Bayesian Estimation for Exponentiated Inverted Weibull distribution under Different Loss Functions

Teena Goyal[a], Piyush Kant Rai[b] and Sandeep K. Maurya[*a]

Department of Mathematics & Statistics, BanasthaliVidyapith[a]
Department of Statistics, Banaras Hindu University [b]

Email: sandeepmaurya.maurya48@gmail.com[*]

Abstract

The aim of the present article is to find the best estimator for the shape and scale parameter of exponentiated inverted Weibull distribution using informative and non-informative prior under squared error, linex and general entropy loss function. The performance of these proposed estimators has been compared on the basis of their simulated risk.

Keywords: Exponentiated inverted Weibull distribution, General entropy loss function, Bayes estimator, M-H algorithm, simulated risk.

1. Introduction

Lifetime distributions are used to describe the real life circumstances in various fields of life like; medical, engineering etc. There are lots of lifetime distributions available in the statistical literature to describe such types of situations. Weibull and exponential distributions are very useful lifetime distributions having some interesting properties. Jiang and Murthy (1999) introduced an approach to characterize the parameters of exponentiated Weibull distribution graphically. Several extensions of Weibull distribution are also made and used in real life applications as Mudholkar and Srivastava (1995) introduced the generalization of the standard Weibull distribution titled as exponentiated Weibull distribution, applied on bus-motor failure data. Mudholkar and Hutson (1996) firstly introduced exponentiated Weibull distribution as a generalization of Weibull family distributions by adding shape parameter. In a similar way, Flaih et al. (2012), introduced a simple generalization of inverted Weibull (IW) and exponentiated inverted exponential (EIE) distribution, named as exponentiated inverted Weibull (EIW) distribution, by adding one more shape parameter exponentially in IW distribution.

Let X be a random variable, said to follow EIW distribution if its probability density function (PDF) is given as

$$f(x;\theta,\beta) = \theta\beta x^{-(\beta+1)}(e^{-x^{-\beta}})^{\theta}; \qquad x>0,(\theta,\beta)>0 \tag{1}$$

and its associated distribution function is given as

$$R(t;\theta,\beta) = 1-(e^{-t^{-\beta}})^{\theta}; \quad t>0,\ (\theta,\beta)>0 \tag{2}$$

Here, θ and β both are shape parameters. Using PDF given in equation (1), if we put $\theta = 1$, then it will be converted to inverse Weibull distribution and if $\beta = 1$ then, it becomes the exponentiated inverted exponential distribution. In general, maximum likelihood estimation is the most widely used method for estimating the parameters of any lifetime distribution. In the recent years, Bayesian approach of estimation has been received great attention from most of the theoreticians and applied researchers. Ahmad et al. (2014) explained the Bayes estimation of EIW distribution under asymmetric loss functions. Hasan and Baizid (2017) obtained Bayes estimates under different loss functions using Gamma prior in case of exponential distribution.

An important element, in the Bayes point estimation problem, is the specification of the loss function. The most popular loss function used in the estimation problem is the squared error loss function (SELF), which can be easily justified on the grounds of minimum variance unbiased estimation. However, the weakness of this loss function is that it is symmetric and gives an equal weight to the overestimation and underestimation of the same magnitude. A number of asymmetric loss functions are also available in the statistical literature, and one of the most widely used asymmetric loss function is the linex loss function (LLF), originally proposed by Varian (1975) and popularized by Zellner(1986). Calabria and Pulcini(1994) have been introduced one more asymmetric loss function, named general entropy loss function (GELF), which is also very popular among the young researchers, nowadays.

The rest of the article is organized as follows: Section 2, describes the technique of Bayes estimation under squared error, linex and general entropy loss function using gamma prior. The techniques of MCMC simulation by Metropolis-Hastings algorithm is discussed in Section 3. In Section 4, Bayes interval i.e. highest posterior density interval is discussed. Section 5 deals with the simulation study which is done on 10,000 samples from PDF given in equation (1). Conclusion is presented in the last section of the article.

2. Bayes Estimation

A statistical estimation problem is a special kind of statistical decision problem in which the decision made by the investigator is the estimate of the value of an unknown parameter of the model. For the estimation of parameters θ and β of EIW distribution, it is assumed that both are independent gamma variates, therefore

$$\pi(\theta) = \frac{a^b}{\Gamma b} e^{-a\theta}\theta^{b-1}; a, b, \theta > 0 \tag{3}$$

$$\pi(\beta) = \frac{c^d}{\Gamma d} e^{-c\beta}\beta^{d-1}; c, d, \beta > 0 \tag{4}$$

Thus, joint prior of θ and β is given by

$$\pi(\theta,\beta) = \frac{a^b c^d}{\Gamma b \Gamma d} e^{-(a\theta+c\beta)}\theta^{b-1}\beta^{d-1}; (a,b,c,d) > 0, (\theta,\beta) > 0 \tag{5}$$

The formula for the joint posterior of θ and β is given by

$$\Pi(\theta,\beta|x) = \frac{\pi(\theta,\beta)L(\theta,\beta|x)}{\int_{\forall\theta}\pi(\theta,\beta)L(\theta,\beta|x)d\theta d\beta} \tag{6}$$

where $L(\theta,\beta|x)$ is the likelihood function of the EIW distribution, given as follows

$$L(\theta,\beta|x) = (\theta\beta)^n \prod_{i=1}^{n} x_i^{-(\beta+1)} \prod_{i=1}^{n} (e^{-x_i^{-\beta}})^{\theta}; x, \theta, \beta > 0 \tag{7}$$

For the Bayes estimation techniques, loss function plays very important role. Here, in this article, we are using three different types of loss functions, as mention above.

