Analysis of Energy for Fluid Flow Motions in Porous Media

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Abstract

The largest transport analysis of non-compressed fluid flow in porous sources based on a volume-measurement of heat transfer was given in various studies. In the present paper there is an analysis and output of statistics based on the concept of medium term. This provides new insights and a way to analyze the flow of chaos in porous media. By taking the time variations of flow structures with the deviation, there are usually two ways to find and study macroscopic calculations. The first method was based on a user-time estimate followed by a volume measurement originally used by Kuwahara in 1998. The second method based on the concept of pre-measurement volume measurement used by Lee & Howell in 1987 and large transport statistics. established by these two methods are equal. Average transport statistics are found in the current paper. Estimated transport rates play an important role in analyzing transit media that can penetrate when turbulent flow occurs in the liquid phase.

Keywords: Power, Hollow Media, Fluid Flow.

1. Introduction

We know that almost every fluid we see in everyday life is chaotic. Some common examples flow in cars, planes, buildings etc. Boundary layers and ambience around and behind other bodies such as cars, planes and buildings are more chaotic. And the flow and burning in the engines, both in the piston engines and the gas engines and combustors, is very chaotic. The air movement in the rooms is chaotic, at least near the walls where the jets are built into the wall. The flow of fluid into the boreholes is often turbulent. Therefore, when we calculate the flow of fluid there is likely to be confusion. In the chaos flow we usually divide the velocities in one time averaged part $\bar{v}_i$, which is independent of time when the mean flow is steady, and one fluctuating part v'_i, so that $v_i = \bar{v}_i + v'_i$. The intensity of turbulence presented is related to the *energy* that is turbulent kinetic energy (TKE). As is apparent from the name of this quantity, the value of TKE directly represents the 'strength' of the turbulence in the flow. The turbulent flow has no suitable definition but it has a number of characteristic features as given:

1.1 Irregularity

Flow of Turbulent is unusual and unplanned but is governed by the Navier-Stokes equation. Flow contains a spectrum of different scales (eddy sizes). We do not have an accurate description of eddy chaos, but we think it is somewhere in the atmosphere at some point in the chaos and is eventually destroyed by the cascade process or by disintegration. It has an element of speed and length i.e., called velocity and length. A region covered by a large eddi may also close smaller eddies. The largest eddies belong to the flow pattern geometry, i.e., the thickness of the boundary layer, the width of the jet, etc. On the other side of the spectra we have very small eddies that disperse viscous forces (pressures) into thermal energy leading to temperature. increase. Even a random commotion determines and is explained by Navier-Stokes equation.

1.2 Diffusivity

In the turbulent flow increases the diffusivity. The disorder increases the exchange of pressure e.g. boundary layers, and thus reduces or delayed the separation in bluff bodies such as cylinders, airfoils and vehicles. Increased diffusivity increases resistance (wall collision) and heat transfer to internal flow such as channels and pipes.

1.3 Large Reynolds Numbers

Flow of chaos occurs with a high number of Reynolds. For example, switching to a turbulent flowin pipes occurs that $\boldsymbol{Re}_D \simeq 2300$, and in boundary layers at $\boldsymbol{Re}_x \simeq$ 11000-500000.

1.4 Dissipation

The turbulent flow disintegrates, meaning that the kinetic energy in the small eddies (dissipatives) is converted into thermal energy. Smaller eddies gain kinetic power in larger eddies. Larger eddies get their power from the biggest eddies and so on. Very large eddies release their energy from the central flow. This process of transferring energy from the largest moving scale (eddies) to the smallest is called the cascade process.

1.5 Continuum

Even if we have small moving scales in the flow they are a much larger process than the molecular scale and can treat flow as continuous. In liquid motion the concept of macroscopic transport of non-compressed fluid flow to porous sources was used by Vafai & Tien [7] in 1981 based on a volume-measurement of heat transfer measurement. The concept of space measurement in perforated media is based on the assumption that although liquid velocity and pressure may be abnormal on the permeability scale, the average local dimensions of these figures vary considerably. Macroscopic statistics are usually obtained by measuring the microscopic area above the Reproductive Elementary Volume (REV) of open media. REV should be a slightly different volume, resulting in sensible local properties. It means that the length of this volume must be large enough than the aperture scale. Also, the system size should be much larger than the REV length limit to avoid non-homogeneity i.e., Three-Dimensional Turbulent flow is always three-dimensional and unsteady. But when the equations are time averaged, we can treat the flow as two-dimensional flow in general.

$$p \ll D \ll L$$

Where p is the pore scale or microscopic length scale, D is the macroscopic length scale and L is the mega-scale or scale of the system as represented by figure-2.

