



A study of space time of the spatially homogeneous and anisotropic Bianchi type I & V cosmological model with general relativity

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Abstract

In the sense of general relativity, we studied a new class of Bianchi Type I and V spatially homogeneous and anisotropic models that have ideal liquid distributions, in case of a time that is different in cosmology and gravity. For starters, two types of cosmologies can be identified with exact solutions of Einstein's field equations. $m \neq 3$ And $m = 3$, respectively. We suggest an alternative law on anisotropy (σ/θ) Scalar (total) is proportional with the scale factor R function, i.e. $\frac{(\sigma/\theta)}{\theta} \propto f(R)$ (where σ is a shear scalar). Detailed discussion is made of the physical properties of models. The models are late isotropic. Cosmological distance parameters have also been defined for both models. The space-time model of Bianchi type I & V gives continuous value to the parameter of deceleration. We derive two different regulations for the average factor in scale with cosmic time, one is power-law-like and the other is exponential. In both types of cosmologies, Bianchi type I & V space-time can obtain precise solutions for Einstein field equations with total fluid and heat conduction. In cosmology with a power law, solutions match a cosmological model that begins to expand with a positive deceleration parameter from the singular state. Our solutions favor the model CDM as well as discussed state finder parameters.

KEYWORDS: Bianchi type- I & V, Distance parameters, Perfect fluid, Λ CDM model, cosmological model, deceleration parameter.

1. INTRODUCTION

Recent observations from different sources [1-6] have led to the standard cosmological model of the late universe rapid expansion. Due to Einstein's overall relative principle, a great negative pressure must be imposed on the universe's material fields to account for the present accelerated period of the universe, known as dark energy. Recent findings indicate that our universe consists of

approximately 68.3% earth (PLANCK DATA), 26.8% dark and 4.9% normal matter at this time. But the lack of proof of the existence of the dark aspect has led to an intense debate, especially in theory. The cosmological symbol – Λ is the most simple and favorite candidate to enunciate this mysterious element, and it acts as an isotropic and homogenous source on the equations of Einstein's field. The energy density of the vacuum can be associated with this. Models with a relic cosmological constant have in recent years attracted considerable attention among researchers as they seem to match current data. The old and unresolved constant **cosmological question [7–10]** is the major difference in forecasting quantum energy density theory and quantum theory observational values. There is a fine-tuning problem with the observed weak, but not zero, cosmic constant and the problem of cosmic equilibrium is a vacuum energy that is of the same magnitude as the actual universe density. A large number of scientists have looked at a changing cosmological constant to find an ad hoc solution to this problem [11-14]. Through such models, cosmological concept is relaxed by the expansion of the universe with its low present value. To adapt and interpret the above-mentioned facts, a number of investigators have proposed physical models with different cosmological constants, particularly time dependent.

At the other hand, the continuous gravity G is the continuous connection between geometry and matter. The consistency of basic constants and the general relativity of the "principle of equivalence" are doubtful from recent observations. Dirac's concept of large quantities was introduced by the notion of variable G and, after this modification, a updated theory of general relativity has been investigated. Astrophysics has many important consequences for G . It is revealed that the cosmology of G -varying is consistent with all currently available cosmology findings [16]. Many scholars [17] have written about cosmologies that vary from one another. The study of the cosmological model forms Bianchi type I & V is more important as the model has various isotropical cases and at every instant of cosmic time it needs arbitrary small anisotropy (σ/θ). The naturally generalizing open models of FRW, which will eventually become isotropical, play an important role in understanding the phenomenon as galaxy formation in the early universe, among specific Bianchi-Type I & V universes. Space time with a perfect fluid source and time dependent cosmologic concept in the sense of spatially uniform anisotropic Bianchi Type I & V Singh[18] investigated possible variation legislation for the Hubbleparameter and Kumar and Yadav [19] investigated some of the accelerating universe models Bianchi-type-I & V with dark energies, **Roy and Prasad[20] researched Bianchi type-V** u Bianchi[20]. Singh and Chaubey [25, 26] acquired the square form of an preferably fluid and viscous liquid Bianchi-V metric structure model, following the research done by Saha [24]. Camci et al. [27] have developed a new technology to manufacture precise EFE solutions with perfect fluid for space-time Bianchis type I & V. In different physical contexts, some other authors [28-29] examined Bianchi type-I&V models. Bali et al [30]