2.1 Bayes estimation under squared error loss function

The squared error loss function (SELF) is very famous symmetric type of loss function, which gives same weight to under and over estimation problem. The mathematical form of SELF is given as

$$L(\hat{\alpha},\alpha)=(\hat{\alpha}-\alpha)^2 \tag{8}$$

Where $\hat{\alpha}$ is the Bayes estimator of parameter α of any given lifetime model. The Bayes estimator under SELF is nothing but the posterior mean. The squared error loss function is widely used by Box and Tiao (1973), Sinha and Kale (1980), Martz and Waller (1982) and Berger (2013). After, some time researchers Zellner and Geisel (1968), Varian (1975), Aitchison and Dunsmore (1980) and Ferguson (2014) pointed out that the use of symmetric loss function is inappropriate in some situations. There may be a situation where a negative error may be more serious than positive error or vice-versa. For example in dam construction, underestimation of peak of water level is more serious than overestimation. In the same fashion, in reliability context, the overestimation of time of blast of bomb is more serious than underestimation. This leads to statistician to think about asymmetric loss function in which under and over estimation have different weights. Varian (1975) suggested an asymmetric loss function, which is linex loss function.

2.2 Bayes estimation under linex loss function

The mathematical form of LLF is given as

$$L(\hat{\alpha},\alpha)=e^{\delta(\hat{\alpha}-\alpha)}-\delta(\hat{\alpha}-\alpha)-1 \tag{9}$$

where $\delta\neq 0$. Here, constant δ determines the shape of the loss function. Sign of the constant shows the direction of asymmetry and magnitude of the shape parameter shows the degree of asymmetry. If δ >0, the overestimation is more serious than underestimation and vice-versa. For δ closed to 0, the linex loss is approximately squared error loss and therefore almost symmetric.Under the linex loss, Bayes estimator of θ is which estimator that minimizes the expected loss. Therefore, the Bayes estimator under linex loss is given by

$$\hat{\alpha}_L=\frac{-1}{\delta}E_\alpha(e^{-\delta\alpha}) \tag{10}$$

Zellner and Geisel (1968) used linex loss for estimating the mean of normal distribution and Basu and Ebrahimi(1991) used linex loss in estimation of reliability under exponential distribution.

2.3 Bayes estimation under general entropy loss function

Among the asymmetric loss functions available in statistical literature, general entropy loss function (GELF) is the frequently used loss function. It was introduced by Calabria and Pulcini(1994) and its mathematical form is given as

$$L(\hat{\alpha},\alpha)=\left(\frac{\hat{\alpha}}{\alpha}\right)^{\delta_1}-\delta_1\log\left(\frac{\hat{\alpha}}{\alpha}\right)-1 \tag{11}$$

Where, δ_1 is the shape parameter of general entropy loss function. The sign and magnitude of the shape parameter δ_1, defines the direction and degree of asymmetry respectively. Also, when δ_1 >0, a positive error cause more serious consequence than negative error. This loss function is the generalization of entropy loss function, as put δ_1 = 1, it gives entropy loss function. The Bayes estimator under GELF is given by

$$\hat{\theta}_G = \left[E_\theta\left(\theta^{-\delta_1}|X\right)\right]^{-1/\delta_1} \tag{12}$$

It is easy to see that, when put δ_1=1, it gives Bayes estimator under weighted SELF and when put δ_1 = -1, it gives Bayes estimator under SELF.

Since, it is not possible to solve integrations involved in posterior distribution and Bayes estimator analytically; we perform Monte Carlo Markov Chain (MCMC) simulation technique.

3. Metropolis-Hastings Algorithm

MCMC is a powerful technique for solving integration by using simulation. This technique has been used in Bayesian inference nowadays in a regular basis. Metropolis et al. (1953) used MCMC first time in statistics to study the equation of a state of a two-dimensional rigid sphere system. Metropolis-Hastings algorithm is a generalization of basic algorithm originated by Metropolis and Ulam(1949). Hastings (1970) generalized the original method by allowing arbitrary proposal distribution and this method become very famous method, Metropolis-Hastings algorithm. A general way of constructing a MCMC sample was given by Metropolis-Hastings (M-H) algorithm. Suppose, we wish to simulate a sample of size n from a posterior density $\pi(\alpha \mid x)$. The Metropolis-Hastings algorithm can be start with the following iterative steps:

1. Set an initial guess value say α^0
2. For t = 1, 2, ⋯, N.
3. Simulate a candidate value $\theta^*$from a proposal density $p(\alpha^*| \alpha)$.
4. Calculate the probability $p_r$that the candidate value α^*will be accepted as the next value in the sequence

$$p_r = min\left(1, \frac{g(\alpha^*|x)p(\alpha|\alpha^*)}{g(\alpha|x)p(\alpha^*|\alpha)}\right).$$

5. Sample a value α^t such that $\alpha^t = \alpha^*$ with probability p_r; otherwise $\alpha^t = \alpha^{t-1}$.

A problem for the choice of initial guess in the proposed algorithm is arises. Contour plot technique can be used for guess value in MCMC process. Also, the choice of variance is an important point; one can follow Ntzoufras (2011) and Gamerman and Lopes (2006) for more details.

4. Highest posterior density interval

Recently in Bayesian inference, researchers summarizes the marginal posterior distribution by $100(1-\alpha)\%$ posterior credible intervals for the parameter of interest. These credible intervals can be easily obtained by analytical solution or MCMC simulation technique (see Edwards et al. (1963)). The HPD credible interval (see Chen and Shao (1999)) of the parameter α is obtained, say for example on the basis of ordered MCMC samples of α as $\alpha_{(1)}, \alpha_{(2)}, \cdots, \alpha_{(N)}$. After that $100(1-\lambda)\%$ credible interval for the parameter α is obtained as $\left((\alpha_{(1)}, \alpha_{[(1-\lambda)N]+1}), \cdots, (\alpha_{[N\lambda]}, \alpha_N)\right)$. Where $[Y]$ denotes the largest integer less than or equal to Y. The shortest interval among all of the Bayesian credible intervals is called highest posterior density (HPD) interval. Box and Tiao (1973) has been discussed that, a HPD interval has two main properties as: The density of any point within interval is either equal to or greater than the density of any point outside the interval.

