# System of Rotating Discs with MR-Fluid with respect to Theory of Rotation

**Prof. Mohammad Miyan**
Head, Department of Mathematics,
Shia P. G. College, University of Lucknow, Lucknow,
Uttar Pradesh, India -226020.
Email: miyanmohd@rediffmail.com

**Abstract:**

*The system of rotating discs is analyzed in areas of various engineering fields like as flywheels, turbine engines, gears and others. This paper analyses the motion of a plain rotating disc for the fluid flow, thereby giving an in-depth development of the fluid flow and modeling approach for the second order rotation with MR-fluid. The solution of differential equation for the motion of system of discs with MR-fluid gives the load carrying capacity which numerically gives various solutions. In this analysis with MR-fluid the magnetic field is applying on the system that results the change in the behavior of fluid with variation of intensity of magnetic field. The load capacity varies positively on increasing values of Hartmann number and also with the intensity of magnetic field.*

**Key words:** Hartmann number, Load capacity, Magnetic field, Reynolds equation, Taylor's number, Viscosity.

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## 1. Introduction

The idea of 2-dimensional classical theories of lubrication [2, 3, and 9] changed into given through the equation of Osborne Reynolds [6, 7]. In the wake of an analysis as the results of Beauchamp Tower experiment [22, 23], he had given a differential equation that stated as Reynolds Equation. The simple mechanism and formation of the fluid film changed into discovered via means of that experiment with few assumptions that the fluid film thickness is much smaller than its axial and longitudinal dimensions and if lubricant layer is to supply stress among the bearing and the shaft then the layer will range the thickness of the fluid film. After sometimes, Osborne Reynolds once more revised his differential equation that changed into advanced model and stated as: Generalized Reynolds Equation [4, 9, 10, and 14]. This differential equation relies upon on viscosity, density, film thickness, transverse and longitudinal velocities. The idea of the rotation [2] of the fluid film approximately an axis, which lies throughout the fluid film, offers a few amazing solutions within the lubrication issues of the fluid mechanics. The starting place of rotation changed via means of a few theorems of vorticity within the rotating fluid dynamics.

The rotation induces aspect of vorticity within the course of rotation of fluid film and outcomes springing up from it are predominant, for big Taylor's Number, it effects in streamlines turning into limited to transverse to course of rotation of the fluid film. The today's prolonged model of the Generalized Reynolds Equation [15, 16] is referred to as the Extended Generalized Reynolds Equation [1, 3] that takes into consideration of outcomes of uniform rotation approximately an axis, which lies throughout the fluid film and relies upon on rotation quantity M [3], that is the root of the classical Taylor's Number.

The generalization of the concept of hydrodynamic lubrication is stated because the Rotatory Theory of Hydrodynamic Lubrication [7]. The idea of the Second Order Rotatory Theory of Hydrodynamic Lubrication [4, 11] changed into given via means of keeping expressions containing as much as $2^{nd}$ powers of M and neglecting big powers of M. The lubrication of discs may be made mechanically equal to gears in the event that they have the identical radius at their touch line and rotate on the identical angular velocities that of the gears. For the machine of discs [24, 25], we are able to take the origin on the floor of disc of radius R on the lines of centers of the discs. The geometry of the machine of discs is given by the figure (1.1) and figure (1.2).

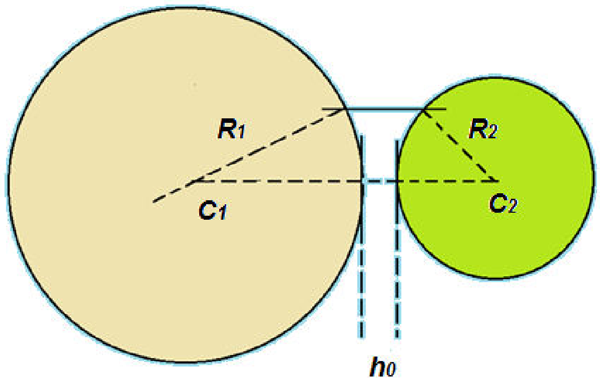


Figure- (1.1) (Geometry of system of discs)

