Motion of Fluid in Rayleigh Step Slider Bearing for Rotatory Lubrication Theory of Second Order

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Abstract

The fluid mechanics theory of viscous lubrication was studied from the Navier-stokes equations by the tactic of consecutive approximations that was based mostly upon the size of film thickness. It had been supported that the primary approximation offers the Reynolds equation. The second order rotatory theory of fluid mechanics lubrication was supported on the expression obtained by holding the terms containing 1st and second powers of rotation number within the extended generalized Reynolds equation. In this paper, there are some new wonderful elementary solutions with the assistance of geometrical figure, expressions, calculated tables and graphs for the step bearing within the second order rotatory theory of fluid mechanics lubrication. The analysis of equations for pressure and load capacity, tables and graphs reveal that pressure and load capacity don't seem to be freelance of viscousness. Conjointly the pressure and load capacity each increase with increasing values of rotation range. The relevant tables and graphs ensure these vital investigations within the present paper.

Keywords: Fluid Film, Pressure, Reynolds equation, Rotation number, Viscosity.

1. Introduction

The analysis of lubrication film was initially worked out by Osborne Reynolds in 1886, on the fluid flow matter through the converging passages, and it was for long accepted that the passages were necessary for film lubrication (Pinkus et al. 1961). In 1946, Fogg analyses the use of the thrust bearings with parallel faces, and given the explanation that the thermal expansion of lubricant generates the thermal wedge. More analysis in this context has been given by Bower in 1946 and Shaw in 1947 (Shaw et al. 1949). The temperature distribution in the bearings was observed by Christopherson in 1941 and Cameron & Wood in 1946 (Cameron, 1981).

The fundamental equations of hydrodynamics were expressed by Cope in 1942 by assuming all the physical properties of the fluid as variables. The flow of lubricants obeys the basic laws of fluid mechanics i.e., the equation of conservation of mass and the momentum conservation equations. The assumption of incompressibility is the perfectly adequate in most of cases. The equation of continuity or the mass conservation equation for an incompressible fluid in Cartesian coordinates is given by

$$div\ v = \partial v_x/\partial x + \partial v_y/\partial y + \partial v_z/\partial z = 0 \qquad (1)$$

The equation of momentum conservation or Navier-Stokes equation for a Newtonian fluid also in rectangular Cartesian coordinates (x_1, x_2, x_3) is the statement of the balance of momentum along each of the three the x_i directions

$$\rho\frac{\partial v_{x_i}}{\partial t} + v.\nabla v_{x_i} = -\frac{\partial p}{\partial x_i} + \eta\nabla^2 v_{x_i} + \rho g_{x_i}, i = 1,2,3 \qquad (2)$$

Here t is time, $v = (v_{x_1}, v_{x_2}, v_{x_3})$ is the velocity field vector and $g = (g_{x_1}, g_{x_2}, g_{x_3})$ is the gravitational acceleration vector. For analysis of the fluid flow in the lubricating films the following assumptions are commonly made.

- ✓ Steady state conditions
- ✓ Constant pressure through film
- ✓ Negligible body forces

- ✓ Negligible inertia forces
- ✓ Laminar flow
- ✓ Constant fluid density
- ✓ Newtonian fluid
- ✓ No slip at boundaries
- ✓ Rigid and smooth solid surfaces
- ✓ Constant viscosity through film

The equations resulting from introduction of above assumptions into the original governing equations of the fluid mechanics constitute the statement of the lubrication theory. The equations of fluid mechanics i.e., the equations of motion and continuity can be combined under the assumptions of the lubrication theory to yield an equation for computing the pressure inside the film, which is said to be Reynolds equation. In a Cartesian coordinate system, let the z-axis be located along the direction of film thickness $h(x, y)$, whereas the span of liquid layer on the $x-y$ plane is much larger than its thickness. Let the fluid motion be driven by relative velocity (U, V) and be restricted to the $x-y$ plane.

