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Diffusion Equations for Dynamical Fluid Flow in Porous Rocks

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ABSTRACT

The multiphase flow in porous media is a subject of great complexities with a long rich history in the field of fluid mechanics. This is a subject with important technical applications, most notably in oil recovery from petroleum reservoirs and so on. The single-phase fluid flow through a porous medium is well characterized by Darcy's law. In the petroleum industry and in other technical applications, transport is modeled by postulating a multiphase generalization of the Darcy's law. In this connection, distinct pressures are defined for each constituent phase with the difference known as capillary pressure, determined by the interfacial tension, micro pore geometry and surface chemistry of the solid medium. For flow rates, relative permeability is defined that relates the volume flow rate of each fluid to its pressure gradient. In the present paper, there is a derivation and analysis about the diffusion equation for the fluid flow in porous rocks and some important results have been founded. The permeability is a function of rock type that varies with stress, temperature etc., and does not depend on the fluid. The effect of the fluid on the flow rate is accounted for by the term of viscosity. The numerical value of permeability for a given rock depends on the size of the pores in the rock as well as on the degree of interconnectivity of the void space. The pressure pulses obey the diffusion equation not the wave equation. Then they travel at a speed which continually decreases with time rather than travelling at a constant speed. The results shown in this paper are much useful in earth sciences and petroleum industry.

Keywords: Darcy's law, Diffusion equation, Multiphase flow, Porous rock.

I. INTRODUCTION

For discussing the fluid flow in porous media, we discuss firstly about the groundwater. The groundwater systems contain a huge quantity of the fresh water present on earth, providing a repository of water that is necessary for both human society and ecological systems. Many times for groundwater range from hundreds to thousands of years, making it a water source that is largely independent of the seasonal caprices associated with many surface water sources.

Due to primary source of drinking water worldwide, protection of this resource is critical to ensure widespread access to reliable sources of clean water. Instances of groundwater contamination are common, and many can be identified with significant risks to public health. But from long residency times often extend to groundwater contamination and pollutants can be involved with long-term deleterious impacts on contaminated resources. Non-aqueous phase liquids (NAPLs) represent a class of contaminants for which existing remediation strategies are particularly inadequate. NAPL contaminated systems are common, arising from improper disposal of solvents used in industry, leakage of underground storage tanks containing petroleum products, spills and byproducts of refinement and coal gasification [1], [7], [8]. NAPLs are immiscible in water, and most are soluble in trace amounts. Once NAPLs have been introduced into a system contamination can persist for decades or even centuries [12], [13]. The development of useful remediation strategies for these systems has been largely unsuccessful, and standard mathematical modeling approaches used to give the flow behavior for these systems are subject to a number of problems occurring, severely limiting their predictive capability [9], [10]. The in use modeling approaches fail too properly account for multiple fluid phases, and more precise mathematical descriptions are required to analyses risks involved in contamination, advance fundamental understanding of system behavior, and develop remediation strategies related with these systems.

The consideration of porous media within a multi-scale framework is an emerging concept that takes advantage of the more mature state of understanding that applies at smaller length scales as a method to give the description of larger scale systems. Many physical systems can be associated with a series of length scales; each one is associated with a particular mathematical formulation which describes the system behavior at that scale. The multi-scale frameworks give the relationship between these different descriptions, which give a series of mathematical formulations.

When these are applied to porous media the approach can be used to tie macroscopic thermodynamic forms and conservation equations to those that apply at the pore-scale, otherwise known as the micro scale. This is useful when macroscopic closure relationships are unreliable or incomplete; microscopic closure relationships are usually better known. Microscopic simulations can therefore be applied to give insights into macroscopic behavior, judges simplifying assumptions, and generate suitable macroscopic closure relationships. These studies rely heavily on computational methods to give the actual solutions for the microscopic analysis of porous medium flows. The computational analysis gives opportunities to incorporate larger and more realistic details of micro-analysis behavior into macroscopic modeling analysis.