recently reviewed the cosmological model for Bianchi type I and V viscous fluid, Singh and Baghel [32] presented Cosmological models of Bianchi Type I & V with general relativity material deceleration parameter. Baghel and Singh [31] studied the Type-I & V Bianchi universes of large-viscous materials and time varying in gravity and cosmology. Within this paper, we found it worthwhile researching the Bianchi form I and V cosmological pattern in the presence of perfect fluid distribution in time, with cosmological and gravitational constants evolving. The cosmological scenario is studied proposing a law of variance in which the scalar (to) anisotropy (Europa) per expansion unit is proportionate to the factor R scale function. For two different cosmology forms, the exact solutions of Einstein field equations are obtained. $m \neq 3$ and $m = 3$ respectively. Models are represented in depth on physical and kinematical features. The simulations asymptotically isotropy and demonstrate that the universe is distributed evenly, Displaying decelerating and accelerating stages. Observational parameters such as cosmological redshift, time backlog, proper wavelength for both versions, and wavelength of light and angular diameters were discussed. The state finding parameters for both models were also addressed and we concluded they were based on Λ CDM model.

2. Equations System and Field

The line dimension describes a space time of Bianchi type I&V in space uniform, anisotropic

$$ds^2 = -dt^2 + A^2(t)dx^2 + e^{2mx} \{ B^2(t)dy^2 + C^2(t)dz^2 \}, \quad (1)$$

Where metric functions are $A(t)$, $B(t)$, $C(t)$ and m is a constant.

Scope equations for Einstein with ($c = 1$) appear

$$R_{ij} - \frac{1}{2}R_k^k g_{ij} = -8\pi G(t)T_{ij} + \Lambda(t)g_{ij}, \quad (2)$$

where R_{ij} , R_k^k , g_{ij} The tensor is Ricci, the metric tensor and Ricci scalar. $G(t)$, $\Lambda(t)$ Is constant, cosmological constant gravity.

T_{ij} is the celestial material's tensor of energy and momentum

$$T_{ij} = (\rho + p)v_i v_j + p g_{ij}, \quad (3)$$

where the energy and the cosmic pressure are density and p and v_i the four vector velocity are defined by $v_i v_i = -1$.

We consider the State (EoS) equation

$$p = \omega\rho, \tag{4}$$

where ω is a constant and $0 \leq \omega \leq 1$.

Field equations (2) are used in asving coordinates to measure (1) and energy-momentum tensor (3) gives

$$\frac{m^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}\dot{C}}{BC} = 8\pi Gp - \Lambda, \tag{5}$$

$$\frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}\dot{C}}{BC} = 8\pi Gp - \Lambda, \tag{6}$$

$$\frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}\dot{B}}{AB} = 8\pi Gp - \Lambda, \tag{7}$$

$$-\frac{3m^2}{A^2} - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{A}\dot{C}}{AC} = 8\pi G\rho - \Lambda, \tag{8}$$

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{9}$$

Where an overhead dot indicates specific derivatives for cosmic time t , here and everywhere

Instead of the absence of the Einstein tensor, we get

$$8\pi G \left[\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + 8\pi\rho\dot{G} + \dot{\Lambda} = 0 \tag{10}$$

The usual energy conservation equation $T_{i;j}^j = 0$ yields

$$\dot{\rho} + (\rho + p) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) = 0 \tag{11}$$

