Motion of Hydrodynamic Fluid through Bearing of Type "$h = h_o\ Exp.(\theta); \theta = f(y)$" for Rotation

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Abstract

The second order rotatory theory of fluid mechanics lubrication was supported on the expression obtained by retentive the terms containing initial and second powers of rotation range within the extended generalized Reynolds equation. Within the present paper, there are some new glorious basic solutions with the assistance of geometrical figures, expressions, calculated tables and graphs for the bearing of type $h = h_o\ Exp.(\theta); \theta = f(y)$ within the second order rotatory theory of fluid mechanics lubrication. The analysis of equations for pressure and load capability, tables and graphs reveal that pressure and load capability aren't freelance of viscosity of fluid and increase linearly, slightly with viscosity. Conjointly the pressure and load capability each increase exponentially with increasing values of rotation range. Within the absence of rotation, the equation of pressure and load capability provides the classical solutions of the classical theory of fluid mechanics lubrication. The relevant tables and graphs make sure these necessary investigations within this paper.

Keywords: Rotation number, Taylor's number, Reynolds equation, Viscosity.

1. Introduction

In the theory of fluid mechanics lubrication, 2- dimensional classical theories (Cameron, 1958, 81), (Chandrashekhar, 1970), (Dowson, 1962) were initially given by Osborne Reynolds (Reynolds, 1886). In 1886, within the wake of a classical experiment by Beauchamp Tower, he developed a vital equation that was proverbial as: Reynolds Equation (Reynolds, 1886). The formation and basic mechanism of fluid film was analyzed by that experiment on taking some necessary assumptions given as:

- The fluid film thickness is extremely low as compare to the axial and longitudinal dimensions of fluid film.
- If the stuff layer is to transmit pressure between the shaft and also the bearing, the layer should have variable thickness.

Later Osborne Reynolds himself derived the improved version of Reynolds Equation proverbial as: "Generalized Reynolds Equation" (Pinkus et al. 1961), (Halling, 1975), that depends on density, viscosity, film thickness, surface and cross velocities. The rotation (Banerjee et al. 1981) of fluid film concerning associate degree axis that lies across the film offers some new ends up in lubrication issues of mechanics. The origin of rotation may be derived by bound general theorems associated with vorticity within the rotating fluid dynamics. The rotation induces an element of vorticity within the direction of rotation of fluid film and also the effects arising from it are predominant, for giant Taylor's number, it ends up in the streamlines changing into confined to plane cross to the direction of rotation of the film.

The new extended version of "Generalized painter Equation" is alleged to be "Extended Generalized painter Equation" (Banerjee et al. 1981,82), that takes under consideration of the results of the uniform rotation concerning associate degree axis that lies across the fluid film and depends on the rotation number M, i.e. the root of the standard Taylor's number. The generalization of the classical theory of fluid mechanics lubrication is thought because the "Rotatory Theory of fluid mechanics Lubrication". The "First Order Rotatory Theory of Hydrodynamic Lubrication" and also the "Second Order Rotatory Theory of Hydrodynamic Lubrication" was given by retentive the terms containing up to initial and second powers of M severally by neglecting higher powers of M (Banerjee et al. 1981, 82). Within the exponentially inclined slider, the film thickness h will increase exponentially with the distance. The geometry of slider bearing of type $h = h_o\, Exp.(\theta); \theta = f(y)$ is given by figure (1).