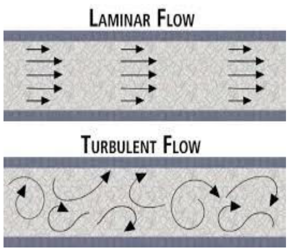

Figure-1. Turbulent flow

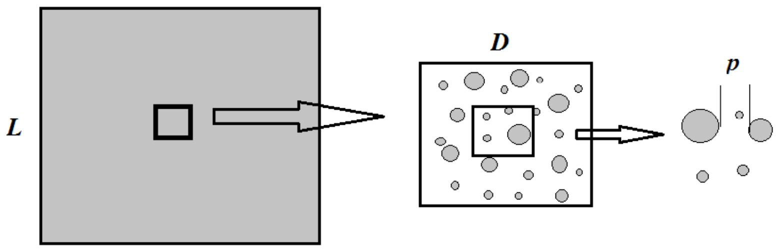

Figure-2. Identification of different length scales.

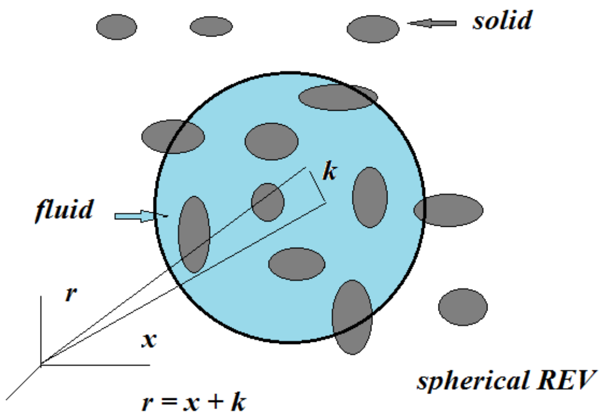

Figure 3. Spherical representative elementary volume (REV).

A schematic representation of a spherical REV consisting of a fixed solid phase saturated with a continuous fluid phase and is shown by the figure-2, here the solid phase is fixed, i.e., the solid phase does not change randomly if different ensembles are considered. The volume of the REV is constant i.e., independent of the space and its value is equal to the sum of the fluid and solid volumes inside the REV [4], i.e.,

$$V = V_s + V_f$$

The spherical representative elementary volume is shown by figure-3. On taking the time fluctuations of the flow properties with spatial deviations, there are generally two methods for deriving and studying the macroscopic equations. The first method based on the time-average operator followed by the volume-averaging initially used by Kuwahara *et al.* [3] in 1998. The second method based on the concept of volume-averaging before time averaging that was used by Lee & Howell [2] in 1987, and the macroscopic transport equations established by these two methods are equivalent. This initial method for the flow variables has been extended to the nonbuoyant heat transfer for the porous media by considering the phenomenon of time variations and spatial deviations was taken by Rocamora & Lemos [5] in 2000.Later, the researches on the natural convection flow on the porous layer, double-diffusive convection for the turbulent flow and heat transfer in the porous media was given by de Lemos *et al.* in 2004. The numerical based analysis for applications of double-decomposition theory to buoyant flow was also reviewed by de Lemos [1].