In Germany and many other countries more than half of the population depends on groundwater as their supply in drinking water [4]. The problems with groundwater quality arise from disposal dumps, leaking storage tanks and accidental spills of substances used in industry. For removing these substances from the subsurface are extremely complicated and difficult, if at all possible [6]. In order to design effective remediation methods it is important to understand the governing physical processes of flow and transport in porous media. The mathematical modeling is one of the important methods that help to get the target. Including the more detailed physics and geometric detail into the mathematical models wants the use of efficient numerical algorithms and large scale parallel computers, both are of major concern in this analysis. Among the most toxic and prevalent materials threatening groundwater quality are so called non-aqueous phase liquids (NAPLs) such as petroleum products, chlorinated hydrocarbons etc. These chemicals have low solubility in water and are to be considered as separate phases in the subsurface.

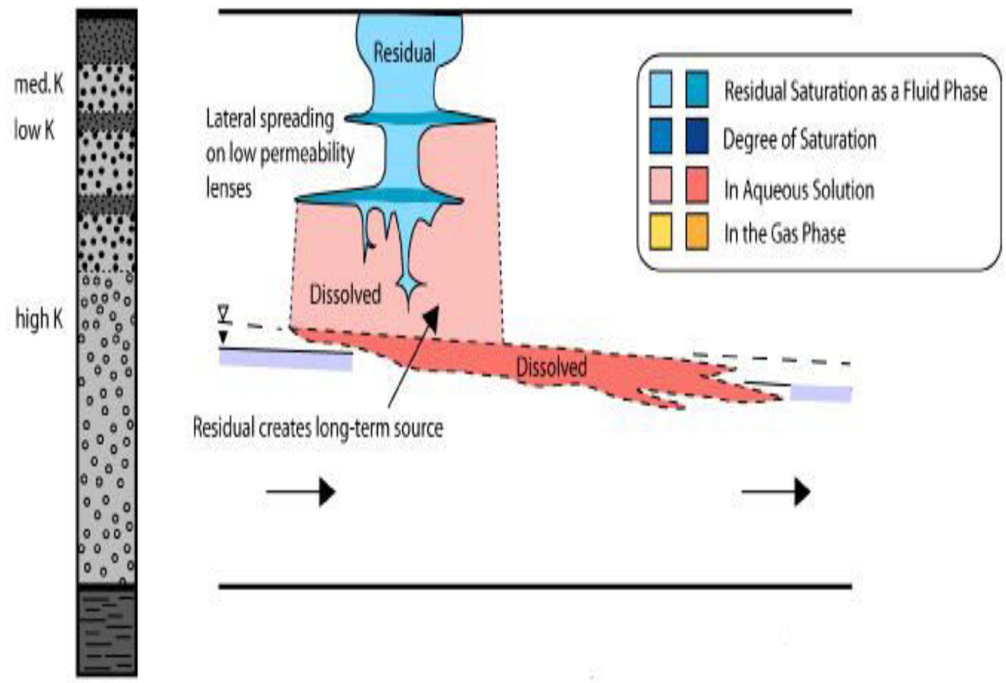

Figure 1 Formation of residual and dissolution

(Source: *Friedrich Schwille, 1988, Lewis Publishers, Chelsea, Michigan*)

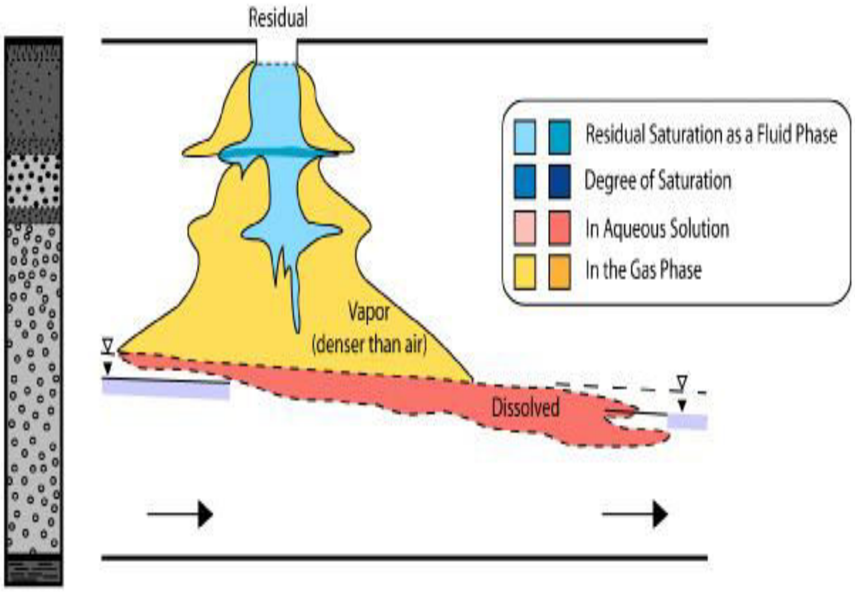

Figure 2 Formation of residual and vapor plume

(Source: *Friedrich Schwille, 1988, Lewis Publishers, Chelsea, Michigan*)