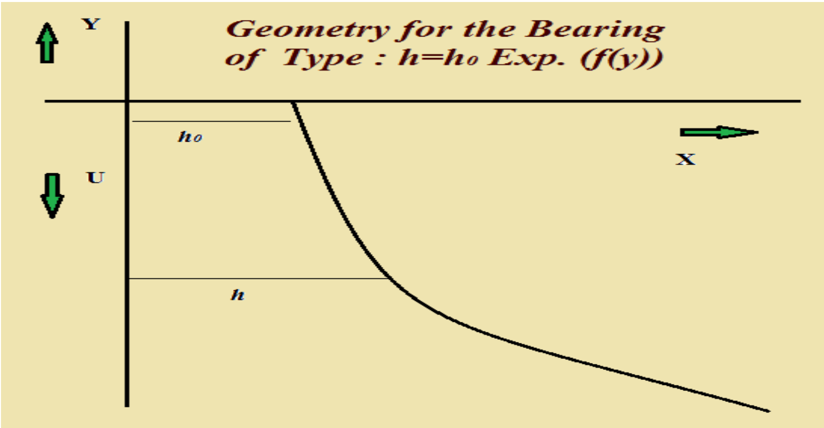

Figure-1 Geometry of the bearing

The figure shows that the runner move in the (-*y*) direction, which implies that the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction. So the terms containing pressure gradient $\partial p/\partial x$ can be neglected as compared to the terms containing $\partial p/\partial y$ in the differential equation of pressure, hence P may be taken as function of y alone.

2. Governing Equations and Boundary Conditions

In the second order rotatory theory of hydrodynamic lubrication the "Extended Generalized Reynolds Equation" [7] is given by equation (1).

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} - \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} + \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$

$$+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} - \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} + \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$

$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} + \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} + \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$

$$-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} + \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} + \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right] =$$

$$= -\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} + \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} + \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right] - \frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} - \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} - \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right] - \rho W^* \quad (2.1)$$

Where *x, y* and *z* are coordinates, *U* is the sliding velocity, *P* is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in *z*-direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* and by retaining the terms containing up to second powers of *M* and neglecting higher powers of *M*, can be written as equation (8). For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right] + \frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right] + \frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right] - \frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$= -\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right] - \frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right] \quad (2.2)$$

Let we assume the bearing to be infinitely long in *y*-direction, which implies that the variation of pressure in *x*-direction is very small as compared to the variation of pressure in *y*-direction i.e., $\frac{\partial P}{\partial x} \ll \frac{\partial P}{\partial y}$, then the equation (2.2) will be

$$\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right] + \frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]$$
$$= -\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right] - \frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \quad (2.3)$$

Taking the pressure distribution as the function of the coordinate along the length of the slider only, we have *P = P(y)*, *h=h(y)* and *U=-U*, we have

$$\frac{d}{dy}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right] = -\frac{d}{dy}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \quad (2.4)$$

The film thickness at any point is given by

$h = h_o\, Exp.(\theta); \theta = f(y) = -\alpha\, y$

$\Rightarrow h = h_o e^{-\alpha y}$ (2.5)

For the determination of pressure the boundary conditions are:

$$P = 0, at\ h = h_o \quad or\ y = 0\ and\ P = 0, at\ h = h_o e^{-\alpha y}\ or\ y = -L \tag{2.6}$$

So we have the differential equation for the pressure will be

$$\frac{dP}{dy} = -\frac{1}{2}M\rho U\left[1 - \left({h_0}^3 - \frac{17M^2\rho^2{h_0}^7}{1680\mu^2}\right)\frac{1}{{h_0}^3 e^{-3\alpha y}} - \frac{17M^2\rho^2{h_0}^3}{1680\mu^2}h_0 e^{-\alpha y}\right] \tag{2.7}$$

3. Determination of pressure

The solution of the differential equation (2.4) under the boundary condition (2.6) gives the pressure for exponentially inclined slider bearings by (3.1).

$$P = M\left(\frac{\rho U}{6\alpha}(e^{3\alpha y} - e^{-3\alpha L})\right) + M^3\left[\frac{17U\rho^3{h_0}^4}{10080\alpha\,\mu^2}(e^{-3\alpha L} + 3\,e^{\alpha L} - 3\,e^{-\alpha y} - e^{3\alpha y})\right] \tag{3.1}$$

4. Determination of load capacity

The load capacity for exponentially inclined slider bearing is given by

$$W = -\int_L^0 P\ dy \tag{4.1}$$

$$W = M\left(\frac{1 - e^{-3\alpha L}}{3\alpha} - Le^{-3\alpha L}\right)\frac{\rho U}{6\alpha} + M^3\left[\left(\frac{17U\rho^3{h_0}^4}{10080\,\alpha\,\mu^2}\right)\left\{Le^{-3\alpha L} + 3Le^{\alpha L} + \frac{3}{\alpha}(1 - e^{\alpha L}) - \frac{1}{3\alpha}(1 - e^{-3\alpha L})\right\}\right] \tag{4.2}$$