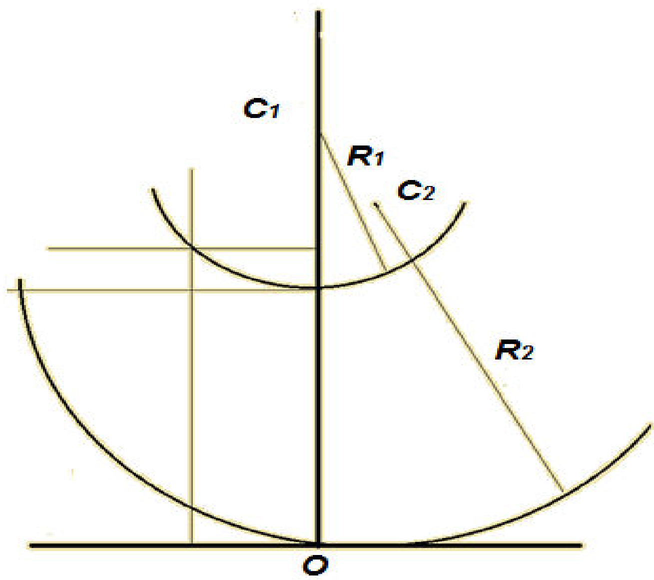


Figure- (1.2) (Film thickness of system of discs)

The fluid film thickness '*h*' will be:

$$h = h_0\left[1 + \frac{y^2}{2h_0}\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right)\right] \quad (1.1)$$

$$\left(\frac{1}{R_1} \mp \frac{1}{R_2}\right) = \frac{1}{R} \quad (1.2)$$

$$tan\theta = \frac{y}{\sqrt{2Rh_0}} \quad (1.3)$$

$$h = h_0 sec^2\,\theta \quad (1.4)$$

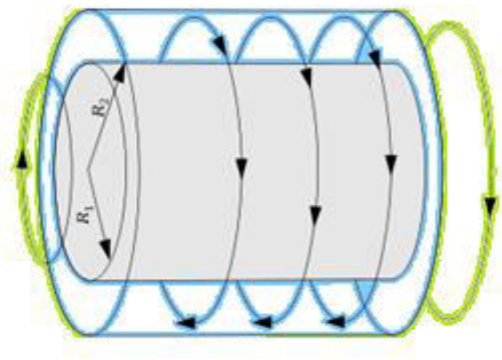


Figure- (1.3) (System of rotation of discs [20]

A magneto rheological fluid (MRF) is a clever fluid whose houses may be controlled with the aid of metallic particles and magnetic fields. These fluids have the capacity transmit energy in a managed way with assistance magnetic subject, which improves their overall performance particularly in regions in which managed fluid motion is needed. The bottom fluid is an inert or non-magnetic carrier fluid wherein metallic debris are suspended. Base fluid must have herbal lubrication and damping properties [1, 5, and 8].

For higher implementation of MRF base fluid generation must have low viscosity and should not change with temperature. This is vital in order that the MRF impact, i.e. the viscosity modifications due to the magnetic area dominant compared to the natural viscosity variation. To apply this generation properly, we need this sort of sort of particle that can be without problems and fast magnetized that is why we use metallic particles. Ferro-particles used in MR-technology are very small. The particle size is approximately inside the order of 1 µm to 7µm [1]. Generally the metallic particles used are carbonyl iron, powdered iron and iron cobalt alloys. Its miles vital to feature certain components to the MR fluid for controlling its houses. Those substances include stabilizers and surfactants. Surfactants are used for discount settling charge of metal debris [12, 13]. whilst the capabilities of additives is to govern viscosity fluids, keep friction between metal particles and ok lessen the charge of fluid thickening because of long-term using fluid and components also increases the lifestyles MR fluid. Typically used components are ferrous oleate and lithium stearate [17, 21].

Rheology is the examination of deformation and flow liquid. Essentially three factors viscosity, shear pressure and strain charges are taken into consideration within the take a look at of glide and deformation. In most fluid packages, there is viscosity a critical function. Within the case of popular drinks, viscosity modifications with a change in other physical homes along with shear stress, temperature, and many others. In the case of MR fluid, the viscosity can be controlled with the help of a magnetic subject. Rheologically, MR fluid can trade from a liquid country to a solid kingdom magnetic discipline power and this transformation is reversible [26-29].

**2. Governing Equations and Boundary Conditions:**

The Extended Generalized Reynolds Equation of the second order rotation, in increasing powers of *M* can expressed as equation (2.1).

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right] \quad (2.1)$$

In the above equation; *x, y* and *z* belongs to the coordinate system, $\mu$ is the dynamic viscosity of the fluid, *U* is the sliding velocity, *P* belongs to pressure, *and* $\rho$ is the density of fluid.