2. Governing Equations

The macroscopic instantaneous transfer equations for the incompressible fluid flow having the constant properties are given as:

$$\nabla.\bar{v} = 0 \qquad (1)$$

$$\rho\, \nabla.(\bar{v}.\bar{v}) = -\nabla P + \mu\nabla^2\bar{v} + \rho\, \bar{g} \qquad (2)$$

$$(\rho\, C_P)\, \nabla.(\bar{v}\, T) = \nabla.(\lambda\, \nabla\, T) \qquad (3)$$

Where $\bar{v}$ is the velocity vector, P is the pressure, μ is the viscosity of the fluid, ρ is the density of the fluid, $\bar{g}$ is the acceleration vector due to gravity, C_P is the specific heat, T is the temperature and λ is the thermal conductivity of the fluid. The mass fraction distribution related to chemical species e is governed by the transport equation given as:

$$\nabla.(\rho\, \bar{v}\, m_e + \bar{J}_e) = \rho\, R_e \qquad (4)$$

Where m_e is the mass fraction of component e, $\bar{v}$ is the mass-averaged velocity of the fluid mixture, so we have

$$\bar{v} = \sum_e m_e \, \bar{v}_e \tag{5}$$

where $\bar{v}_e$ is the velocity of species e. The mass diffusion flux $\bar{J}_e$ is due to velocity slip of the species e and is given as:

$$\bar{J}_e = \rho_e \, (\bar{v}_e - \bar{v}) = -\rho \, D_e \, \nabla \, m_e \tag{6}$$

where D_e is the diffusion coefficient of species e for the mixture. The equation (6) is also known as the Fick's law. The R_e represents the generation rate of species per unit mass.

If the density ρ varies with the temperature T for the natural convection flow, the remaining density based on the Boussinesq concept will be given as:

$$\rho_T \cong \rho \, [1 - \beta(T - T_r)] \tag{7}$$

where T_r is the temperature at reference value and β is the thermal expansion coefficient and is defined as:

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T}\right)_P \tag{8}$$

By using the equation (2) and (7), we get

$$\rho \, \nabla . (\bar{v} \, \bar{v}) = -(\nabla \, P)^* + \mu \, \nabla^2 \, \bar{v} - \rho \, \bar{g} \beta \, (T - T_r \,) \tag{9}$$

Where $(\nabla \, P)^* = \nabla \, P - \rho \, \bar{g}$, represents the modified pressure gradient.

From equation (3), we have the equation for fluid as:

$$\left(\rho \, C_p\right)_F \nabla . (\bar{v} \, T_F) = \nabla . (\lambda_F \, \nabla \, T_F) + \, S_F \tag{10}$$

Also from equation (3), we have the equation for solid or porous matrix as:

$$\nabla . \left(\lambda_p \, \nabla \, T_p\right) + \, S_p = 0 \tag{11}$$

where the suffix F and p are used for fluid and porous matrix respectively. The factor S_F or S_p vanishes in the absence of heat generation. The volume-averaging in the porous medium was given primarily by Slattery in 1967 [6] and later by others. It makes the concept of *REV* (representative elementary volume) and by using the concept, the equations are integrated.

2.1 Volume and Time Average Operators

The volume average of the general property term φ over REV for the porous medium was given by Gray *et al.* [2] in 1977 and is written as:

$$[\varphi]_V = \frac{1}{\delta V} \int \varphi \, dV \tag{12}$$

where $[\varphi]_V$ is taken for any point surrounded by REV of size δV. The average is given as:

$$[\varphi_F]_V = \phi \, [\varphi_F]_i \tag{13}$$

where the suffix 'i' is used for the intrinsic average and ϕ is the porosity of the medium and is defined as:

$$\phi = \frac{\delta V_F}{\delta V}$$

$$\varphi = [\varphi]_i + \varphi_i \tag{14}$$

in addition to the condition that

$$[\varphi_i]_i = 0 \tag{15}$$

where φ_i is the spatial deviation of φ for the intrinsic average φ_i. To derive the flow equations, we have to know the relation between the volume average of derivatives and derivatives of volume average. The relation between these two was presented by Gray *et al.* [2] in 1977. So we have

$$[\nabla\varphi]_V = \nabla\{\phi(\varphi)_i\} + \frac{1}{\delta V}\left[\iint \hat{n}\ \varphi\ ds\right]_{\alpha_i} \tag{16}$$

$$[\nabla.\varphi]_V = \nabla.\{\phi(\varphi)_i\} + \frac{1}{\delta V}\left[\iint \hat{n}\,.\varphi\ ds\right]_{\alpha_i} \tag{17}$$