In the theory of hydrodynamic lubrication, the two dimensional classical theories (Cameron, 1981) were first given by Osborne Reynolds (Reynolds, 1886). In 1886, as the result of a classical experiment by Beauchamp Tower, he formulated a differential equation, which was known as: Reynolds Equation (Reynolds, 1886).The formation and basic mechanism of the fluid film was analyzed by that experiment on taking some important assumptions given as:

- ➢ The fluid film thickness is very small as compare to the axial and longitudinal dimensions of fluid film.
- ➢ If the lubricant layer is to transmit pressure between the shaft and the bearing, the layer must have varying thickness.

Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as: "Generalized Reynolds Equation" (Dowson, 1962), which depends on density, viscosity, film thickness, surface and transverse velocities. The rotation (Banerjee et al. 1981) of fluid film about an axis that lies across the film gives some new results in lubrication problems of fluid mechanics. The origin of rotation can be traced by certain general theorems related to vorticity in the rotating fluid dynamics. The rotation induces a component of vorticity in the direction of rotation of fluid film and the effects arising from it are predominant, for large Taylor's Number, it results in the streamlines becoming confined to plane transverse to the direction of rotation of the film (Banerjee et al. 1981).

The new extended version of "Generalized Reynolds Equation" is said to be "Extended Generalized Reynolds Equation", which takes into account of the effects of the uniform rotation about an axis that lies across the fluid film and depends on the rotation number M, i.e. the square root of the conventional Taylor's Number. The generalization of the classical theory of hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication". The "First Order Rotatory Theory of Hydrodynamic Lubrication" and the "Second Order Rotatory Theory of Hydrodynamic Lubrication" was given by retaining the terms containing up to first and second powers of M respectively by neglecting higher powers of M (Banerjee et al. 1981).

2. Governing Equations and Boundary Conditions

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation" is given by equation (1). Let us consider the mathematical terms as follows:

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$

$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$

$$=-\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}-\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]-\rho W^* \tag{2.1}$$

Where *x, y* and *z* are coordinates, *U* is the sliding velocity, *P* is the pressure, *ρ* is the fluid density, *μ* is the viscosity and W^* is fluid velocity in *z*-direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* and by retaining the terms containing up to second powers of *M* and neglecting higher powers of *M*, can be written as equation (2). For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2.2}$$

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \tag{2.3}$$

The step bearing was first used by Lord Rayleigh [4] in 1918. He used the calculus of variation to see which film shape had the biggest load-carrying capacity. He found the best was two parallel zones.

The geometry of step bearing is given by the figure- 1. The figure shows that the entry zone has a gap h_1 and the exit gap is h_2.

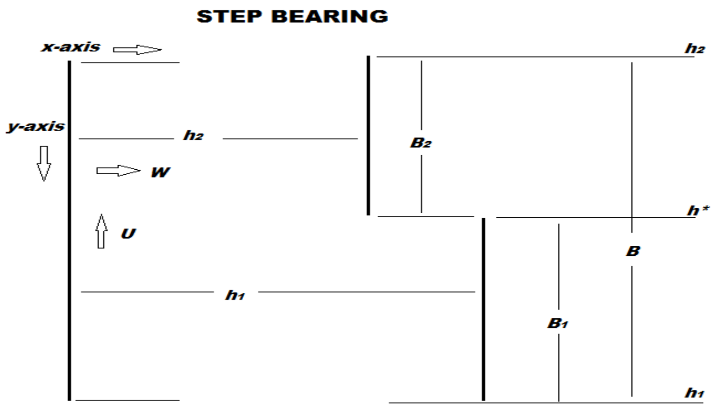

Figure-1 (Geometry of Step Bearing)

The figure shows that the runner move in the (-*y*) direction, which implies that the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction. So the terms containing pressure gradient $\partial p/\partial x$ can be neglected as compared to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence *P* may be taken as function of *y* alone.

Taking $h=h(y),\ U=-U,\ P=P(y)$ (2.4)

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{dP}{dy}\right] = \frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \quad (2.5)$$

When h is constant then the resultant pressures will be zero. Hence a bearing having a constant film thickness has no load capacity. However, if the film is parallel but has step in it such as shown in the figure, the bearing will produce hydrodynamic forces.