Equation (10) in conjunction with (11)

$$8\pi\rho\dot{G} + \dot{\Lambda} = 0 \tag{12}$$

Here we can see that G is a constant, whether it is stable or negative. Let R be the average Bianchi –V universe scale factor, that is

$$R^3 = \sqrt{-g} = ABC \tag{13}$$

From (9), we obtain

$$A^2 = BC \quad (14)$$

We will consider (5)–(7) and (14)

$$\frac{\dot{A}}{A} = \frac{\dot{R}}{R} \quad (15)$$

$$\frac{\dot{B}}{B} = \frac{\dot{R}}{R} - \frac{a}{R^3} \quad (16)$$

$$\frac{\dot{C}}{C} = \frac{\dot{R}}{R} + \frac{a}{R^3} \quad (17)$$

Where the integration is constant.

Equations (15)–(17) yield when combined.

$$A = \alpha_1 R \quad (18)$$

$$B = \alpha_2 R \exp\left(-a \int \frac{dt}{R^3}\right) \quad (19)$$

$$C = \alpha_3 R \exp\left(a \int \frac{dt}{R^3}\right) \quad (20)$$

where α_1, α_2 and α_3 are constants of integration such that $\alpha_1\alpha_2\alpha_3 = 1$.

The generalized parameter (q) is defined as

$$q = -1 + \frac{d}{dt} \left(\frac{1}{H} \right) = - \left(1 + \frac{\dot{H}}{H^2} \right) \quad (21)$$

Where,

$$H = \frac{\dot{R}}{R} = \frac{1}{3} (H_1 + H_2 + H_3) \quad (22)$$

Here $H_1 = \dot{A}/A, H_2 = \dot{B}/B = H_3 = \dot{C}/C =$ Hubble directional in the x, y and z direction. Hubble directional Hubble Parameters.

The anisotropy parameter (\bar{A}) is defined as

$$\bar{A} = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \quad (23)$$

We add an extension of volume to scalar shear and scalar as

$$\theta = v^i_{;i} \text{ and } \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij}$$

where σ_{ij} is shear tensor given by

$$\sigma_{ij} = v_{i;j} + \frac{1}{2} (v_{i;k} v^k v_j + v_{j;k} v^k v_i) + \frac{1}{3} v^k_{;k} (g_{ij} + v_i v_j) \quad (24)$$

Expressions for both θ and σ are the result of the bianchi type-V metric.

$$\theta = 3 \frac{\dot{R}}{R} \quad (25)$$

and

$$\sigma^2 = \frac{1}{6} \left\{ \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right)^2 \right\} \quad (26)$$

The Hubble parameter H , shear scalar σ and q deceleration parameters can also be expressed as equations (5)–(8), (11) and (14)

$$\frac{m^2}{R^2} + H^2(2q - 1) - \sigma^2 = 8\pi G\rho - \Lambda \quad (27)$$

$$-\frac{3m^2}{R^2} + 3H^2 - \sigma^2 = 8\pi G\rho - \Lambda \quad (28)$$

And,

$$\dot{\rho} + 3(\rho + p)H = 0 \quad (29)$$

From (28), we obtain

$$3 \frac{\sigma^2}{\theta^2} = 1 - \frac{24\pi G\rho}{\theta^2} - \frac{9m^2}{R^2\theta^2} - \frac{3\Lambda}{\theta^2} \quad (30)$$

Therefore, $0 < \frac{\sigma^2}{\theta^2} < \frac{1}{3}$, $0 < \frac{8\pi G\rho}{\theta^2} < \frac{1}{3}$ for $\Lambda \geq 0$.

The existence of anisotropical diseases decreases the upper limits, whereas a negative disease gives more room.

We're seeing (27) and (28)

$$\frac{d\theta}{dt} = \Lambda - 4\pi G(\rho + 3p) - 2\sigma^2 - \frac{\theta^2}{3} \quad (31)$$

This is Raychaudhuri 's distribution equation. Equation (31) indicates that the expansion rate decreases over time and that the presence of positive decreases while negative Λ increases this rate.