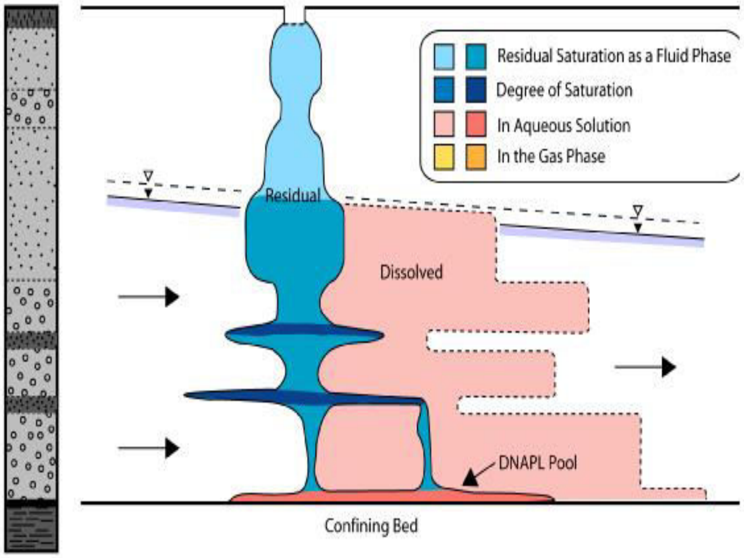

Figure 3 DNAPL below the water table

(Source: *Friedrich Schwille, 1988, Lewis Publishers, Chelsea, Michigan*)

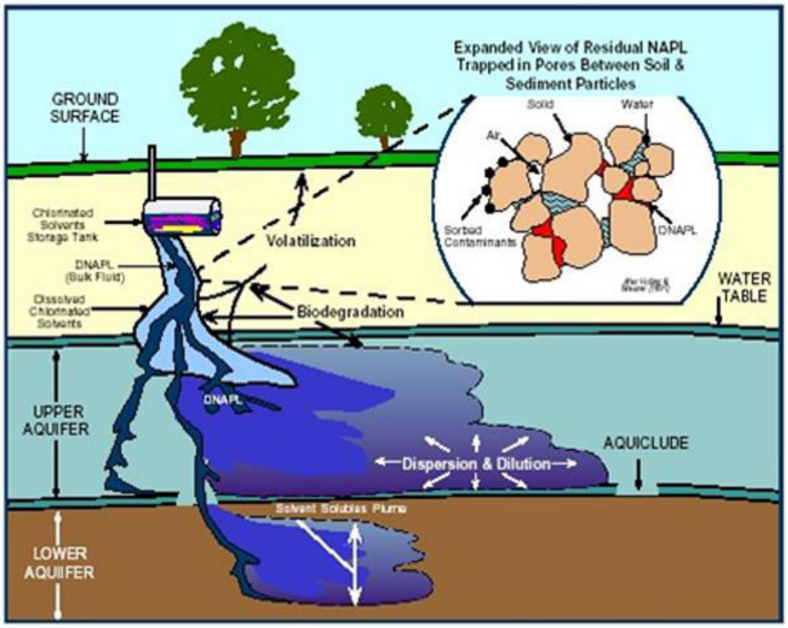

Figure 4 Summary of processes in the DNAPL spill.

(Source: *Pope, D. F., and J. N. Jones, 1999.*

Monitored Natural Attenuation of Chlorinated Solvents.