1. The density of any point within interval is either equal to or greater than the density of any point outside the interval.

2. For a given probability say $(1-\lambda)$, the interval is of shortest length.

5. Simulation study

In order to compare the Bayes estimators under different loss functions with gamma prior, we use the Monte Carlo simulation technique in the following way:

In the simulation study, we have used gamma prior *Gamma*(*a*,*b*), where we have taken the values of hyper parameters a and b in two ways; firstly, we considered gamma prior as non-informative or flat prior by taking $a = 0.1$, $b = 0.1$; secondly, we have taken $a = 1$, $b = 4$, that means gamma prior as informative prior. For purpose of Bayesian estimation, three different loss functions, both symmetric and asymmetric have been used. In symmetric loss function, squared error loss function (SELF) and in asymmetric loss function, linex loss function (LLF) and general entropy loss functions (GELF) have been used. In case of LLF, value of the shape parameter δ, we have taken both positive ($\delta = 0.5, 2$ & 3.5) as well as negative ($\delta = -0.5, -2$ & -3.5) value as its sign shows the direction of asymmetry (discussed above in topic 2.2). In the similar fashion, in case of GELF, we have taken three different values for positive ($\delta_1 = 0.5, 2$ & 3.5) and three different values for negative ($\delta_1 = -0.5, -2$ & -3.5) case for the shape parameter of GELF.

In this section, simulation technique is performed to compare the method of estimation on the basis of Bayes risk. For this purpose, we have taken sample size variations as $n = 10, 30, 50$ and 80 to represent small, moderate and large sample sizes from EIW distribution with the various combinations of scale and shape parameter θ and β = (1,2), (2,2), (3,2) and (2,1) with replication number 10,000.

In the following resulting tables (Table 1-10), $\hat{\theta}_S$, $\hat{\theta}_{Li}$ and $\hat{\theta}_{Gi}$ stands for Bayes estimates under SELF, LLF and GELF respectively. R_S, $R_{\theta Li}$ and $R_{\theta Gi}$ stands for the Bayes risk of estimates of θ under SELF, LLF and GELF respectively. Here, $i = 1, 2, \cdots, 6$ as δ and δ_1 taking 6 different values for LLF and GELF. Similar notations are used for the estimates and Bayes risk for parameter β. Also, in Table 9 and 10, $\text{hpd}_{\theta LL}$ and $\text{hpd}_{\theta UL}$ stands for the lower limit and upper limit of the HPD interval for θ respectively. Similar notations used for the parameter β.

Table 1: Bayes estimators of θ under different loss functions for various values of (θ,β) and n, when prior is $Gamma(1,4)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	$\hat{\theta}_S$	$\hat{\theta}_{L1}$	$\hat{\theta}_{L2}$	$\hat{\theta}_{L3}$	$\hat{\theta}_{L4}$	$\hat{\theta}_{L5}$	$\hat{\theta}_{L6}$	$\hat{\theta}_{G1}$	$\hat{\theta}_{G2}$	$\hat{\theta}_{G3}$	$\hat{\theta}_{G4}$	$\hat{\theta}_{G5}$	$\hat{\theta}_{G6}$
10	1,2	0.8542	0.8369	0.7906	0.7508	0.8725	0.9361	1.0180	0.7940	0.7329	0.6703	0.8342	0.8940	0.9539
	2,2	1.1355	1.1054	1.0268	0.9620	1.1682	1.2853	1.4392	1.0548	0.9727	0.8889	1.1088	1.1887	1.2677
	3,2	1.2682	1.2297	1.1316	1.0526	1.3105	1.4674	1.6813	1.1752	1.0811	0.9854	1.2373	1.3297	1.4213
	2,1	1.1878	1.1545	1.0687	0.9985	1.2239	1.3554	1.5301	1.1031	1.0169	0.9288	1.1597	1.2436	1.3264
30	1,2	0.9598	0.9517	0.9287	0.9073	0.9682	0.9952	1.0272	0.9349	0.9100	0.8850	0.9515	0.9766	1.0022
	2,2	1.5936	1.5717	1.5108	1.4563	1.6164	1.6909	1.7762	1.5525	1.5113	1.4698	1.5799	1.6208	1.6615
	3,2	2.0008	1.9630	1.8614	1.7740	2.0410	2.1786	2.3461	1.9439	1.8869	1.8299	1.9818	2.0388	2.0957
	2,1	1.6326	1.6096	1.5457	1.4886	1.6566	1.7355	1.8261	1.5906	1.5483	1.5058	1.6186	1.6605	1.7022
50	1,2	0.9807	0.9754	0.9602	0.9459	0.9861	1.0036	1.0246	0.9648	0.9490	0.9334	0.9753	0.9914	1.0082
	2,2	1.7423	1.7264	1.6809	1.6387	1.7587	1.8106	1.8672	1.7150	1.6877	1.6602	1.7332	1.7604	1.7876
	3,2	2.3045	2.2728	2.1853	2.1074	2.3376	2.4467	2.5727	2.2633	2.2221	2.1810	2.2907	2.3319	2.3732
	2,1	1.7609	1.7446	1.6982	1.6552	1.7776	1.8308	1.8888	1.7334	1.7058	1.6780	1.7517	1.7792	1.8066
80	1,2	0.9884	0.9849	0.9748	0.9652	0.9920	1.0039	1.0190	0.9780	0.9677	0.9576	0.9849	0.9956	1.0070
	2,2	1.8323	1.8213	1.7893	1.7588	1.8436	1.8784	1.9152	1.8144	1.7964	1.7783	1.8264	1.8443	1.8622
	3,2	2.5228	2.4983	2.4292	2.3656	2.5479	2.6286	2.7177	2.4938	2.4648	2.4359	2.5131	2.5421	2.5711
	2,1	1.8520	1.8407	1.8079	1.7766	1.8635	1.8993	1.9373	1.8338	1.8155	1.7972	1.8459	1.8641	1.8822