If we suppose that the disc is uniform at the lower part of surface transverse to the fluid film at sliding less point and *U*=+*U* (constant). Also considering that the pressure variation in *x*-direction is too low in comparison to the variation in its perpendicular direction. So the expressions having terms of pressure gradient $\partial p/\partial x$ can be omitted in comparison to the terms having $\partial p/\partial y$ in the final differential equation, so that *P* can be taken as function of only *y*. Taking *h=h(y), U=+U, P=P(y)*;

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]$$
$$-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \quad (2.2)$$

We have

$$M^2=T_a=\frac{4\Omega^2L^2}{\mu^2} \quad (2.3)$$

$$H_a=LB\sqrt{\frac{\sigma}{\mu}} \quad (2.4)$$

Where,

$T_a$ =Taylor's number

$H_a$=Hartmann number

$\Omega$ =Characteristic angular velocity

$L$ =Characteristic length scale perpendicular to the direction of rotation

B=Magnetic field intensity

$\sigma$ =Electric cunductivity

Hence the differential equation for the motion of system in the fluid can be expressed as:

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17\,\Omega^2 H_a^4\rho^2 h^4}{420\mu^2\mathrm{B}^4\sigma^2}\right)\rho\frac{\partial P}{\partial y}\right]=-\frac{d}{dy}\left[\frac{\Omega\mathrm{B}^2\rho^2 U}{\sigma H_a^2}L^3\left\{-\frac{h^3}{12\mu}\left(1-\frac{17\,\Omega^2 H_a^4\rho^2 h^4}{420\mu^2\mathrm{B}^4\sigma^2}\right)\right\}\right] \quad (2.5)$$

The imposed boundary conditions are as follows:

(i) *P=0 at h=h₀ or P=0 at y=0 or P=0 at θ=0*

(ii) *P=dP/dθ=0 at y=y₁ or θ=γ* (say) (2.6)

### 3. Determination of Load Capacity

The solution of the differential equation [18, 19] under the boundary conditions imposed is given by

$$P=-\sqrt{\frac{Rh_0}{2}}\,M\rho U\left[\frac{17\,\Omega^2\rho^2\mathrm{B}^2L^4\,{h_0}^4}{420\mu^2}\,F(\theta)-\tan\theta\,F(\gamma)\right] \quad (3.1)$$

Where $F(\theta)$ is given by

$$F(\theta)=\tan\theta\left[\frac{1}{9}\,sec^8\theta+\frac{8}{63}\,sec^6\theta+\frac{48}{315}\,sec^4\theta+\frac{192}{945}\,sec^2\theta+\frac{384}{945}\right] \quad (3.2)$$

The load capacity for the system of discs can be written as:

$$W=\int_{\gamma}^{0}P\,dy=\int_{\gamma}^{0}P\,sec^2\theta\,\sqrt{2Rh_0}\,d\theta \quad (3.3)$$

$$W=-\frac{17M\rho^3{h_0}^3UR\Omega^2}{420\mu^2}\mathrm{B}^2L^4\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^8\gamma+\frac{8}{315}sec^6\gamma+\frac{16}{315}sec^4\gamma+\frac{192}{945}tan^2\gamma\right)-\frac{Rh_0M\rho U}{2}\,tan^2\gamma \quad (3.4)$$

$$W=-\frac{17M\rho^3{h_0}^3UR\Omega^2}{420\mathrm{B}^2\sigma^2}H_a^4\left(\frac{1}{10}+\frac{1}{90}sec^{10}\gamma+\frac{1}{63}sec^8\gamma+\frac{8}{315}sec^6\gamma+\frac{16}{315}sec^4\gamma+\frac{192}{945}tan^2\gamma\right)-\frac{Rh_0M\rho U}{2}\,tan^2\gamma \quad (3.5)$$

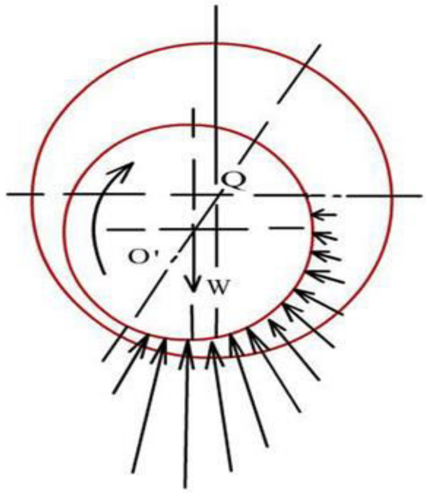


Figure- (4.1) (Load Capacity [19])

**4. Calculation Tables and Graphs:**

The numerical data for variations in the load carrying capacity with respect to magnetic field and Hartmann number can be calculated that is given by the table-4.1 and table-4.2 respectively.

