x} \gg \frac{\partial P}{\partial y}$, then the equation (2) will be

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right]$$

(16)

Taking *h, U, P* has given

$$h = h(x),\ U = -U,\ P = P(x) \tag{17}$$

The film thickness in angular coordinates is given as

$$h = e_0 \cos\theta \tag{18}$$

By rotating the angular coordinate 90°, in the direction of motion, we have

$$h= e_0 \sin\theta,\ x = R\theta \tag{19}$$

In view of above conditions, the equation (16) can be written as:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$

(20)

On integrating and using the boundary conditions (8), (9) the equation of pressure after neglecting the higher powers of M and retaining the terms up to M^2 is given as:

$$P=\frac{3\mu UR}{{e_0}^2}\left[F_1(\theta_1)-F_1(\theta)\right]+\frac{M^2\rho^2{e_0}^2\,RU}{280\mu}\left[F_2(\theta_1)-F_2(\theta\right]$$

(21)

Where $F_1(\theta)$ and $F_2(\theta)$ are given by expressions:

$$F_1(\theta)=2cot\theta-sin\theta_2\left(cosec\theta\ cot\theta-logtan\frac{\theta}{2}\right) \tag{22}$$

$$F_2(\theta)=17\left(\frac{sin2\theta}{4}-\frac{\theta}{2}-sin\theta_2\ cos\theta\right)-7\left\{sin^5\theta_2\ \left(logtan\frac{\theta}{2}-cosec\theta\ cot\theta\right)-\theta+\frac{sin2\theta}{2}\right\}$$

(23)

With the help of equations (12) and (13), the components of load capacities are given by

$$W_x = \frac{3\mu ULR^2}{{e_0}^2}\left[F_1(\theta_1)cos\theta_1 - F_1(\theta_1)cos\theta_2 + G_1(\theta_1) - G_1(\theta_2)\right] + \frac{M^2\rho^2{e_0}^2R^2\,UL}{280\mu}\left[F_2(\theta_1)cos\theta_1 - F_2(\theta_1)cos\theta_2 + G_2(\theta_1) - G_2(\theta_2)\right] \quad (24)$$

$$W_y = \frac{3\mu ULR^2}{{e_0}^2}\left[F_1(\theta_1)sin\theta_2 - F_1(\theta_1)sin\theta_1 + H_1(\theta_1) - H_1(\theta_2)\right] + \frac{M^2\rho^2{e_0}^2R^2\,UL}{280\mu}\left[F_2(\theta_1)sin\theta_2 - F_2(\theta_1)sin\theta_1 + H_2(\theta_1) - H_2(\theta_2)\right] \quad (25)$$

Where $G_1(\theta)$, $G_2(\theta)$, $H_1(\theta)$ and $H_2(\theta)$ are given by the expressions:

$$G_1(\theta) = -2sin\theta + sin\theta_2\, log\, sin\theta - sin\theta_2\left(log\, sin^2\frac{\theta}{2} - 2cos^2\frac{\theta}{2}\, log\, tan\frac{\theta}{2}\right) \quad (26)$$

$$G_2(\theta) = \frac{3}{2}(sin\theta - \theta\, cos\theta) - \frac{1}{2}sin^3\theta - \frac{17}{4}sin\theta_2\, cos2\theta + 7sin^5\,\theta_2\left(log\, sin^2\frac{\theta}{2} - 2cos^2\frac{\theta}{2}\, log\, tan\frac{\theta}{2} - log\, sin\theta\right) \quad (27)$$

$$H_1(\theta) = -2\left(cos\theta + log\, tan\frac{\theta}{2}\right) - sin\theta_2\left(cot\theta + sin\theta\, log\, tan\frac{\theta}{2}\right) \quad (28)$$

$$H_2(\theta) = \frac{3}{2}(cos\theta + \theta\, sin\theta) + \frac{1}{2}cos^3\theta + \frac{17}{2}\, sin\theta_2\left(\theta + \frac{sin2\theta}{2}\right) + 7sin^5\,\theta_2\left(sin\theta\, log\, tan\frac{\theta}{2} + cot\theta\right) \quad (29)$$

3. Calculated Table

Let us take the values of mathematical terms in C.G.S. system as follows:

$\rho = 0.9$, $\mu = 0.0002$, $e_0 = 0.3$, $R = 3.35$, $U = 500$, $\theta_1 = 20^0$, $\theta = 50^0$, $\theta_2 = 160^0$.