The film thicknesses are taken as:

$h=h_1$ in the region B_1, (2.6)

$h=h_2$ in the region B_2. (2.7)

The boundary conditions for the determination of pressure are:

$P=0$, when $y=0$ (2.8)

$P=P_c$, $dP/dy=0$ at $h=h^*$ (2.9)

Where h^* is determined by equating the two values of P_c derived in regions B_1 and B_2 respectively.

3. Solution of Differential Equation

Integrating equation (5) under the boundary conditions (8) and (9), we get the equations for pressure. The pressure for the region B_1 is given by

$$P = M\,\frac{\rho U B_2}{2}\left\{\frac{{h_1}^3-{h_2}^3}{{h_1}^3B_2+{h_2}^3B_1}\right\}y$$

$$+M^2\left[\frac{17\,B_2\,U\,\rho^3 y}{3360\mu^2}\frac{\frac{(h_1{}^3-h_2{}^3)(h_1{}^7B_2-h_2{}^7B_1)-(h_1{}^7-h_2{}^7)(h_1{}^3B_2+h_2{}^3B_1)}{(h_1{}^3B_2+h_2{}^3B_1)}}{\left\{M\left(1+\frac{17\rho^2}{1680\,\mu^2}\frac{(h_1{}^7B_2-h_2{}^7B_1)}{(h_1{}^3B_2+h_2{}^3B_1)}M^2\right)\right\}}\right] \quad (3.1)$$

The pressure for the region B_2 is given by

$$P = M\frac{\rho U B_1}{2}\left\{\frac{h_1{}^3-h_2{}^3}{h_1{}^3B_2+h_2{}^3B_1}\right\}y + M^2\left[\frac{17\,B_1\,U\,\rho^3 y}{3360\mu^2}\frac{\frac{(h_1{}^3-h_2{}^3)(h_1{}^7B_2-h_2{}^7B_1)-(h_1{}^7-h_2{}^7)(h_1{}^3B_2+h_2{}^3B_1)}{(h_1{}^3B_2+h_2{}^3B_1)}}{\left\{M\left(1+\frac{17\rho^2}{1680\,\mu^2}\frac{(h_1{}^7B_2-h_2{}^7B_1)}{(h_1{}^3B_2+h_2{}^3B_1)}M^2\right)\right\}}\right] \quad (3.2)$$

The load capacity W for step bearing is given by

$$W = M\frac{\rho UL}{2}\left\{1-\frac{(17M^2\rho^2h_2{}^7-1680\mu^2h_2{}^3)(B_1+B_2)}{17M^2\rho^2(h_1{}^7B_2-h_2{}^7B_1)-1680\mu^2(h_1{}^3B_2+h_2{}^3B_1)}\right\}\int_0^{B_1} y\,dy + M\frac{\rho UL}{2}\left\{\frac{(17M^2\rho^2h_1{}^7-1680\mu^2h_1{}^3)(B_1+B_2)}{17M^2\rho^2(h_1{}^7B_2-h_2{}^7B_1)-1680\mu^2(h_1{}^3B_2+h_2{}^3B_1)}-1\right\}\int_0^{B_2} y\,dy \quad (3.3)$$

$$W = M\frac{\rho U L B_1 B_2 (B_1+B_2)}{4}\left\{\frac{h_1{}^3-h_2{}^3}{(h_1{}^3B_2+h_2{}^3B_1)}+M^2\frac{17\rho^2}{1680\,\mu^2}\frac{\frac{(h_1{}^3-h_2{}^3)(h_1{}^7B_2-h_2{}^7B_1)-(h_1{}^7-h_2{}^7)(h_1{}^3B_2+h_2{}^3B_1)}{(h_1{}^3B_2+h_2{}^3B_1)}}{\left(1+\frac{17\rho^2}{1680\,\mu^2}\frac{(h_1{}^7B_2-h_2{}^7B_1)}{(h_1{}^3B_2+h_2{}^3B_1)}M^2\right)}\right\} \quad (3.4)$$

4. Numerical Simulation

By taking the values of different mathematical terms in *C.G.S.* system the calculated tables and graphical representations are as follows:

$U = 80, \rho = 1, \frac{L}{B} = 1,\ h_1 = 0.0269, h_2 = 0.0167, e = 0.2, \mu = 0.0002, B_1 = 1, B_2 = 0.5$