5. Numerical Simulation and Discussion

By taking the values of different mathematical terms in ***C.G.S.*** system the calculated tables and graphical representations are as follows:

Table: 5.1

$U = 500, \rho = 0.9, L = 15, y = 7.5, h = 0.015, h_o = 0.01, \mu = 0.0002$

S.NO.	*M*	*P*	*W*
1.	0.1	1536.413947	14781.27479
2.	0.2	3072.827575	29550.61501
3.	0.3	4609.240567	44296.08607
4.	0.4	6145.652604	59005.75339
5.	0.5	7682.063366	73667.68240
6.	0.6	9218.472537	88269.93852
7.	0.7	10754.87980	102800.5872
8.	0.8	12291.28483	117247.6938
9.	0.9	13827.68731	131599.3237
10.	1.0	15364.08693	145843.5425

Table: 5.2

$U = 500, \rho = 0.9, L = 15, y = 7.5, h = 0.015, h_o = 0.01, M = 0.1$

S.NO.	μ	*P*	*W*
1.	0. 00015	1536.413906	14779.72772
2.	0.00020	1536.413947	14781.27479
3.	0.00025	1536.413966	14781.99087
4.	0.00030	1536.413976	14782.37985
5.	0.00035	1536.413983	14782.61439
6.	0.00040	1536.413987	14782.76662
7.	0.00045	1536.413990	14782.87098
8.	0.00050	1536.413992	14782.94563
9.	0.00055	1536.413993	14783.00087
10.	0.00060	1536.413994	14783.04288

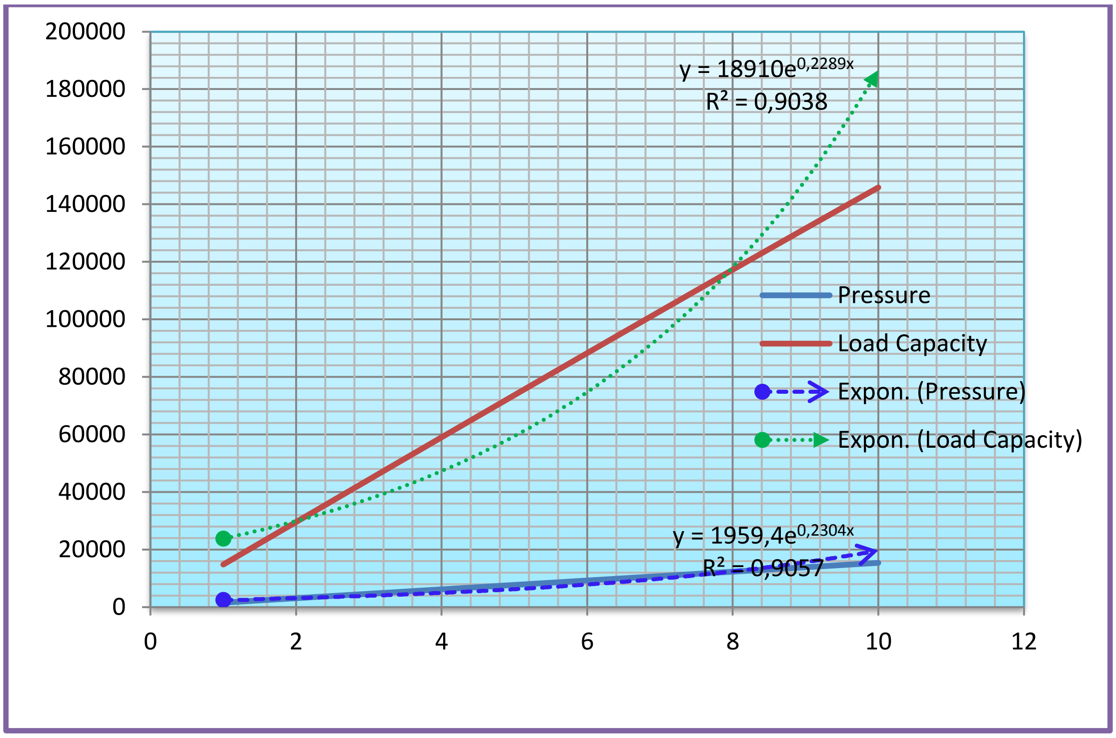

Figure-2 (Variation of *P* and *W* against *M* for μ=0.0002)