In case of negligible Λ , Let us examine Raychaudhuri 's equation by considering the four velocity vector u^i 's time-like geodesics Let such geodesics be permanent. Then, we have

$$\dot{u}^i = 0 \quad (32)$$

Let t be a parameter of time through a typical geodesic $u^i = \frac{dx^i}{dt}$. Then from (31), for $G = 1$, we have

$$\theta_{,t} u^t = \frac{\partial \theta}{\partial x^t} \cdot \frac{\partial x^t}{\partial t} = \frac{d\theta}{dt} = -\frac{1}{3}\theta^2 - 2\sigma^2 - 4\pi(\rho + 3p) \quad (33)$$

we assume that $2\sigma^2 + 4\pi(\rho + 3p)$ is more than a constant positive $(1/3)\xi^2$. The following differential equation then governs the action of θ

$$\frac{d\theta}{dt} = -\frac{1}{3}(\theta^2 + \xi^2) \quad (34)$$

Equation (34) integrates to give

$$\theta = \theta_0 - \xi \tan \left[\frac{\xi (t - t_0)}{3} \right] \quad (35)$$

where θ_0 is the value of θ at $t = t_0$.

We note from (35) that θ is infinite when t is diminished from t_0 to $t_0 - \frac{3\pi}{2\xi}$. For example, t indicates the correct time along the geodesic lines then reveals that the expansion of the geodesic is happily endless in the past. An endless value of average amounts to a cross and a sort of explosion like a big bang at that time.

3. Solution of Field Equations

We suppose that anisotropy (σ/θ) per unit extension scalar (θ) , in order to obtain the Scalar Factor $R(t)$ value, is commensurate with the Scale factor (R) function. This gives

$$\frac{\sigma/\theta}{\theta} = \frac{b}{R^n} \quad (36)$$

Where the consists of $b (> 0)$ and $n(\text{figure } 0)$.

We get equations (25), (26) and (36)

$$R = \left(\frac{3-n}{6} \sqrt{\frac{a}{b}} t + k \right)^{2/3-n}, n \neq 3 \quad (37)$$

$$= k_0 \exp \left(\frac{1}{3} \sqrt{\frac{a}{b}} t \right), n = 3 \quad (38)$$

where k, k_0 are constants of integration.

4. State finder Parameters

The CDM is regarded as a typical cosmological model at the current level. The basic cosmological observer data supports the Cold Dark Material (CDM) principle. Cosmological constant in general relativity plays the function of Dark energy. When q is , cosmic acceleration and cosmic decline are positive for q in the context of universe expansion. In certain cases, it is difficult to obtain knowledge of the complex characteristics of the state parameter equation. For information on the dynamic properties of the state equation, more order derivatives of the scale factor are needed. These state-finding parameters can effectively differentiate between different kinds of dark power with Radius scale factor second and third derivatives in the universe, and provide a clear diagnosis that a particular model fits into fundamental observer data. The above diagnostic pair has the shape below.

$$r = \frac{\ddot{R}}{RH^3} \quad (39)$$

And,

$$s = \frac{r-1}{3(q-\frac{1}{2})} \quad (40)$$

R is the radius of radius scale, H is the parameter Hubble parameter, and q is the function of deceleration. The dimensional and geometrical parameters r, s are both as they come directly from the cosmic component. The geometric variables are more consistent than physical depending on the model. The parameters of the state-finder are further steps outside of the HUBBLE parameter,

which depend on R and q, which depends on R. For Λ The value $\{r, s\} = 1, 0$ of CDM, state finder parameters.

For the model (42), we obtain

$$r = \frac{(n-1)(n-2)}{2}$$

We have $r = 1$ for $n = 0$. From this, we have $s = 0$.

Thus the approaching $\{r, s\} = \{1, 0\}$ of the CDM model.