Report Number EPA/600/F-98/022. Office of Researchand Development,

U.S. Environmental Protection Agency, Washington, D.C. May 1999)

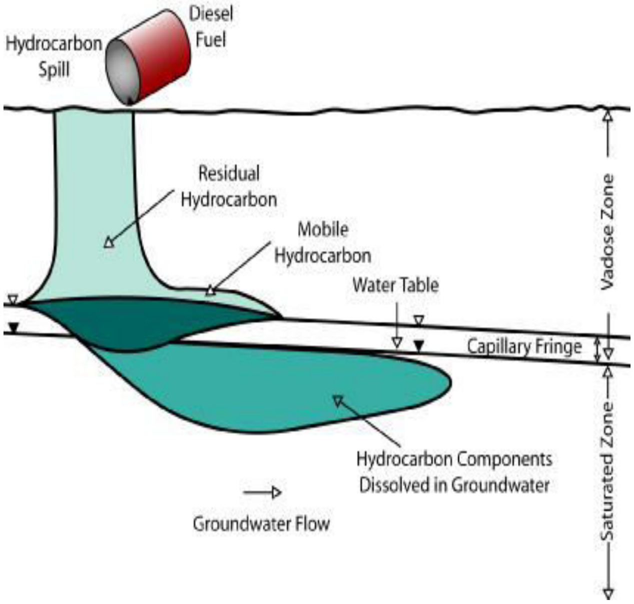

Figure 5 LNAPL behavior at the water table

(Source: *C.W.Fetter, 1992, Contaminant Hydrogeology, Macmillan Publishing Company, New York*)

The above figures taken from different sources illustrate the qualitative flow behavior of different NAPLs in the subsurface. In the case, when a light NAPL (LNAPL) with density smaller than water is released then it migrates downward through the unsaturated zone until it reaches the water table where it continues to spread horizontally. Now, these substances contain volatile components that are then transported in the air phase. When the supply of LNAPL stops, a certain amount of it remains immobile in the soil at residual saturation. The flow of a dense NAPL (DNAPL) is heavier than water and its flow behavior in the unsaturated zone are same. But due to its bigger density it moves downward also through the saturated zone. But due to capillary effects heterogeneities in the soil play an important role in multiphase flows. The regions of smaller pores are not penetrated by the fluid until a critical fluid saturation has accumulated. The size of these regions may variates from centimeters leading to an irregular lateral spreading of the NAPL to meters with the formation of DNAPL pools. The NAPLs give a long term problem to the quality of ground water. The initial infiltration may occur in hours or days while the solution process may occur in years. The small concentrations of NAPL on the order of 10 make the water not suitable for drinking. By the large number of processes mentioned in list it is evident that mathematical modeling of remediation processes can be very difficult. In the simplest example of two phases immiscible flow, the mathematical model is related to two coupled non-linear partial differential equations depend on time. But the detailed geometry of a porous medium is impossible to determine its complicated structure is effectively known by several parameters in the mathematical equations. It is the fundamental problem of all porous medium flow models for finding these parameters. But due to the heterogeneity of the porous medium on different length scales these effective parameters are the scale dependent. The different techniques have been found to address this problem. Here we have mention stochastic modeling [5], upscaling [2] and parameter identification [11].So far we concentrated on groundwater remediation problems as our motivation for the consideration of multiphase fluid flow in porous media. In addition there are other important applications for these models such as oil reservoir exploitation and security analysis of underground waste repositories. The latter application is often complicated by the existence of fractures in hard rock [3].

For flow in porous media, the Darcy's equation has been applied. The Darcy equation is generally based on the principle of a linear relation between the velocity and the pressure gradient in the porous media. The linear factor is expressed as porosity and is representing the resistant to flow in the solid media. The flow process in porous media is governed by several physical phenomena like as viscous forces and the forces coming from surface tensions between solid and fluid, but also surface tensions between different phases of the fluid. The flow process is involved in the principle modeled by use of the momentum equation, but it takes more simulation effort to solve the momentum equation than use the Darcy's equation. For this reason the Darcy's equation is most commonly applied in simulations of fluid flow through porous media.

II. GOVERNING LAWS AND EQUATIONS

The basic law governing the flow of fluids through porous media is Darcy's law, which was formulated by the Henry Darcy in 1856 on the basis of his experiments on vertical water filtration through sand beds. The detailed derivation related to diffusion was given by R. W Zimmerman in 2002 [14]. Darcy found that his data could be described by

$$Q = \frac{C\, A\, \Delta(P - \rho g z)}{L} \tag{1}$$

Where

P = pressure [Pa]
ρ = density [kg/m^3]
g = gravitational acceleration [m/s^2]
z = vertical coordinate (measured downwards) [m]
L = length of sample [m]
Q = volumetric flow rate [m^3/s]
C = constant of proportionality [$m^2/Pa\ s$]
A = cross-sectional area of sample [m^2]