Table2: Bayes estimators of β under different loss functions for various values of (θ,β) and n, when prior is $Gamma(1,4)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	$\hat{\beta}_S$	$\hat{\beta}_{L1}$	$\hat{\beta}_{L2}$	$\hat{\beta}_{L3}$	$\hat{\beta}_{L4}$	$\hat{\beta}_{L5}$	$\hat{\beta}_{L6}$	$\hat{\beta}_{G1}$	$\hat{\beta}_{G2}$	$\hat{\beta}_{G3}$	$\hat{\beta}_{G4}$	$\hat{\beta}_{G5}$	$\hat{\beta}_{G6}$
10	1,2	1.4850	1.4452	1.3394	1.2504	1.5276	1.6740	1.8491	1.4030	1.3174	1.2272	1.4581	1.5380	1.6151
	2,1	0.8214	0.8089	0.7743	0.7433	0.8344	0.8776	0.9298	0.7766	0.7302	0.6818	0.8066	0.8507	0.8947
	3,2	1.1928	1.1666	1.0959	1.0347	1.2204	1.3137	1.4250	1.1260	1.0566	0.9838	1.1708	1.2361	1.2994
	2,2	1.3186	1.2862	1.1998	1.1264	1.3530	1.4707	1.6134	1.2439	1.1663	1.0850	1.2940	1.3669	1.4377
30	1,2	1.8345	1.8159	1.7624	1.7121	1.8535	1.9131	1.9767	1.8040	1.7730	1.7415	1.8244	1.8546	1.8843
	2,1	0.9372	0.9320	0.9172	0.9030	0.9425	0.9594	0.9796	0.9210	0.9048	0.8884	0.9318	0.9481	0.9649
	3,2	1.6024	1.5878	1.5456	1.5060	1.6173	1.6640	1.7138	1.5749	1.5471	1.5188	1.5933	1.6205	1.6474
	2,2	1.7236	1.7065	1.6575	1.6116	1.7410	1.7958	1.8545	1.6938	1.6636	1.6330	1.7137	1.7432	1.7723
50	1,2	1.8993	1.8877	1.8537	1.8210	1.9111	1.9473	1.9849	1.8810	1.8625	1.8437	1.8932	1.9115	1.9295
	2,2	1.8339	1.8227	1.7900	1.7586	1.8452	1.8801	1.9163	1.8156	1.7971	1.7785	1.8278	1.8460	1.8640
	3,2	1.7383	1.7283	1.6990	1.6708	1.7484	1.7795	1.8119	1.7210	1.7036	1.6860	1.7325	1.7497	1.7667
	2,1	0.9651	0.9618	0.9524	0.9434	0.9685	0.9795	0.9934	0.9552	0.9454	0.9356	0.9618	0.9719	0.9827
80	1,2	1.9426	1.9352	1.9131	1.8916	1.9502	1.9731	1.9965	1.9311	1.9195	1.9079	1.9388	1.9503	1.9617
	2,2	1.8937	1.8865	1.8651	1.8443	1.9010	1.9232	1.9460	1.8822	1.8707	1.8591	1.8899	1.9013	1.9126
	3,2	1.8266	1.8199	1.8001	1.7807	1.8334	1.8541	1.8752	1.8156	1.8045	1.7933	1.8230	1.8340	1.8449
	2,1	0.9791	0.9770	0.9708	0.9649	0.9814	0.9891	0.9998	0.9727	0.9665	0.9604	0.9770	0.9837	0.9911

Table 3: Bayes risk of estimators of θ under different loss functions for variations in (θ, β) at different sample size n, when prior is $Gamma(1, 4)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	R_S	$R_{\theta L1}$	$R_{\theta L2}$	$R_{\theta L3}$	$R_{\theta L4}$	$R_{\theta L5}$	$R_{\theta L6}$	$R_{\theta G1}$	$R_{\theta G2}$	$R_{\theta G3}$	$R_{\theta G4}$	$R_{\theta G5}$	$R_{\theta G6}$
10	1,2	0.0361	0.0048	0.0905	0.3118	0.0042	0.0579	0.1648	0.0091	0.1883	0.7026	0.0089	0.0823	0.1712
	2,2	0.7553	0.0871	1.0886	2.6552	0.1017	1.8967	5.6216	0.0471	0.6848	1.9060	0.0510	0.8463	2.6394
	3,2	3.0002	0.2976	2.7580	5.8122	0.4838	7.9184	9.3160	0.0949	1.1729	2.9187	0.1163	2.5106	9.4547
	2,1	0.6721	0.0787	1.0167	2.5278	0.0889	1.5522	4.3236	0.0416	0.6208	1.7652	0.0450	0.7103	2.1504
30	1,2	0.0193	0.0024	0.0385	0.1186	0.0024	0.0389	0.1171	0.0030	0.0519	0.1715	0.0036	0.0413	0.1145
	2,2	0.1920	0.0240	0.3718	1.0734	0.0239	0.3675	1.0250	0.0091	0.1471	0.4513	0.0097	0.1388	0.4054
	3,2	1.0288	0.1161	1.3861	3.3092	0.1423	2.9813	9.6967	0.0230	0.3326	0.9251	0.0251	0.4315	1.4423
	2,1	0.1686	0.0211	0.3346	0.9858	0.0209	0.3188	0.8921	0.0080	0.1302	0.4041	0.0086	0.1194	0.3461
50	1,2	0.0135	0.0017	0.0266	0.0809	0.0017	0.0272	0.0813	0.0019	0.0319	0.1019	0.0024	0.0279	0.0802
	2,2	0.0957	0.0120	0.1945	0.5917	0.0119	0.1818	0.5213	0.0041	0.0671	0.2096	0.0045	0.0617	0.1808
	3,2	0.5233	0.0621	0.8412	2.1836	0.0686	1.2246	3.8775	0.0105	0.1599	0.4666	0.0113	0.1792	0.5669
	2,1	0.0901	0.0113	0.1830	0.5586	0.0112	0.1725	0.4994	0.0038	0.0627	0.1964	0.0043	0.0576	0.1688
80	1,2	0.0092	0.0011	0.0182	0.0553	0.0012	0.0183	0.0540	0.0012	0.0204	0.0643	0.0015	0.0187	0.0545
	2,2	0.0533	0.0067	0.1093	0.3378	0.0066	0.1019	0.2971	0.0021	0.0343	0.1075	0.0024	0.0317	0.0933
	3,2	0.2680	0.0328	0.4859	1.3591	0.0341	0.5616	1.6856	0.0050	0.0782	0.2352	0.0054	0.0808	0.2485
	2,1	0.0489	0.0062	0.1003	0.3111	0.0061	0.0940	0.2768	0.0019	0.0311	0.0978	0.0022	0.0286	0.0843