We also have $r = 1$ and therefore $s = 0$ for the model.

That is why $\{r, s\} = \{1, 0\}$ follows the CDM model as well.

5. Precise solutions

Case 1: Power-law solution (when $n \neq 0$)

$$A(t) = l_1(nlt + c_1)^{1/n} \exp \left[\frac{X_1}{3l(n-3)} (nlt + c_1)^{(n-3)/n} \right], \quad (41)$$

$$B(t) = l_2(nlt + c_1)^{1/n} \exp \left[\frac{X_2}{3l(n-3)} (nlt + c_1)^{(n-3)/n} \right], \quad (42)$$

$$C(t) = l_3(nlt + c_1)^{1/n} \exp \left[\frac{X_3}{3l(n-3)} (nlt + c_1)^{(n-3)/n} \right]. \quad (43)$$

The scalar function solution ϕ , obtained from

$$\phi = \left[\frac{\phi_0(r+2)}{2l(n-3)} \right]^{2/(r+2)} \cdot (nlt + c_1)^{2(n-3)/n(r+2)}. \quad (44)$$

H1 is known as thermal flow function by means of the measured function values of eqs (41)–(43).

$$h_1 = mX_1(nlt + c_1)^{-3/n}. \quad (45)$$

The solutions listed above are true for $n \neq 3$. The scalars are α and β as dynamics

$$\theta = 3l(nlt + c_1)^{-1} \quad (46)$$

$$\sigma^2 = \frac{(X_1^2 + X_2^2 + X_3^2)}{18} (nlt + c_1)^{-6/n}. \quad (47)$$

The parameters of the directional Hubble are as follows:

$$H_1 = l(nlt + c_1)^{-1} + \frac{X_1}{3} (nlt + c_1)^{-3/n}, \quad (48)$$

$$H_2 = l(nlt + c_1)^{-1} + \frac{X_2}{3} (nlt + c_1)^{-3/n}, \quad (49)$$

$$H_3 = l(nlt + c_1)^{-1} + \frac{X_3}{3} (nlt + c_1)^{-3/n}. \quad (50)$$

Also the average generalized Hubble's parameter is given by

$$H = l(nlt + c_1)^{-1}. \quad (51)$$

The anisotropy parameter Am is given by

$$Am = \frac{(X_1^2 + X_2^2 + X_3^2)}{27l^2} (nlt + c_1)^{2(n-3)/n}. \quad (52)$$

The spatial volume is given as

$$V = (nlt + c_1)^{3/n} \exp(2mx). \quad (53)$$

$$\rho = 3l^2(nlt + c_1)^{-2} + \left[\frac{1}{2} \omega \phi_0^2 - \frac{(X_1^2 + X_2^2 + X_3^2)}{18} \right] (nlt + c_1)^{-6/n} - \frac{3m^2}{l_1^2} (nlt + c_1)^{-2/n} \exp \left[\frac{-2X_1}{3l(n-3)} (nlt + c_1)^{(n-3)/n} \right], \quad (54)$$

$$p = (2n-3)l^2(nlt + c_1)^{-2} + \left[\frac{1}{2} \omega \phi_0^2 - \frac{(X_1^2 + X_2^2 + X_3^2)}{18} \right] \times (nlt + c_1)^{-6/n} + \frac{m^2}{l_1^2} (nlt + c_1)^{-2/n} \times \exp \left[\frac{-2X_1}{3l(n-3)} (nlt + c_1)^{(n-3)/n} \right], \quad (55)$$

provided $n \neq 3$.

