INCLINED SLIDER BEARING UNDER THE EFFECTS OF SECOND ORDER ROTATION

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Abstract

The 2nd order rotatory principle of hydrodynamic lubrication changed into based at the expression received by keeping the terms containing 1st and 2nd powers of rotation number in the existing generalized Reynolds equation. In the existing paper, there are a few new tremendous essential solutions with the assist of geometrical figures, expressions, calculated tables and graphs for the slider bearings in the 2nd order rotatory principle of hydrodynamic lubrication. The solution of equations for stress and viscosity, tables and graphs screen stress and viscosity aren't impartial of viscosity and growth barely with viscosity. Also the stress and viscosity each growth with growing values of rotation no. In the absence of rotation, the equation of stress and viscosity offers the classical answers of the classical principle of hydrodynamic lubrication. The applicable tables and graphs affirm those critical investigations in the present paper.

Keywords: Continuity, Density, Film thickness, Reynolds equation, Rotation variety, Taylor's number, Viscosity.

1. INTRODUCTION

The idea of dimensional classical theories of lubrication [4], [10] becomes given with the aid of using Osborne Reynolds [11]. In the wake of a test with the aid of using Beauchamp Tower [12], he had given a differential equation that becomes stated as Reynolds Equation [11]. The fundamental mechanism and formation of the fluid movie become found with the aid of using that test with the aid of using thinking about a few assumptions that the movie thickness of fluid is a great deal smaller than its axial and longitudinal dimensions and if lubricant layer is to provide strain among the bearing and the shaft then the layer will range the thickness of the fluid movie. After a few lengths Osborne Reynolds once more revised his personal differential equation that become the a great deal advanced model and become stated as: Generalized Reynolds Equation [7], [10]. The differential equation relies upon on viscosity, density, movie thickness, and transverse and floor velocities. The idea of the rotation [1] of the fluid movie approximately an axis, which lies throughout the fluid movie, offers a few notable answers within side the lubrication issues of the fluid mechanics. The starting place of rotation becomes found with the aid of using a few theorems of vorticity within side the rotating fluid dynamics. The rotation induces factor of vorticity within side the route of rotation of fluid movie and outcomes bobbing up from it are predominant, for huge Taylor's Number, it consequences in streamlines turning into restricted to the aircraft transverse to route of rotation of the fluid movie. The modern prolonged model of the Generalized Reynolds Equation [7], [10] is referred to as the Extended Generalized Reynolds Equation [1],[3] that takes under consideration of outcomes of uniform rotation approximately an axis, which lies throughout the fluid movie and relies upon on rotation range M [1], that's the rectangular root of the classical Taylor's Number. The generalization of the principle of hydrodynamic lubrication is stated because the Rotatory Theory of Hydrodynamic Lubrication [1], [3]. The idea of the Second Order Rotatory Theory of Hydrodynamic Lubrication [3], [8] become given with the aid of using keeping phrases containing as much as 2nd powers of M and with the aid of using neglecting better powers of M. The lubrication of discs may be made kinematic ally equal to gears in the event that they have the identical radius at their touch line and rotate on the identical angular velocities because the gears. For the device of discs, we can take the starting place on the floor of disc of radius R on the road of facilities of the 2 discs.

The film thickness 'h' is given by

$$h = h_0\left[1 + \frac{y^2}{2h_0}\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right)\right] \quad (1.1)$$

$$\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right) = \frac{1}{R} \quad (1.2)$$

$$tan\theta = \frac{y}{\sqrt{2Rh_0}} \quad (1.3)$$

$$h = h_0 sec^2\,\theta \quad (1.4)$$

Let us suppose that the disc is stationary at the lower surface transverse to the fluid film where sliding is absent and U=+U (constant).
Suppose the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction.
So the terms containing pressure gradient $\partial p/\partial x$ can be neglected in comparison to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence P may be taken as function of y alone.