Any consistent set of units can be used in Darcy's law, such as SI units, C.G.S. units etc. But in the oil industry it is common to use "oilfield units", that are inconsistent. The Darcy's law is mathematically same as other linear transport laws, such as Ohm's law for electrical conduction, Fick's law for solute diffusion and Fourier's law for heat conduction. By the fluid mechanics we know that Bernoulli's equation contains the terms:

$$\frac{P}{\rho} - g\,z + \frac{V^2}{2} = \frac{1}{\rho}\left(P - \rho\,g\,z + \rho\frac{v^2}{2}\right) \tag{2}$$

where P/ρ is related to the enthalpy per unit mass, $g\,z$ is the gravitational energy per unit mass, $v^2/2$ is the kinetic energy per unit mass. But the fluid velocities in a reservoir are small then the third term can be neglected. And we see that the term $(P - \rho\,g\,z)$ represents a term of energy type. That seems reasonable that the fluid can flow from regions of higher to lower energy so, the driving force for flow must be the gradient of $(P - \rho\,g\,z)$. But due to Darcy's analysis, it has been found that all other factors being equal, Q is inversely proportional to the fluid viscosity. It is therefore suitable to factor out μ, and put $C = k/\mu$, where k is known as the permeability. On taking the volumetric flow per unit area, $q = Q/A$. Now the Darcy's law can be written as:

$$q = \frac{Q}{A} = \frac{k}{\mu}\frac{\Delta(P - \rho g z)}{L} \tag{3}$$

where the flux q has the dimensions of $[m/s]$. It is perhaps easier to say of these units as $[m^3/m^2 s]$.

For transient processes in which the flux varies from the point to point, we can write a differential form of Darcy's law. In the vertical direction, this equation can be written as:

$$q_v = \frac{Q}{A} = -\frac{k}{\mu}\frac{d(P - \rho g z)}{dz} \tag{4}$$

where the suffix v is taken for vertical flow. The minus sign is taken since the fluid flows in the direction from higher potential to lower potential. The differential form of Darcy's law for one-dimensional horizontal flow can be

$$q_h = \frac{Q}{A} = -\frac{k}{\mu}\frac{d(P - \rho g z)}{dx} = -\frac{k}{\mu}\frac{dP}{dz} \tag{5}$$

where the suffix v is taken for vertical flow. In most rocks the permeability k_h in the horizontal plane is different than the vertical permeability, k_v ; in most of the cases, $k_h > k_v$. The permeabilities in any two orthogonal directions within the horizontal plane have the difference. So, in this course we shall generally take: $k_h = k_v$. The permeability is the function of rock type, that varies with stress, temperature etc., but it does not depend on the fluid; the effect of the fluid on the flow rate is accounted for by the term of viscosity in the above equations. The permeability has units of m^2, but in mathematical use it is conventional to use "Darcy" units, defined as:

$1\,Darcy = 0.987 \times 10^{-12}\ \text{m}^2 \approx 10^{-12}\ m^2$

The Darcy unit is defined such that a rock having a permeability of 1 *Darcy* would transmit 1 c.c. of water with viscosity 1 *cP per second*, through a region of 1 sq. cm. cross-sectional area, if the pressure drop along the direction of flow were 1 *atm per cm.* Many soils and sands that mathematicians must deal with have permeabilities on the order of a few *Darcies*. The original purpose of the "*Darcy*" definition was thus to avoid the need for using small prefixes such as 10^{-12} etc. But a Darcy is nearly a round number in SI units, so conversion between the two is easy. The numerical value of k for a given rock depends on the size of the pores in the rock, d as well as on the degree of interconnectivity of the void space. So that

$k \approx d^2/1000$

where d denotes the diameter of the pores. The permeabilities of different types of rocks and soils vary over many orders of magnitude. However, the permeabilities of petroleum reservoir rocks tend to be in the range of 0.001-1.0 *Darcies*. So it is convenient to refer the permeability of reservoir rocks in units of "*milliDarcies*" (*mD*), which equal 0.001 *Darcies*. In some reservoirs, the permeability is due mainly to an interconnected network of fractures. The permeabilities of fractured rocks tend to be in the range 1 *mD* to 10 *Darcies*. In a fractured reservoir, the reservoir scale permeability is not closely related to the core scale permeability that anyone can measure it.