The calculated values of pressure and load capacity are given by the table.

S. No.	M	P	W
1	0.1	80.85488142	45449.4097
2	0.2	327.5560264	296635.8569
3	0.3	738.7246014	427591.0792
4	0.4	1314.360606	761970.4658
5	0.5	2054.464041	1191886.946
6	0.6	2959.034906	1717340.766
7	0.7	4028.073201	2338331.607
8	0.8	5261.578926	30.54859.437

9	0.9	6659.552081	3866924.387
10	1.0	8221.992666	4774526.387

4. Graphical Representations

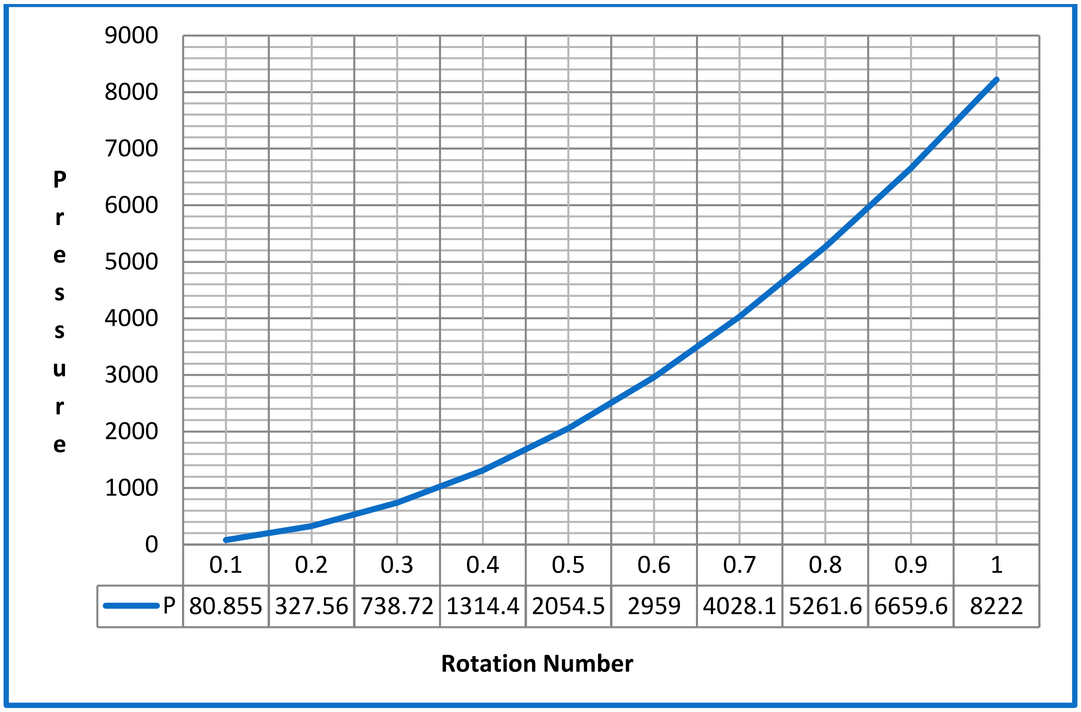

Figure 4.Variation of Pressure with respect to Rotation Number

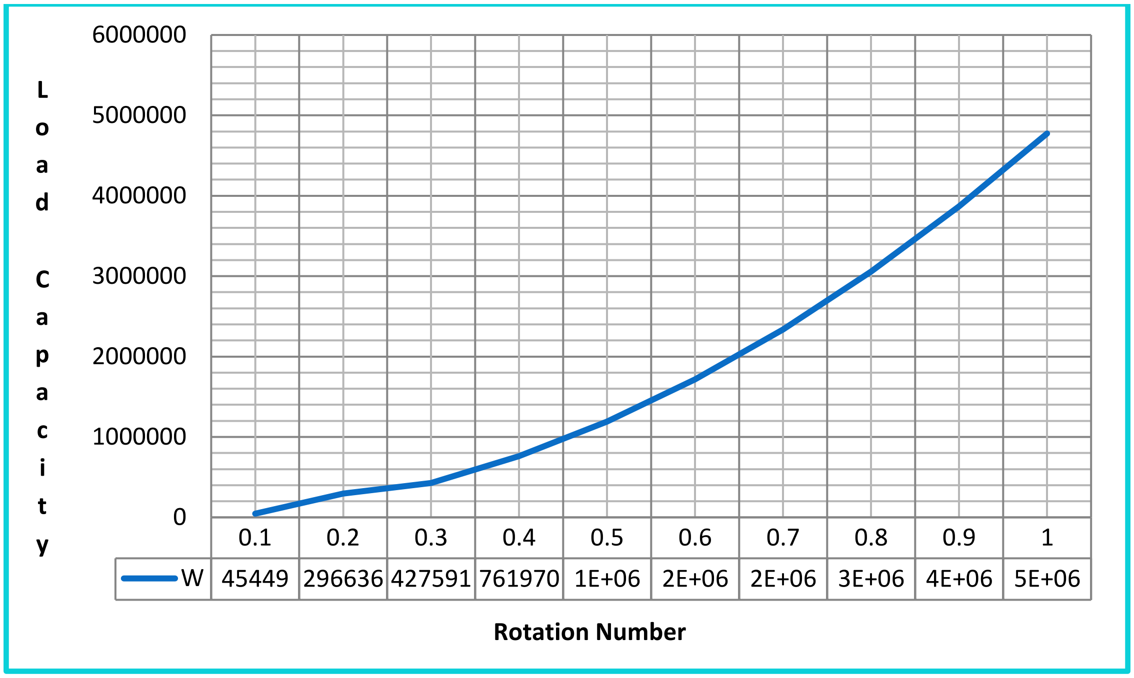

Figure 5.Variation of Load Capacity with respect to Rotation Number

5. Discussions

In this paper two types of assumptions are taken, first by assuming that the bearing is infinitely long along y-axis, so the pressure distribution along x-axis can be neglected. Here the film thickness h and pressure P can be taken as the function of y. After using the boundary conditions the equation of pressure and load capacity contains the terms of odd powers of M. On neglecting the terms of higher than second powers of M, we get the only term of first power of M. So we only get the first order solution by using this assumption. Here the pressure and load capacity are linear in M, so both increases linearly with the increasing values of M. We can't find the classical solution in this case.

If we assume that the bearing is infinitely long along x-axis, so the pressure distribution along y-axis can be neglected. Here the film thickness h and pressure P will be the function of x. After using the boundary conditions the equation of pressure and load capacity contains the terms of even powers of M. On neglecting the terms of higher than second powers of M, we get the equation of pressure and load capacity containing only term of M^2 and the term of M is absent. The relevant table, graphs (4) and (5) show that the pressure and load capacity both increase with the increasing values of M. We can find the classical solution by taking M=0.

6. Conclusions

The pressure and load capacity for fitted bearing in the second order rotatory theory of hydrodynamic lubrication are not independent of viscosity μ; which can be seen by the derived equations of pressure and load capacity. The pressure and load capacity both increases with the increasing values of *M*. On taking (*M*=0) in the expression of pressure and load capacity, we get the classical solutions.

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