Table 4: Bayes risk of estimators of β under different loss functions for variations in (θ, β) at different sample size n, when prior is $Gamma(1, 4)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	R_S	$R_{\beta L1}$	$R_{\beta L2}$	$R_{\beta L3}$	$R_{\beta L4}$	$R_{\beta L5}$	$R_{\beta L6}$	$R_{\beta G1}$	$R_{\beta G2}$	$R_{\beta G3}$	$R_{\beta G4}$	$R_{\beta G5}$	$R_{\beta G6}$
10	1,2	0.3081	0.0389	0.6057	1.7109	0.0378	0.5372	1.3615	0.0175	0.2963	0.9410	0.0167	0.2432	0.6592
	2,2	0.4975	0.0595	0.8147	2.1166	0.0646	1.0968	3.2248	0.0288	0.4434	1.3024	0.0293	0.4741	1.4343
	3,2	0.6776	0.0780	0.9816	2.4224	0.0919	1.8371	6.5959	0.0401	0.5770	1.6047	0.0430	0.7625	2.5496
	2,1	0.0534	0.0068	0.1115	0.3475	0.0066	0.1000	0.2775	0.0117	0.2080	0.6948	0.0108	0.1495	0.3789
30	1,2	0.0710	0.0090	0.1477	0.4628	0.0088	0.1353	0.3978	0.0029	0.0488	0.1566	0.0028	0.0422	0.1213
	2,2	0.1083	0.0136	0.2189	0.6627	0.0134	0.2062	0.5894	0.0047	0.0777	0.2427	0.0046	0.0715	0.2093
	3,2	0.1811	0.0222	0.3301	0.9334	0.0231	0.3861	1.2048	0.0081	0.1262	0.3766	0.0082	0.1328	0.4107
	2,1	0.0162	0.0020	0.0331	0.1026	0.0020	0.0312	0.0891	0.0026	0.0434	0.1398	0.0025	0.0365	0.1021
50	1,2	0.0416	0.0052	0.0849	0.2634	0.0052	0.0809	0.2413	0.0015	0.0254	0.0803	0.0015	0.0231	0.0678
	2,2	0.0536	0.0068	0.1103	0.3415	0.0066	0.1022	0.2970	0.0021	0.0347	0.1091	0.0021	0.0319	0.0939
	3,2	0.0874	0.0109	0.1699	0.5046	0.0110	0.1759	0.5306	0.0036	0.0569	0.1738	0.0036	0.0566	0.1719
	2,1	0.0095	0.0012	0.0192	0.0591	0.0012	0.0184	0.0528	0.0014	0.0227	0.0719	0.0013	0.0203	0.0583
80	1,2	0.0240	0.0030	0.0486	0.1504	0.0030	0.0473	0.1431	0.0008	0.0136	0.0427	0.0008	0.0128	0.0380
	2,2	0.0300	0.0038	0.0617	0.1916	0.0037	0.0579	0.1709	0.0011	0.0180	0.0564	0.0011	0.0168	0.0499
	3,2	0.0451	0.0056	0.0903	0.2748	0.0056	0.0887	0.2652	0.0017	0.0277	0.0855	0.0017	0.0270	0.0814
	2,1	0.0059	0.0007	0.0119	0.0365	0.0007	0.0114	0.0325	0.0008	0.0132	0.0414	0.0008	0.0122	0.0353