If the fluid is in static equilibrium then $q = 0$, so the differential equation will be

$$\frac{d(P - \rho g z)}{dx} = 0 \Rightarrow P - \rho g z = \text{constant} \tag{6}$$

If we take $z = 0$ i.e., at sea level, where the fluid pressure is equal to the atmospheric pressure, then we have

$$P_s = P_a + \rho\,g\,z \tag{7}$$

where P_s, P_a represent the static pressure and atmospheric pressure respectively. But we always measure the pressure above the atmospheric pressure, so we can neglect the term P_a in the above equation (7). We can see by comparing equation (7) with equation (4) that only the pressure above and beyond the static pressure given by equation (2) plays a role in driving the flow. So the term ρgz is useless, as it only contributes to the static pressure and but does not contribute to the driving force for the flow. Then after removing the term, the equation for correct pressure will be

$$P_c = P - \rho gz \tag{8}$$

Now the Darcy's law in the terms of corrected pressure for the horizontal flow can be written as:

$$q = \frac{Q}{A} = -\frac{k}{\mu}\frac{dP_c}{dx} \tag{9}$$

Instead of using sea level i.e., $z = 0$, we can take $z=z_0$ as a datum i.e., the amounts of initial oil in place lie above and below $z=z_o$. So we get

$$P_c = P - \rho\, g\,(z - z_o) \tag{10}$$

The choice of the datum level is immaterial, in the sense that it gives a constant term to the corrected pressure so it does not contribute to pressure gradient. The pressure P_c defined in equation (10) can be interpreted as the pressure of a analytical fluid at depth $z=z_o$ that will be in equilibrium with the fluid and exists at the actual pressure at depth z. The Darcy's law is supposed to be a macroscopic law that is intended to be meaningful over regions that are much bigger than the size of a single pore. Now we can discuss about the permeability at a point in the reservoir, we cannot be referring to the permeability at a mathematically infinitesimal point since the given point can be in a sand grain and not in the pore space. The permeability property is actually defined for a porous medium, not for a single pore. So the permeability is the property that is in a meaning of averaged out on a certain region of space surrounded the point (x,y,z). Now the region must be sufficiently large to encompass a significant number of pores. The pressure P that is used in Darcy's law is actually an average pressure taken on a small region of space.

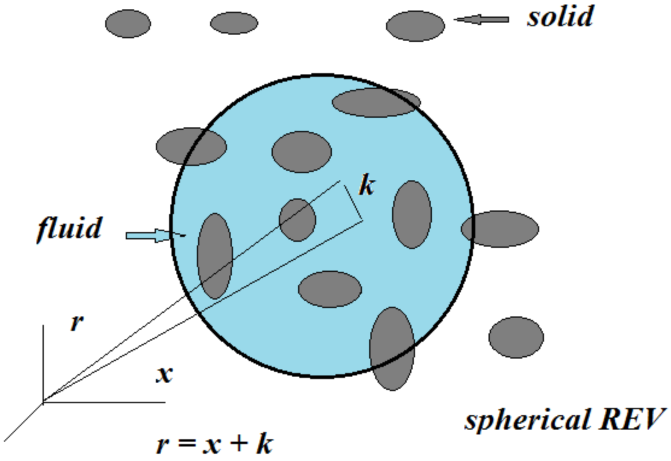

Figure 6. Spherical representative elementary volume (REV).

The spherical representative elementary volume is shown by figure-6. When we refer to the pressure at a certain location in the reservoir, we do not distinguish between two nearby points such as these. So, the entire region shown in the figure will be represented by an average pressure which is taken over the indicated circular region, that is known as a Representative Elementary Volume (REV). Similarly, the permeability of the rock is defined over the REV length scale. About the size of REV, we can say that it must be at least one order of magnitude larger than the pore size.