2. GOVERNING EQUATIONS

The Extended Generalized Reynolds Equation [7] for the second order rotatory theory of hydrodynamic lubrication is given by equation (2.1).

$$\begin{aligned}
&\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} - sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right] \\
&+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} - sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right] \\
&+\frac{\partial}{\partial x}\left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} + sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right] \\
&\qquad -\frac{\partial}{\partial y}\left[-\frac{h}{M} + \sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} + sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right] \\
&= -\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} + sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} + cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\right] \\
&-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{sinh\,h\sqrt{\frac{M\rho}{2\mu}} - sin\,h\sqrt{\frac{M\rho}{2\mu}}}{cosh\,h\sqrt{\frac{M\rho}{2\mu}} - cos\,h\sqrt{\frac{M\rho}{2\mu}}}\right)\right] \\
&\qquad - \rho W^*
\end{aligned} \quad (2.1)$$

Where *x, y* and *z* are coordinates, μ is the viscosity, U is the sliding velocity, P is the pressure, ρ is the fluid density, and W^* is fluid velocity in *z*-direction.

The Extended Generalized Reynolds Equation for the second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number M can be written as equation (2.2).

For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right] \tag{2.2}$$

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2.3}$$

Taking *h=h(y), U=+U, P=P(y)* (2.4)

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{dP}{dy}\right]=-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2.5}$$

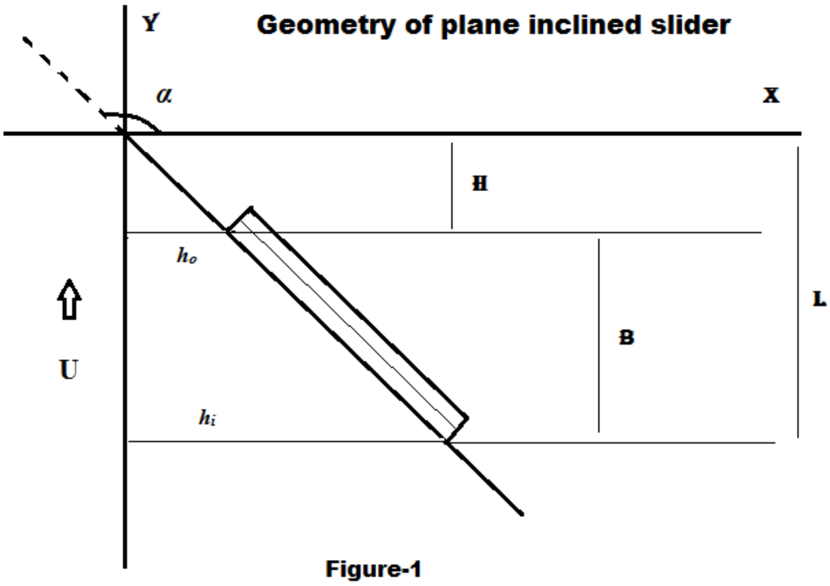

Figure-1 (Geometry of plane inclined slider)

Taking n as:

$$n = \frac{h_i - h_o}{h_o} \quad (1)$$

The film thickness h can be expressed at any point as:

$$h = h_o\left(1 + \frac{ny}{L}\right) \quad (2)$$

$$h = y \cot \alpha, \quad (3)$$

Where α is the angle of inclination of the pad.

$$\cot \alpha = \frac{h_o}{H} = \frac{h_i}{L} = \frac{dh}{dy} \quad (4)$$

$$\frac{dh}{dy} = \frac{nh_o}{L} = \frac{h_i - h_o}{L} \quad (5)$$

$$\frac{h_o}{H} = \frac{h_i}{L} = \frac{h_i - h_o}{L-H} = \frac{L\frac{dh}{dy}}{L-H} \quad (6)$$

2. Governing equations and boundary conditions

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation", given by Banerjee et. al. [1], is given by equation (7). Let us consider the mathematical terms as follows:

$$A = \sinh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) - \sin\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$B = \cosh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) + \cos\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$C = \sinh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) + \sin\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$D = cosh\left(h\sqrt{\frac{M\rho}{2\mu}}\right) - cos\left(h\sqrt{\frac{M\rho}{2\mu}}\right)$$

$$K = \sqrt{\frac{2\mu}{M\rho}}$$

$$F_1 = -\left(\frac{KA}{MB}\right)\frac{\partial P}{\partial x}$$

$$F_2 = -\left(\frac{KA}{MB}\right)\frac{\partial P}{\partial y}$$

$$F_3 = -\left(\frac{h}{M}\right) + \left(\frac{KC}{MB}\right)\frac{\partial P}{\partial y}$$