Table5: Bayesestimatorsofθunderdifferentlossfunctionsforvariousvaluesof$(\theta,\beta)$and$n$, when prior is $Gamma(0.1,0.1)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	$\hat{\theta}_S$	$\hat{\theta}_{L1}$	$\hat{\theta}_{L2}$	$\hat{\theta}_{L3}$	$\hat{\theta}_{L4}$	$\hat{\theta}_{L5}$	$\hat{\theta}_{L6}$	$\hat{\theta}_{G1}$	$\hat{\theta}_{G2}$	$\hat{\theta}_{G3}$	$\hat{\theta}_{G4}$	$\hat{\theta}_{G5}$	$\hat{\theta}_{G6}$
10	1,2	1.0981	1.0600	0.9689	0.8980	1.1462	1.3173	1.5129	1.0056	0.9117	0.8160	1.0674	1.1594	1.2513
	2,2	2.4132	2.1938	1.8304	1.6105	2.9084	4.2251	4.9881	2.1804	1.9544	1.7343	2.3348	2.5724	2.8164
	3,2	3.7718	3.1814	2.4398	2.0647	5.6229	8.9542	10.1876	3.3122	2.8878	2.4967	3.6146	4.0983	4.6124
	2,1	2.4029	2.1872	1.8266	1.6075	2.8878	4.1774	4.9364	2.1724	1.9482	1.7293	2.3253	2.5606	2.8023
30	1,2	1.0283	1.0183	0.9900	0.9638	1.0386	1.0720	1.1107	0.9996	0.9709	0.9420	1.0187	1.0475	1.0766
	2,2	2.1037	2.0615	1.9495	1.8542	2.1490	2.3080	2.4942	2.0459	1.9879	1.9296	2.0845	2.1421	2.1996
	3,2	3.2056	3.0816	2.7931	2.5787	3.3544	3.9679	4.5420	3.0942	2.9838	2.8746	3.1683	3.2805	3.3938
	2,1	2.0945	2.0527	1.9418	1.8471	2.1392	2.2961	2.4796	2.0370	1.9792	1.9212	2.0753	2.1326	2.1898
50	1,2	1.0141	1.0081	0.9912	0.9751	1.0201	1.0395	1.0623	0.9968	0.9797	0.9627	1.0083	1.0257	1.0436
	2,2	2.0538	2.0307	1.9659	1.9068	2.0776	2.1545	2.2403	2.0208	1.9876	1.9544	2.0428	2.0757	2.1086
	3,2	3.1123	3.0457	2.8714	2.7252	3.1846	3.4440	3.7431	3.0497	2.9872	2.9249	3.0914	3.1542	3.2172
	2,1	2.0555	2.0324	1.9673	1.9078	2.0795	2.1567	2.2427	2.0224	1.9891	1.9557	2.0445	2.0776	2.1105
80	1,2	1.0099	1.0061	0.9953	0.9850	1.0137	1.0263	1.0418	0.9989	0.9882	0.9775	1.0062	1.0174	1.0292
	2,2	2.0342	2.0203	1.9803	1.9425	2.0483	2.0925	2.1394	2.0140	1.9937	1.9734	2.0274	2.0476	2.0678
	3,2	3.0731	3.0336	2.9240	2.8252	3.1143	3.2502	3.4050	3.0351	2.9970	2.9590	3.0604	3.0984	3.1365
	2,1	2.0335	2.0195	1.9792	1.9411	2.0477	2.0921	2.1394	2.0131	1.9927	1.9722	2.0267	2.0470	2.0672

Table6: Bayesestimatorsofβunderdifferentlossfunctionsforvariousvaluesof$(\theta,\beta)$and$n$, when prior is $Gamma(0.1,0.1)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	$\hat{\beta}_S$	$\hat{\beta}_{L1}$	$\hat{\beta}_{L2}$	$\hat{\beta}_{L3}$	$\hat{\beta}_{L4}$	$\hat{\beta}_{L5}$	$\hat{\beta}_{L6}$	$\hat{\beta}_{G1}$	$\hat{\beta}_{G2}$	$\hat{\beta}_{G3}$	$\hat{\beta}_{G4}$	$\hat{\beta}_{G5}$	$\hat{\beta}_{G6}$
10	1,2	2.2798	2.1929	1.9735	1.7996	2.3753	2.7225	3.1010	2.1674	2.0495	1.9243	2.2428	2.3521	2.4576
	2,2	2.2605	2.1757	1.9608	1.7901	2.3537	2.6936	3.0707	2.1495	2.0330	1.9097	2.2240	2.3321	2.4365
	3,2	2.2300	2.1496	1.9433	1.7772	2.3174	2.6302	2.9817	2.1222	2.0089	1.8887	2.1946	2.2994	2.4003
	2,1	1.1394	1.1173	1.0569	1.0038	1.1626	1.2397	1.3300	1.0833	1.0246	0.9626	1.1209	1.1757	1.2289
30	1,2	2.0885	2.0657	2.0002	1.9390	2.1119	2.1855	2.2645	2.0558	2.0226	1.9889	2.0777	2.1100	2.1419
	2,2	2.0764	2.0538	1.9890	1.9285	2.0996	2.1724	2.2506	2.0439	2.0108	1.9773	2.0656	2.0978	2.1296
	3,2	2.0746	2.0520	1.9874	1.9270	2.0976	2.1701	2.2478	2.0421	2.0091	1.9756	2.0638	2.0959	2.1276
	2,1	1.0443	1.0384	1.0213	1.0049	1.0504	1.0694	1.0907	1.0276	1.0108	0.9938	1.0388	1.0555	1.0724
50	1,2	2.0474	2.0344	1.9963	1.9596	2.0606	2.1013	2.1435	2.0284	2.0091	1.9897	2.0411	2.0600	2.0788
	2,2	2.0468	2.0338	1.9958	1.9592	2.0600	2.1006	2.1428	2.0278	2.0085	1.9892	2.0405	2.0594	2.0781
	3,2	2.0447	2.0318	1.9939	1.9575	2.0579	2.0983	2.1404	2.0258	2.0066	1.9873	2.0384	2.0573	2.0760
	2,1	1.0256	1.0221	1.0121	1.0024	1.0292	1.0406	1.0542	1.0157	1.0058	0.9959	1.0223	1.0324	1.0429
80	1,2	2.0277	2.0198	1.9964	1.9734	2.0357	2.0601	2.0850	2.0160	2.0042	1.9923	2.0238	2.0355	2.0471
	2,2	2.0315	2.0235	2.0001	1.9773	2.0395	2.0638	2.0888	2.0198	2.0080	1.9962	2.0276	2.0392	2.0508
	3,2	2.0283	2.0203	1.9970	1.9741	2.0362	2.0606	2.0855	2.0166	2.0048	1.9930	2.0244	2.0360	2.0476
	2,1	1.0180	1.0158	1.0094	1.0033	1.0203	1.0281	1.0383	1.0116	1.0054	0.9993	1.0159	1.0225	1.0297