**4.1** Table: 4.1

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = \ 60^0,\ \Omega = 23.88059,\ M = 0.1,\ \sigma = 1, L = 1$

| *S. No.* | *B* | *W* |
|---|---|---|
| 1. | 0.5 | 334740.702 |
| 2. | 1.0 | 1338960.794 |
| 3. | 1.5 | 3012660.948 |
| 4. | 2.0 | 5355841.163 |
| 5. | 2.5 | 8368501.439 |

**4.2** Table 4.2

$U = 80,\ \rho = 1.0,\ R = 3.35,\ h_o = 0.0167,\ \mu = 0.0002,\ \theta = 30^0,\ \gamma = \ 60^0,\ \Omega = 23.88059,\ M = 0.1,\ \sigma = 1, B = 5$

| *S. No.* | $H_a$ | *W* |
|---|---|---|
| 1. | 0 | 0.67134 |
| 2. | 40 | 4.099119 |

| | | |
|---|---|---|
| 3. | 60 | 8.383843 |
| 4. | 80 | 14.382456 |
| 5. | 100 | 22.094959 |

The graphical representations of the variation of pressure with respect to the magnetic field $B$ and Hartmann number $H_a$ are shown by the figure 4.1 and figure 4.2 respectively.

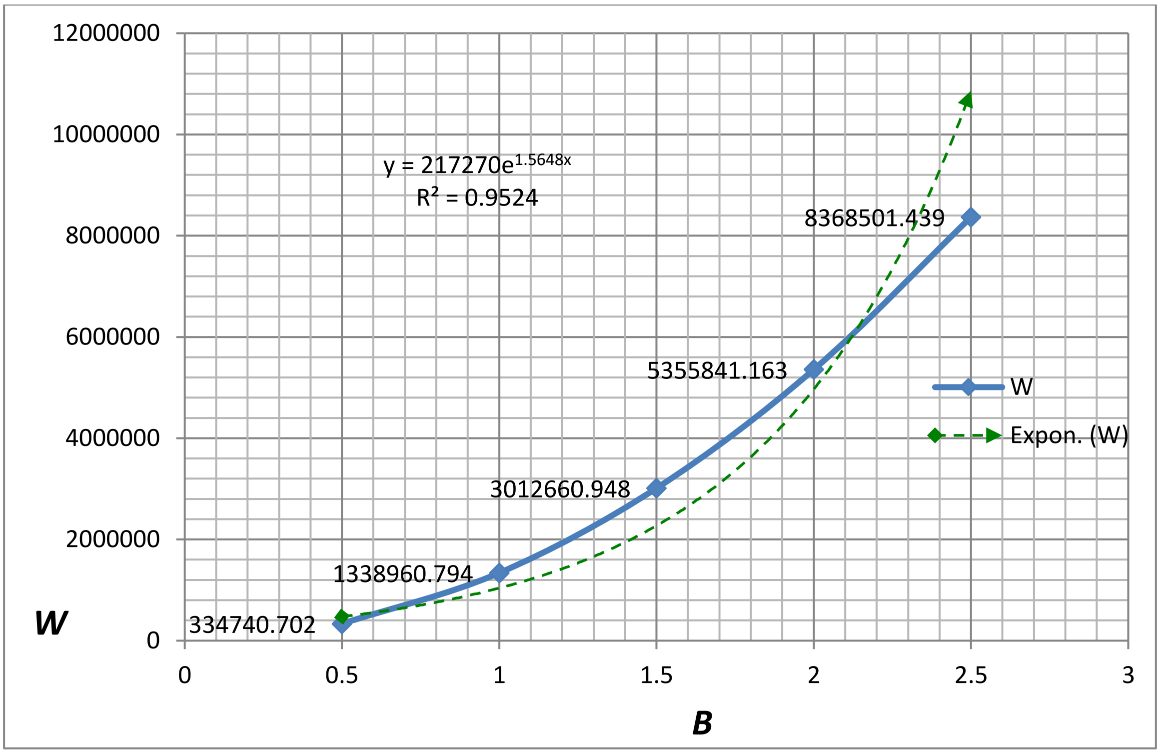


**Figure-4.1** (Variation of ***W*** with respect to Magnetic field ***B*** with exponential trend line)

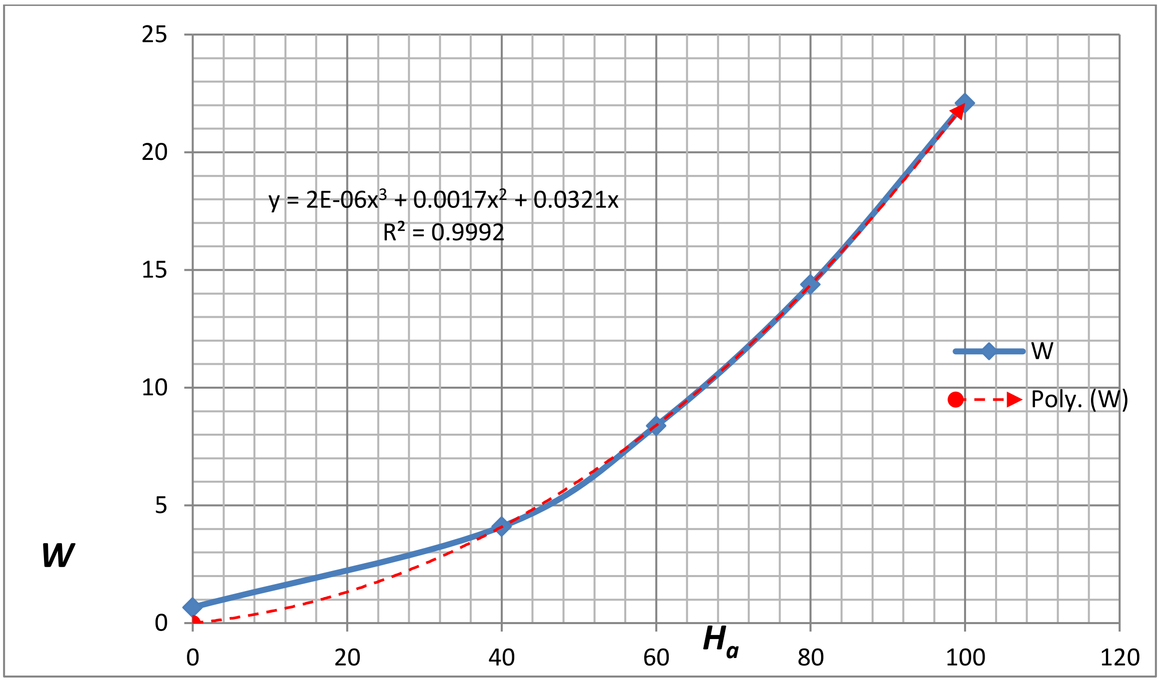


**Figure-4.2** (Variation of ***W*** with respect to Hartmann number $\boldsymbol{H_a}$ with polynomial trend line)

**5. Results and Discussions:**

The variation of load capacity *W* with respect to magnetic field *B* is shown by the table (4.1) and graph (4.1). The figure-4.1 shows the exponential trend line by $y=21727\ e^{1.564x}$ with $R^2=0.947$. The variation of load capacity *W* with respect to Hartmann number $H_a$ by the table (4.2) and graph (4.2) with polynomial trend line $y=0.001x^2+0.032x;\ R^2=0.998$. The figure-4.1, 4.2 indicate that the load capacity *W,* vary with magnetic field and Hartmann number.

**6. Conclusions:**

The solution of the pressure equation, the equation for Load capacity is derived. The numerical values of the Load capacity with respect to magnetic field intensity ***B*** is given in the table and graph representing the Load capacity variation is also demonstrated. The comparative analysis of the Load capacity have been completed by using the geometrical figures, derived equations, numerical values and graphs for lubricating discs with respect to the theory of lubrication. The analysis of equation for Load capacity, table and graphs show that pressure is not independent of dynamic viscosity $\mu$ and increase with increasing values of magnetic field intensity *B*, Hartmann number $H_a$, rotation number *M*, density of used fluid $\rho$, velocity of fluid *U*, characteristic length of the bearing *L* and film thickness $h_0$.

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**8. Conflict of Interests**

The author declare that there is no conflict of interests.

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