$$F_4 = -\left(\frac{h}{M}\right) + \left(\frac{KC}{MB}\right)\frac{\partial P}{\partial x}$$

$$F_5 = \frac{\partial(A/D)}{\partial y} - \frac{\partial(C/B)}{\partial x}$$

$$\frac{\partial(F_1 + F_3)}{\partial x} + \frac{\partial(F_2 - F_4)}{\partial y} = \frac{\rho UKF_5}{2} - \rho W^* \tag{7}$$

Where *x, y* and *z* are coordinates, *U* is the sliding velocity, *P* is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in *z*-direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* and by retaining the terms containing up to second powers of *M* and neglecting higher powers of *M*, can be written as equation (8). For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial(F_1 + F_3)}{\partial x} + \frac{\partial(F_2 - F_4)}{\partial y} = \frac{\rho UKF_5}{2} \tag{8}$$

Let we assume the bearing to be infinitely long in *y*-direction, which implies that the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction i.e., $\frac{\partial P}{\partial x} \ll \frac{\partial P}{\partial y}$, then the equation (8) will be

$$\frac{\partial F_3}{\partial x} + \frac{\partial F_2}{\partial y} = \frac{\rho UKF_5}{2} \tag{9}$$

Taking the pressure distribution as the function of the coordinate along the length of the slider only, we have *P = P(y)*, we have

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right] = -\frac{d}{dy}\left[\frac{M\rho^2 U}{2}\left\{-\frac{h^3}{12\mu}\left(1 - \frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{10}$$

For the determination of pressure the boundary conditions are:

$P=0$ at $h=h_o$ and $P=0$ at $h=h_o\,(1+n)$

So we have the differential equation for the pressure will be

$$\frac{dP}{dy} = -\frac{1}{2}M\rho U\left[1-\left({h_0}^3-\frac{17M^2\rho^2{h_0}^7}{1680\mu^2}\right)\frac{1}{h^3}-\frac{17M^2\rho^2{h_0}^3}{1680\mu^2}h\right] \tag{11}$$

The solution of the differential equation (11) under the boundary conditions gives the pressure for plane inclined slider bearings by (12).

$$P = \frac{M\rho U}{2}\left[\begin{array}{c}\left\{\left(\frac{n(n+1)^2-1}{n(n+1)^2}\right)L-y+\frac{L^3}{n(ny+L)^2}\right\} \\ -\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}\left\{\frac{L^3}{n(ny+L)^2}-\left(y+\frac{ny^2}{2L}\right)-\left(\frac{2-n(n+1)^2(n+2)}{2n(n+1)^2}\right)L\right\}\end{array}\right] \tag{12}$$

The load capacity for plane inclined slider bearing is given by

$$W = -\int_L^0 P\,dy \tag{13}$$

$$W = \frac{M\rho UL^2}{12(n+1)^2}\left[3(n^2+2n+3)-\frac{17M^2\rho^2{h_0}^4}{1680\mu^2}(2n^3+7n^2+8n+9)\right] \tag{14}$$

5. Calculation tables, geometries and graphs

By taking the values of different mathematical terms in C.G.S. system as: $\mu = 0.0002, U = 500, \rho = 0.9, L = 15, n = 1, y = 7.5, h = 0.015, h_i = 0.02, h_o = 0.01$, we can calculate the values of pressure and load capacity shown in the table.