$$\left[\frac{\partial\varphi}{\partial t}\right]_V = \frac{\partial}{\partial t}\{\phi(\varphi)_i\} - \frac{1}{\delta V}\left[\iint \hat{n}\,.(\bar{v}_i\,\varphi)\ ds\right]_{\alpha_i} \tag{18}$$

where α_i, $\bar{v}_i$ and $\hat{n}$ are interfacial area, velocity and unit vector normal to α_i respectively. If the porous substrate is fixed then $\bar{v}_i = 0$. But if the medium is rigid and heterogeneous then δV_F depends on the space and doesn't depend on time as taken by Gray *et al.* [2]. The time average of φ is given as:

$$\bar{\varphi} = \frac{1}{\delta t}\int_t^{t+\delta t} \varphi\ dt \tag{19}$$

where δt is very small time interval as compared to $\bar{\varphi}$ but sufficient to calculate the turbulent fluctuations of φ. Now the time decomposition will be taken as:

$$\varphi = \bar{\varphi} + \varphi' \tag{20}$$

with the condition that

$$\overline{\varphi'} = 0 \tag{21}$$

where φ' is the time fluctuation of φ with respect to $\bar{\varphi}$.

3. Time-Averaged Transport Equation

Let us consider the following:

$$v = \bar{v} + v_1, T = \bar{T} + T_1, P = \bar{P} + P_1 \tag{22}$$

The equations (1), (2) and (9) will be

$$\nabla.\bar{v} = 0 \tag{23}$$

$$\rho\nabla.(\bar{v}\,\bar{v}) = -(\nabla\bar{P})^* + \mu\,\nabla^2\bar{v} + \nabla.(-\rho\overline{v_1 v_1}) - \rho\,\bar{g}\,\beta\,(\bar{T} - T_r) \tag{24}$$

$$(\rho\, C_p)\,\nabla.(\bar{v}\,\bar{T}) = \nabla.(K_e\,\nabla\,\bar{T}) + \nabla.(-\rho\, C_p\,\overline{(v_1 T_1)}) \tag{25}$$

Taking,

$$\frac{\{\nabla\,\bar{v} + (\nabla\bar{v})_T\}}{2} = \overline{D_m} = \text{mean deformation tensor} \tag{26}$$

$$\frac{(\bar{v}_1.\bar{v}_1)}{2} = K_e = \text{turbulent kinetic energy per unit mass} \tag{27}$$

By using the eddy-diffusivity concept, we have from equation (24),

$$-\rho\,\overline{(v_1 v_1)} = \mu_t\, 2\overline{D_m} - \frac{2}{3}\,\rho K_e \hat{A} \tag{28}$$

where $\mu_t, \hat{A}$ are the turbulent viscosity and unity tensor respectively.

Again by using the eddy-diffusivity concept for the turbulent heat flux for equation (25), we have

$$-\rho\, C_p\, \overline{(v_1 T_1)} = C_p\, \frac{\mu_t}{\sigma_t}\, \nabla\, \bar{T} \tag{29}$$

where σ_t is the turbulent Prandlt number. The transport equation for turbulent kinetic energy will be founded by taking the multiplication of the difference between the instantaneous and the time-averaged momentum equations by v_1. Again, using the time-average operator, the equation takes the form:

$$\rho\,\nabla.(\bar{v}\,K_e) = -\rho\,\nabla.\left\{v_1 \frac{P_1}{\rho} + u\right\} + \mu\nabla^2\, K_e + P_K + Q_K - \rho\, e_1 \tag{30}$$

where

$$P_K = -\rho\,\overline{(v_1 v_1)}$$

$$\nabla\,\bar{v} = \text{generation rate of } K_e \text{ due to the mean velocity gradient}$$

$$Q_K = -\rho\,\beta\,\bar{g}.\overline{(v_1 T_1)} \tag{31}$$

$$e_1 = \text{dissipation rate of } K_e$$

The term Q_K is the buoyancy generation rate of K_e.

$$u = \frac{v_1.v_1}{2} \tag{32}$$

4. Conclusions

The paper gives a new method for the analysis of kinetic energy for the turbulent flow in the porous media by using the time-averaged transport equation. This might be better when studying transport over highly permeable media where the turbulent flow occurs in the fluid phase. The analysis gives opportunities for environmental and engineering flows from these derivations.

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