S. No.	*M*	*P*	*W*
1.	0.1	2.0565596	2.3136315
2.	0.2	4.1131680	4.6172160
3.	0.3	6.1337808	6.8946132
4.	0.4	8.1358800	9.1528792
5.	0.5	10.100488	11.363063

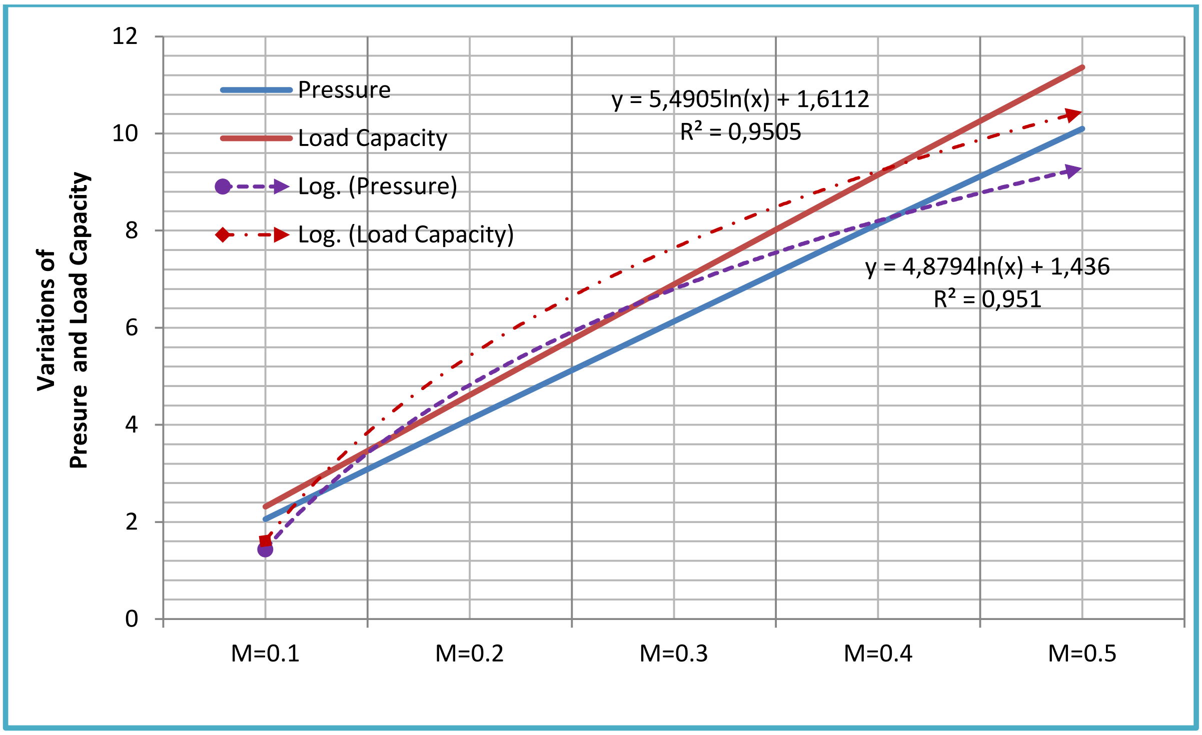

Figure-2 (Variations of Pressure and Load Capacity with respect to rotation number *M*)

5. Conclusion

The variation of pressure and load capacity for *Step Bearings* with respect to rotation number *M*, when viscosity is constant; are shown by table and graph.

The logarithmic equations for the pressure and load capacity have been found and are as follows:

$$P = 4.879 log_e M + 1.436; R^2 = 0.951 ; \mu = 0.0002 \qquad (5.1)$$

$$W = 5.490 log_e M + 1.611; R^2 = 0.950 ; \mu = 0.0002 \qquad (5.2)$$

Hence in the second order rotatory theory of hydrodynamic lubrication, the pressure and load capacity for step bearings both increases with increasing values of *M,* when viscosity is taken as constant. The equations for pressure and load capacity show that both of them are not independent of viscosity, it varies with μ.

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