S.No.	M	P	W
1.	0.1	234.3618400	3634.675200
2.	0.2	234.3675975	3634.762613
3	0.3	234.3702624	3634.803072
4.	0.4	234.3717100	3634.825050
5.	0.5	234.3725829	3634.838302
6.	0.6	234.3731494	3634.846903
7.	0.7	234.3735378	3634.852800
8.	0.8	234.3738156	3634.857018
9.	0.9	234.3740212	3634.860139
10.	1.0	234.3741775	3634.862573

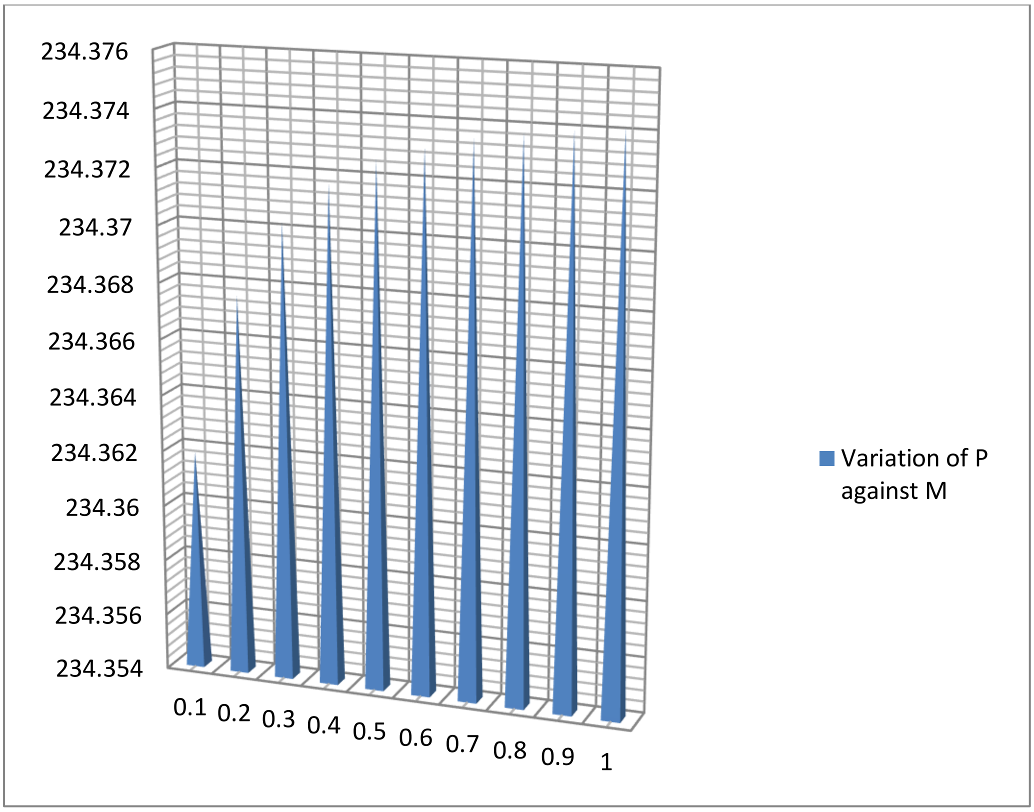

Figure-1

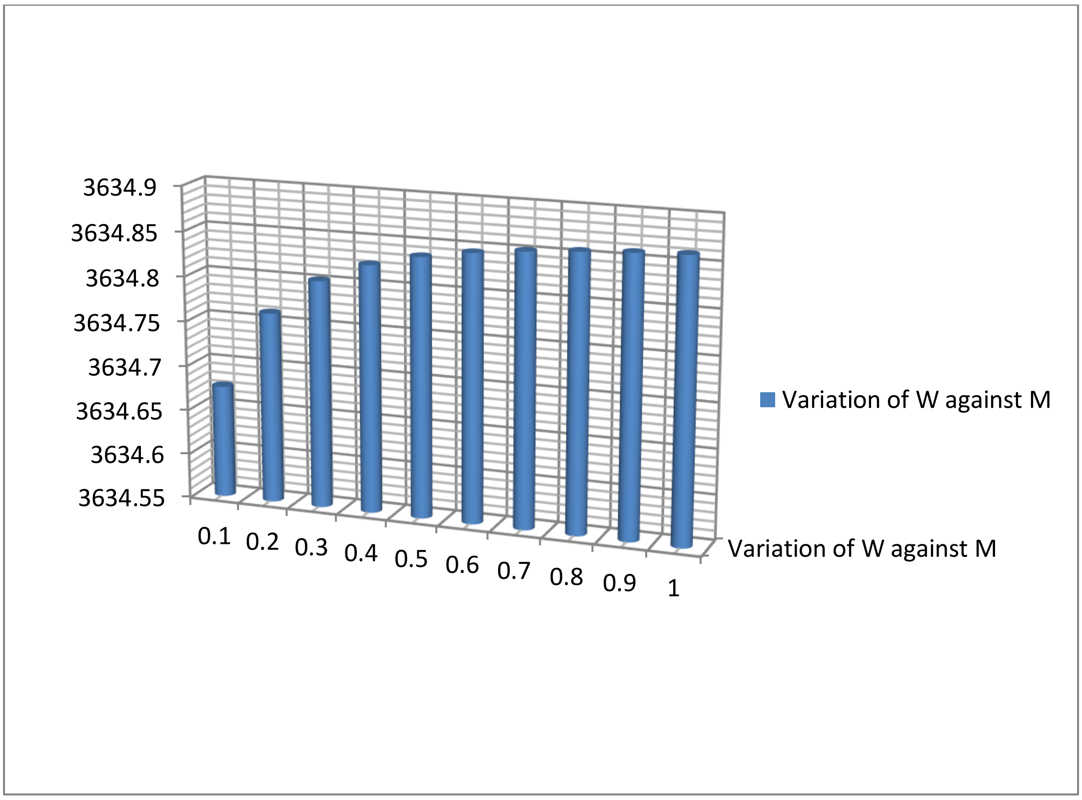

Figure-2

S.No.	M	P	W
1.	0.1	1536.413947	14781.27479
2.	0.2	3072.827575	29550.61501
3	0.3	4609.240567	44296.08607
4.	0.4	6145.652604	59005.75339
5.	0.5	7682.063366	73667.68240
6.	0.6	9218.472537	88269.93852
7.	0.7	10754.87980	102800.5872
8.	0.8	12291.28483	17247.6938
9.	0.9	13827.68731	131599.3237
10.	1.0	15364.08693	145843.5425

S.No.	µ	P	W
1.	0.00015	1536.413906	14779.72772
2.	0.00020	1536.413947	14781.27479
3	0.00025	1536.413966	14781.99087
4.	0.00030	1536.413976	14782.37985
5.	0.00035	1536.413983	14782.61439
6.	0.00040	1536.413987	14782.76662
7.	0.00045	1536.413990	14782.87098
8.	0.00050	1536.413992	14782.94563
9.	0.00055	1536.413993	14783.00087
10.	0.00060	1536.413994	14783.04288