Table 7: Bayes risk of estimators of θ under different loss functions for variations in (θ, β) at different sample size n, when prior is $Gamma(0.1, 0.1)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	R_S	$R_{\theta L1}$	$R_{\theta L2}$	$R_{\theta L3}$	$R_{\theta L4}$	$R_{\theta L5}$	$R_{\theta L6}$	$R_{\theta G1}$	$R_{\theta G2}$	$R_{\theta G3}$	$R_{\theta G4}$	$R_{\theta G5}$	$R_{\theta G6}$
10	1,2	0.1098	0.0126	0.1582	0.3921	0.0149	0.2938	1.0329	0.0143	0.2425	0.7757	0.0186	0.1931	0.5191
	2,2	0.6468	0.0595	0.4337	1.0259	0.1121	2.6533	7.4664	0.0132	0.1991	0.5871	0.0185	0.2179	0.6466
	3,2	1.9405	0.1208	0.7814	2.4405	0.5518	9.0310	20.6393	0.0143	0.2030	0.6016	0.0205	0.2629	0.8123
	2,1	0.6270	0.0577	0.4266	1.0231	0.1078	2.6016	7.3700	0.0133	0.1990	0.5872	0.0186	0.2200	0.6531
30	1,2	0.0311	0.0038	0.0561	0.1587	0.0040	0.0682	0.2168	0.0040	0.0652	0.2035	0.0049	0.0596	0.1737
	2,2	0.1339	0.0155	0.1953	0.4964	0.0180	0.3476	1.2144	0.0038	0.0606	0.1825	0.0050	0.0619	0.1884
	3,2	0.3785	0.0398	0.4100	1.0999	0.0558	1.2954	4.3035	0.0045	0.0700	0.2070	0.0060	0.0759	0.2350
	2,1	0.1276	0.0148	0.1872	0.4842	0.0172	0.3335	1.1701	0.0037	0.0591	0.1786	0.0049	0.0603	0.1836
50	1,2	0.0181	0.0022	0.0340	0.0992	0.0023	0.0379	0.1171	0.0023	0.0376	0.1168	0.0029	0.0354	0.1045
	2,2	0.0732	0.0087	0.1219	0.3314	0.0096	0.1717	0.5765	0.0022	0.0351	0.1066	0.0028	0.0355	0.1082
	3,2	0.2056	0.0233	0.2839	0.7562	0.0283	0.5732	2.0135	0.0027	0.0420	0.1258	0.0035	0.0440	0.1359
	2,1	0.0726	0.0087	0.1203	0.3261	0.0095	0.1708	0.5741	0.0022	0.0347	0.1051	0.0028	0.0351	0.1073
80	1,2	0.0113	0.0014	0.0218	0.0646	0.0014	0.0232	0.0706	0.0014	0.0231	0.0712	0.0018	0.0222	0.0660
	2,2	0.0434	0.0053	0.0772	0.2178	0.0056	0.0963	0.3141	0.0013	0.0212	0.0645	0.0017	0.0214	0.0654
	3,2	0.1204	0.0142	0.1902	0.5159	0.0160	0.2973	1.0185	0.0016	0.0337	0.1040	0.0021	0.0330	0.1004
	2,1	0.0452	0.0055	0.0802	0.2255	0.0058	0.1001	0.3259	0.0014	0.0282	0.0867	0.0017	0.0278	0.0846

Table 8: Bayes risk of estimators of β under different loss functions for variations in (θ, β) at different sample size n, when prior is $Gamma(0.1, 0.1)$

			δ=0.5	δ=2	δ=3.5	δ=-0.5	δ=-2	δ=-3.5	δ_1=0.5	δ_1=2	δ_1=3.5	δ_1=-0.5	δ_1=-2	δ_1=-3.5
n	(θ,β)	R_S	$R_{\beta L1}$	$R_{\beta L2}$	$R_{\beta L3}$	$R_{\beta L4}$	$R_{\beta L5}$	$R_{\beta L6}$	$R_{\beta G1}$	$R_{\beta G2}$	$R_{\beta G3}$	$R_{\beta G4}$	$R_{\beta G5}$	$R_{\beta G6}$
10	1,2	0.3090	0.0339	0.3379	0.7155	0.0431	0.8664	2.9186	0.0072	0.1121	0.3336	0.0073	0.1164	0.3518
	2,2	0.2940	0.0323	0.3279	0.7195	0.0410	0.8298	2.8166	0.0070	0.1099	0.3317	0.0071	0.1125	0.3396
	3,2	0.2611	0.0286	0.2962	0.6907	0.0366	0.7527	2.5973	0.0065	0.1026	0.3140	0.0065	0.1045	0.3172
	2,1	0.0751	0.0088	0.1135	0.2723	0.0100	0.1842	0.6267	0.0070	0.1083	0.3237	0.0070	0.1129	0.3412
30	1,2	0.0752	0.0090	0.1248	0.3320	0.0098	0.1738	0.5745	0.0022	0.0344	0.1041	0.0022	0.0349	0.1067
	2,2	0.0735	0.0088	0.1226	0.3296	0.0096	0.1696	0.5599	0.0021	0.0342	0.1039	0.0021	0.0344	0.1049
	3,2	0.0730	0.0087	0.1216	0.3270	0.0095	0.1690	0.5590	0.0021	0.0342	0.1038	0.0022	0.0345	0.1054
	2,1	0.0192	0.0023	0.0351	0.1002	0.0024	0.0411	0.1304	0.0022	0.0350	0.1063	0.0022	0.0352	0.1065
50	1,2	0.0416	0.0051	0.0742	0.2085	0.0053	0.0915	0.2963	0.0012	0.0199	0.0604	0.0013	0.0201	0.0615
	2,2	0.0431	0.0053	0.0774	0.2179	0.0055	0.0941	0.3025	0.0013	0.0206	0.0629	0.0013	0.0207	0.0634
	3,2	0.0420	0.0051	0.0755	0.2132	0.0054	0.0917	0.2955	0.0013	0.0201	0.0614	0.0013	0.0202	0.0617
	2,1	0.0107	0.0013	0.0203	0.0595	0.0013	0.0222	0.0688	0.0013	0.0204	0.0621	0.0013	0.0203	0.0613
80	1,2	0.0256	0.0031	0.0478	0.1385	0.0033	0.0544	0.1724	0.0008	0.0125	0.0381	0.0008	0.0126	0.0384
	2,2	0.0255	0.0031	0.0475	0.1377	0.0032	0.0543	0.1727	0.0008	0.0124	0.0378	0.0008	0.0125	0.0383
	3,2	0.0257	0.0042	0.0667	0.2029	0.0042	0.0682	0.2110	0.0010	0.0159	0.0491	0.0010	0.0157	0.0478
	2,1	0.0064	0.0011	0.0170	0.0520	0.0011	0.0170	0.0515	0.0010	0.0160	0.0493	0.0010	0.0155	0.0466