The Darcy's law in itself does not give sufficient information to solve time-dependent i.e., transient problems that involving subsurface flow. So to find the complete governing equation that applies to these problems, we will first find the mathematical expression for the principle of conservation of mass. The conservation of mass equation states the balance between rate of mass change in an arbitrary volume and inflow of mass through the boundary surface area. In integral form, this can be expressed as follows:

$$\frac{\partial}{\partial t}\iiint \rho\,\phi\,dV + \iint \rho\,\bar{v}.\bar{n}\,dS = \iiint q\,dV \tag{11}$$

In this equation the double and triple integrals are taken over surface and volume respectively and $\bar{v}, \bar{n}, \rho, q$ and ϕ represent the velocity vector, unit normal vector, fluid density, external mass flow rate and porosity respectively. The right hand side term of the equation (11) can be changed into a volume integral form by the using the Gauss' divergence theorem as:

$$\iint \rho\,\bar{v}.\bar{n}\,dS = \iiint \nabla.(\rho\,\bar{v})\;dV \tag{12}$$

So, for a fixed control volume, the integral form of the conservation law will be

$$\iiint \left[\frac{\partial(\rho\,\phi)}{\partial t} + \nabla.(\rho\,\bar{v}) - q\right] dV \tag{13}$$

Now the differential form the conservation equation for mass can be written in coordinate invariant form as:

$$\frac{\partial(\rho\,\phi)}{\partial t} + \nabla.(\rho\,\bar{v}) = q \tag{14}$$

For the multiphase flow, it is necessary to account for the saturation of each phase. So that the equation (14) within each phase α can be written as:

$$\frac{\partial(\rho_\alpha\,S_\alpha \phi)}{\partial t} + \nabla.(\rho_\alpha\,\bar{v}_\alpha) = q_\alpha \tag{15}$$

III. DIFFUSION EQUATION IN CARTESIAN COORDINATES

The transient flow of a fluid through a porous medium is governed by a certain type of partial differential equation known as a diffusion equation. The detailed derivation and discussion related to diffusion was given by R. W. Zimmerman in 2002 [14] *i.e.,* discussed in this paper. So to derive the equation, we combine Darcy's law with the law of mass conservation and an equation that describes the process for which the fluid is stored inside a porous rock. Now using the differentiation for the product function $(\rho\,\phi)$, we get

$$D(\rho\,\phi) = \rho\,D\phi + \phi\,D\rho, D \equiv \frac{d}{dt} \tag{16}$$

$$= \rho\frac{d\phi}{dP}\frac{dP}{dt} + \phi\frac{d\rho}{dP}\frac{dP}{dt}$$

$$= \phi\left[\left(\frac{1}{\phi}\frac{d\phi}{dP}\right)\frac{dP}{dt} + \rho\left(\frac{1}{\rho}\frac{d\rho}{dP}\right)\frac{dP}{dt}\right]$$

$$D(\rho\,\phi)\; = \rho\,\phi\,\left(c_R + c_f\right) D(P) \tag{17}$$

where $c_R,\;c_f$ are the compressibility of the rock and the fluid respectively.

The equation for mass conservation for the fluid flow is taken as:

$$-\frac{d(\rho\,q)}{dx} = \frac{d(\rho\,\phi)}{dt} \tag{18}$$

By using the Darcy's law, we have

$$-\frac{d(\rho\,q)}{dx} = -\frac{d}{dx}\left[-\frac{\rho k}{\mu}\frac{dP}{dx}\right]$$

$$-\frac{d(\rho\,q)}{dx} = \frac{k}{\mu}\left[\rho\frac{d^2\,P}{dx^2} + \frac{d\rho}{dx}\frac{dP}{dx}\right]$$

$$-\frac{d(\rho\, q)}{dx} = \frac{k}{\mu}\left[\rho\frac{d^2\, P}{dx^2} + \frac{d\rho}{dP}\frac{dP}{dx}\frac{dP}{dx}\right]$$

$$-\frac{d(\rho\, q)}{dx} = \frac{k\rho}{\mu}\left[\frac{d^2\, P}{dx^2} + \frac{1}{\rho}\frac{d\rho}{dP}\left(\frac{dP}{dx}\right)^2\right]$$

$$-\frac{d(\rho\, q)}{dx} = \frac{k\rho}{\mu}\left[\frac{d^2\, P}{dx^2} + c_f\left(\frac{dP}{dx}\right)^2\right] \tag{19}$$

From the equations (17) and (19), we get

$$\frac{d^2\, P}{dx^2} + c_f\left(\frac{dP}{dx}\right)^2 = \frac{1}{K}\rho\,\phi\,\left(c_R + c_f\right) D(P) \tag{20}$$

But we know that,

$$\frac{d^2\, P}{dx^2} \gg c_f\left(\frac{dP}{dx}\right)^2$$

On neglecting the term $c_f\left(\frac{dP}{dx}\right)^2$, $i.e.$

$$c_f\left(\frac{dP}{dx}\right)^2 \approx c_f\left[\frac{\mu\, Q}{2\,\pi\, k\, H\, R}\right]^2 \tag{21}$$

$$\frac{d^2\, P}{dx^2} \approx \frac{\mu\, Q}{2\,\pi\, k\, H\, R^2} \tag{22}$$

So we can neglect the nonlinear term in equation (20), we get the diffusion equation as given

$$\frac{dP}{dt} = \frac{K}{\phi\,\mu\,(c_\phi + c_f)}\frac{d^2P}{dx^2} \tag{23}$$

where $\left(c_\phi + c_f\right) = c$ is the total compressibility.