From eqs (54) and (55), we find that

$$\frac{dp}{d\rho} = \frac{M}{N}, \quad (56)$$

Where

$$\begin{aligned}
M = & -2nl^3(2n-3)(nlt+c_1)^{-3} \\
& + \left[\frac{(X_1^2 + X_2^2 + X_3^2)l}{3} - 3\omega\phi_0^2l \right] (nlt+c_1)^{-(6/n)-1} \\
& - \frac{2m^2l}{l_1^2} (nlt+c_1)^{-(2/n)-1} \exp \left[\frac{-2X_1}{3l(n-3)} (nlt+c_1)^{(n-3)/n} \right] \\
& - \frac{2m^2X_1}{3l_1^2} (nlt+c_1)^{-5/n} \exp \left[\frac{-2X_1}{3l(n-3)} (nlt+c_1)^{(n-3)/n} \right], \quad (57)
\end{aligned}$$

$$\begin{aligned}
N = & -6nl^3(nlt+c_1)^{-3} \\
& + \left[\frac{(X_1^2 + X_2^2 + X_3^2)l}{3} - 3\omega\phi_0^2l \right] (nlt+c_1)^{-(6/n)-1} \\
& + \frac{6m^2l}{l_1^2} (nlt+c_1)^{-(2/n)-1} \exp \left[\frac{-2X_1}{3l(n-3)} (nlt+c_1)^{(n-3)/n} \right] \\
& + \frac{2m^2X_1}{l_1^2} (nlt+c_1)^{-5/n} \exp \left[\frac{-2X_1}{3l(n-3)} (nlt+c_1)^{(n-3)/n} \right]. \quad (58)
\end{aligned}$$

In order to assess the region for solution 's physical validity, we must have $dp / dc \sim a$ 1. It leads to an incredibly complicated long and complex disparity for the physical truth.

6. CONCLUSION

In this paper we examined **space-time Bianchi of the spatial homogeneous and anisotropic** form I & V with perfect fluid with **G and cosmological term in general**, depending on time. For field equations a special variation law was overcome, **in which the anisotropy** of the field (σ/θ) is a function of the scalar (θ) **scale factor** R by unit expansion. There are two cosmological types in the model. **for $n = 3$ and $n \neq 3$ respectively. Cosmology for $n = 3$** implies the universe's extension of control law, while $n = 3$ reflects the universe's exponential growth. The expansion of energy law reveals the unique pattern, whereby scalar volume and spatial factors disappear at $T = 0$. In the original epoch there is an infinite energy density, pressure and Hubble parameter. As $T \rightarrow \infty$, is different, and μ , p and H are less than zero. The scale of T is different. In the beginning, A^- and σ^2 are very high but decrease with cosmic time and disappear as $T \rightarrow \infty$. In the late period of its creation the model reveals an isotropic state. With rigid fluid $(\omega = 1)$ and radiations $(\omega = 1/3)$, all of **this behavior of the universe is observed**. That is the case It is possible to show that both the rigid fluid and the radiation era, $|\Lambda| = \infty$ at $T = 0$ and $|\Lambda| \rightarrow 0$ as $T \rightarrow \infty$, **which shows that Λ is a decreasing function of cosmic time T** . The exponential solution represents the universe's uniqueness free model. The model represents a uniform expansion in this case and increases exponentially with the time volume; so that we can solve a variety of cosmological

problems. Flattery, horizon, hegemony and so on. All other criteria are physical. \dot{H} , $\dot{\Lambda}$ at $T = 0$ is constant exponentially reduced with cosmic T time and late is negligible except for H , which is constant during evolution. In $|G| = \text{constant at } T = 0 \text{ and } |G| \rightarrow \infty \text{ for large } T$. And G increases over time exponentially. Also, $|\Lambda|$ Consistent at $T = 0$ and decreases in time at late time to cosmological constant both for rigid fluid ($\omega = 1$) and for Radiation. ($\omega = \frac{1}{3}$)eras respectively. There too, the universe isotropic ally isotropic in the model late. It is in line with the feedback. The limitation q gives $dH/dt = 0$. This is the model that gives the maximum value and rapidly expanding pace to the humble parameter of the real universe. We have also addressed state-screening systems and other cosmological distance parameters for cosmology. Λ CDM model. Our model therefore agrees with recent remarks.

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