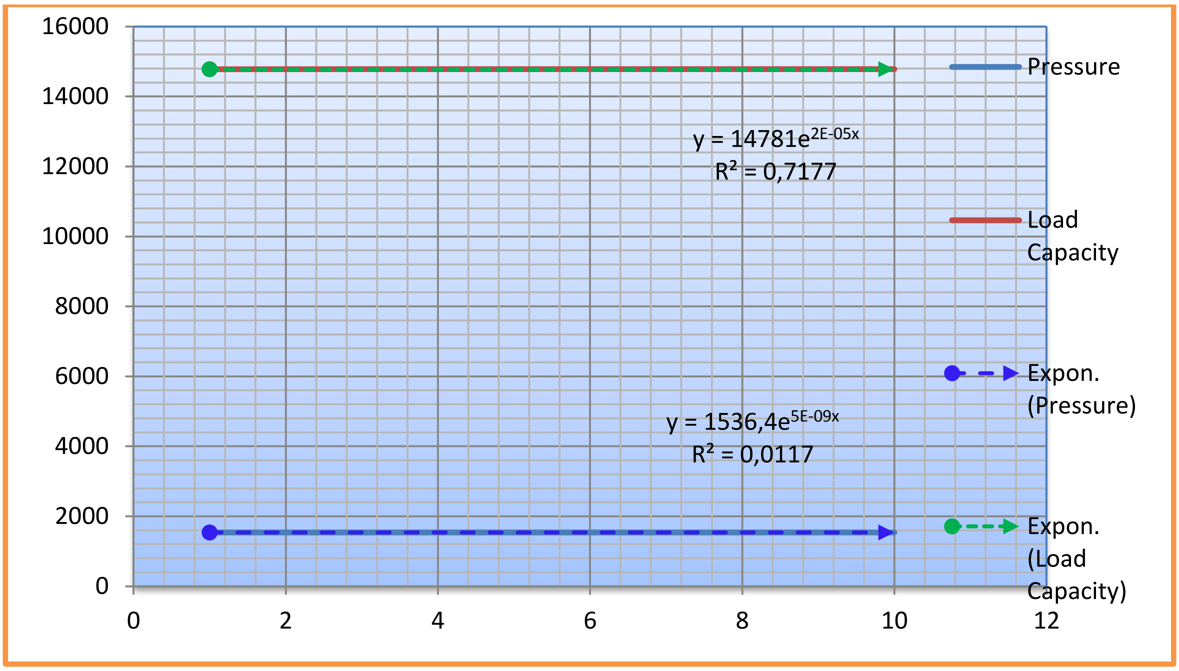

Figure-3 (Variation of P and W against μ for M=0.1)

By the method of least square approximation, the trend line equations for the pressure and load capacity are as follows:

$P= 1959\ e^{0.23M}$, R^2=0.905 (μ=0.0002) (5.1)

$P= 1536\ e^{5E-9\mu}$, R^2=0.011 (M=0.1) (5.2)

$W= 1891\ e^{0.228M}$, R^2=0.903 (μ=0.0002) (5.3)

$W= 14781\ e^{2E-5\mu}$, R^2=0.717 (M=0.1) (5.4)

The equations (5.1) and (5.3) show that the pressure and load capacity both increases with respect to rotation number for the viscosity of fluid μ=0.0002. The equations (5.2) and (5.4) show that there is negligible variation in the pressure and load capacity for small change in viscosity. But these are not independent of viscosity.

6. Conclusion

The variation of pressure and load capacity for the bearings of type $h = h_o\ Exp.(\theta); \theta = f(y)$ with relevancy rotation range M, once viscosity is constant; are shown by tables and graphs. Thence within the second order rotatory theory of fluid mechanics lubrication, the pressure and load capacity for these bearings each will increase exponentially with increasing values of M, once viscosity is taken as constant. The variation of pressure and load capacity for these bearings with relevancy viscosity, once M is constant; also are shown by tables and graphs. This shows that they're not freelance of viscosity μ and slightly will increase linearly with μ, once M is constant. On taking (M=0) within the expression of pressure and load capacity, we tend to get the classical solutions.

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