The parameter which governs the rate at which fluid pressure diffuses through the rock is the hydraulic diffusivity D_H , that is defined by

$$D_H = \frac{k}{\phi\,\mu\,(c_\phi + c_f)} \tag{24}$$

The distance d at which a pressure disturbance will travel during an elapsed time t is given as

$$d = \sqrt{4\, D_H t} \tag{25}$$

For the multi-phase flow if we have assume that the pores of the rock are filled with two components, oil and water, and often also contain some hydrocarbons in the gaseous phase. So we have to find the governing flow equations for an oil and water system, in the general form. From the rock properties module that Darcy's law can be generalized for two-phase flow by including a relative permeability factor for each phase, we have

$$q_w = \frac{-kk_{rw}}{\mu_w}\frac{dP_w}{dx} \tag{26}$$

$$q_o = \frac{-kk_{ro}}{\mu_o}\frac{dP_o}{dx} \tag{27}$$

where the subscripts w and o are used for oil and water respectively. The two relative permeability functions are supposed to be known functions of the phase saturations. For the oil-water system, the two saturations are necessarily related to each other by the relationship

$$S_w + S_o = 1 \tag{28}$$

The pressures in the two phases at every point in the reservoir must be different. If the reservoir is oil-wet then the two pressures will be given by

$$P_o - P_w = P_c\, S_o \tag{29}$$

where the capillary pressure P_c is given by the rock-dependent function of oil saturation.

But the volume of the oil in a given region is equal to the total pore volume multiplied by the oil saturation then by the equations of the conservation of mass for the two phases can be written by inserting a saturation factor in the storage term as given

$$-\frac{d(\rho_0 q_0)}{dx} = \frac{d(\phi \rho_0 S_0)}{dt} \tag{30}$$

$$-\frac{d(\rho_w w)}{dx} = \frac{d(\phi \rho_w S_w)}{dt} \tag{31}$$

The densities of the two phases are related to their respective phase pressures with the equation of state as given by

$$\rho_0 = \rho_0(P_0) \tag{32}$$

$$\rho_w = \rho_w(P_w) \tag{33}$$

where the temperature are taken as constant.

Lastly, the porosity must be the function of the phase pressures Po and Pw. The above two pressures independently affect the porosity. Now, the capillary pressure P_c is generally small so that

$$Po \approx Pw$$

From which we can use the pressure-porosity relationship that would be obtained under single-phase conditions, *i.e.,*

$$\phi = \phi\,(P_0) \tag{34}$$

If the fluid is taken as slightly compressible or if the pressure variations are small then the equations of state are written as

$$\rho(P_o) = \rho_{0i}[1 + c_o\,(P_o - P_{oi})] \tag{35}$$

where the subscript "i" is used for the initial state, and the compressibility c_o is taken as a constant.

IV. DISCUSSION AND RESULTS:

The permeability is a function of rock type that varies with stress, temperature etc., and does not depend on the fluid. The effect of the fluid on the flow rate is accounted for by the term of viscosity. The numerical value of k for a given rock depends on the diameter of the pores in the rock "d" as well as on the degree of interconnectivity of the void space. The parameter that governs the rate at which fluid pressure diffuses through a rock mass is the hydraulic diffusivity which is defined by

$$D_H = \frac{k}{\phi\,\mu\,c}$$

The distance d at which a pressure disturbance will travel during an elapsed time t is given as

$$d = \sqrt{4\,D_H t}$$

The time required for a pressure disturbance to travel a distance d is found by

$$t = \frac{\phi\,\mu\,c\,d^2}{4k}$$

The pressure pulses obey a diffusion equation not a wave equation. So, they travel at a speed that continually decreases with time rather than travelling at a constant speed.

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