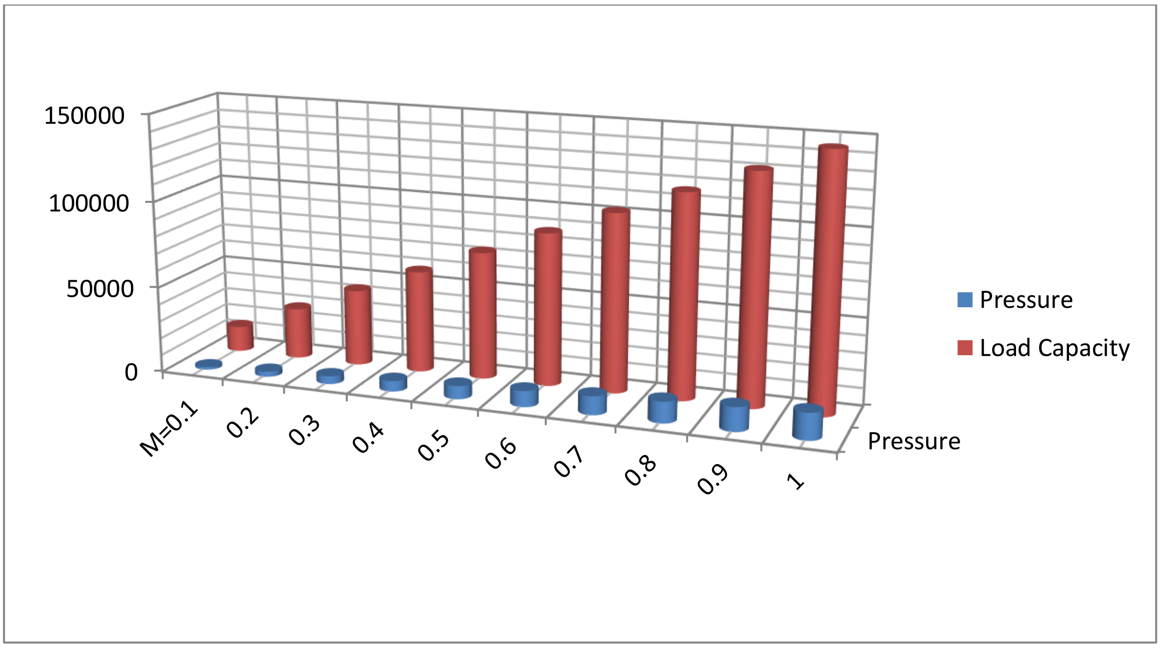

Figure-3 (Variation of *P* and *W* against *M* for μ=0.0002)

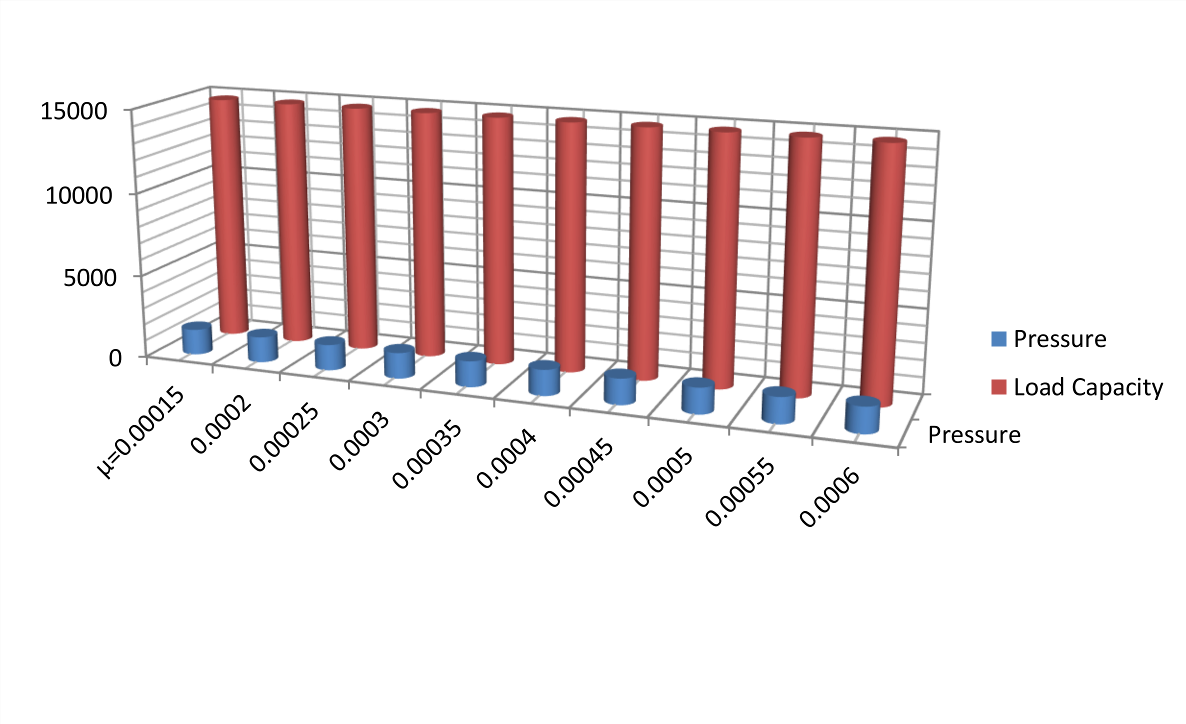

Figure-4 (Variation of P and W against μ for M=0.1)

6. CONCLUSIONS

The variation of pressure and load capacity for plane inclined slider bearings with respect to rotation number M, when viscosity is constant; are shown by tables and graphs. Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure and load capacity for plane inclined slider bearings both increase with increasing values of M, when viscosity is taken as constant. The equations of pressure and load capacity show that they are not independent of viscosity μ and slightly increases with μ, when M is constant. On taking (M=0) in the expression of pressure and load capacity, we get the classical solutions.

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