Table 9: HPD interval for variations in n and θ, β along with length, when prior is $Gamma(1,\ 4)$

n	(θ,β)	$hpd_{\theta LL}$	$hpd_{\theta UL}$	length	$hpd_{\beta LL}$	$hpd_{\beta UL}$	length
10	1,2	0.3837	1.3662	0.9826	0.7441	2.2601	1.5160
	2,2	0.5068	1.8333	1.3265	0.6524	2.0185	1.3661
	3,2	0.5568	2.0646	1.5078	0.5936	1.8238	1.2303
	2,1	0.5306	1.9197	1.3891	0.4103	1.2396	0.8293
30	1,2	0.6231	1.3041	0.6809	1.3089	2.3565	1.0476
	2,2	1.0375	2.1675	1.1299	1.2221	2.2274	1.0053
	3,2	1.2725	2.7703	1.4978	1.1383	2.0703	0.9320
	2,1	1.0624	2.2182	1.1558	0.6641	1.2055	0.5414
50	1,2	0.7066	1.2565	0.5499	1.4809	2.3119	0.8310
	2,2	1.2611	2.2274	0.9663	1.4242	2.2399	0.8157
	3,2	1.6300	2.9991	1.3691	1.3515	2.1249	0.7734
	2,1	1.2748	2.2507	0.9759	0.7486	1.1768	0.4282
80	1,2	0.7655	1.2108	0.4453	1.6062	2.2747	0.6685
	2,2	1.4276	2.2344	0.8069	1.5630	2.2214	0.6584
	3,2	1.9244	3.1281	1.2036	1.5078	2.1431	0.6353
	2,1	1.4434	2.2606	0.8172	0.8056	1.1475	0.3418

Table 10: HPD interval for variations in n and θ, β along with length, when prioris $Gamma(0.1,\ 0.1)$

n	(θ,β)	$hpd_{\theta LL}$	$hpd_{\theta UL}$	length	$hpd_{\beta LL}$	$hpd_{\beta UL}$	Length
10	1,2	0.4308	1.7329	1.3021	1.1700	3.3214	2.1514
	2,2	0.8911	3.7051	2.8140	1.1609	3.2967	2.1358
	3,2	1.1920	6.1372	4.9452	1.1550	3.2572	2.1023
	2,1	0.8904	3.7023	2.8119	0.5851	1.6640	1.0789
20	1,2	0.6512	1.4038	0.7526	1.5005	2.6535	1.1531
	2,2	1.3333	2.8348	1.5015	1.4924	2.6407	1.1484
	3,2	1.9028	4.4392	2.5364	1.4920	2.6394	1.1473
	2,1	1.3281	2.8229	1.4948	0.7484	1.3283	0.5799
50	1,2	0.7227	1.3053	0.5827	1.6033	2.4818	0.8785
	2,2	1.4721	2.6161	1.1440	1.6021	2.4802	0.8781
	3,2	2.1329	4.0521	1.9191	1.6010	2.4776	0.8766
	2,1	1.4736	2.6202	1.1467	0.8003	1.2431	0.4428

80	1,2	0.7788	1.2412	0.4624	1.6804	2.3691	0.6888
	2,2	1.5790	2.4785	0.8996	1.6836	2.3721	0.6885
	3,2	2.3073	3.8125	1.5052	1.6808	2.3689	0.6881
	2,1	1.5785	2.4798	0.9013	0.8405	1.1891	0.3486

Form these tables (i.e. Table1-5), we can conclude that:

1. Risk is minimum in case of linex loss function for each variation of (θ,β) and n under both priors (see Table 3, 4, 5 and 6).
2. Risk for the Bayes estimates of θ and β increases as the value of the shape parameter θ increases for LLF and GELF both in case of positive as well as negative shape parameter of loss functions (see Table 3, 4, 5 and 6) under both the priors.
3. Form Table 3 and 7, we can conclude that the risk is minimum for parameter θ under *Gamma* $(1,4)$ prior as compared to *Gamma* $(0.1,0.1)$ prior for all combinations of θ,β and n, that are taken in this study.
4. Form Table 4and 8, we can conclude that the risk is minimum for parameter β under *Gamma* $(0.1,0.1)$ prior as compared to *Gamma* $(1,4)$ prior for all combinations of θ,β and n, that are taken in this study.
5. The length of the HPD interval is increases as increase in the value of the shape parameter θ, for all variations of sample size n, i.e. n = 10, 30, 50 & 80 (see Table 9 and 10) under both the priors.
6. The length of the HPD interval decreases as increase in the sample size n for both the parameters, see Table 9 and 10, for each combination of θ and β under both the priors *Gamma*$(0.1,0.1)$ and *Gamma*$(1, 4)$.
7. Also, the length of the HPD interval decreases as increases the sample size for fixed parameter θ, with both type of priors i.e. *Gamma* $(0.1, 0.1)$ and *Gamma* $(1, 4)$, see Table 9 and 10.

6. Conclusion

In this article, Bayes estimators are calculated for three different values of θ and β and four different values of sample size as n = 10, 30, 50 and 80 by using gamma as a prior distribution with two sets of hyper parameters as $a = 1$, $b = 4$ and $a = 0.1$, $b = 0.1$ as informative and non-informative prior. All estimators are calculated under three different loss functions named as squared error, linex loss and general entropy loss function for EIW distribution. After this study, we can conclude that, Bayes estimators under linex loss are better among the all other estimators with lowest value of loss function shape parameter δ. In other words, as we increase the value δ, the risk of Bayes estimator will increase.

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