Analysis of Kinetic Energy in the Porous Media

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Abstract

The macroscopic transport analysis for the incompressible fluid flow within the porous media supported the volume-average technique for the heat transfer was given within the numerous researches. Within the present paper there is the analysis and derivations of equations supported the construct of time-average. This offers a latest new ideas and technique for the analysis of flow in porous media. The time-averaged transport equations play a vital role on analyzing the transportation over the extremely semi permeable media wherever the flow happens within the fluid flow.

Keywords: Porous media, Turbulent flow, Transport equation.

1. Introduction

The idea of macroscopic transportation for the incompressible fluid flow within the porous media was employed by Vafai & Tien (Vafai et al. 1981) in 1981 and Whitaker in 1999 (Whitaker, 1999), supported the volume-average methodology for the heat transfer by Hsu & Cheng (Hsu eta. 1990) in 1990. The idea of space average in porous media is predicated on the idea that though fluid velocities and pressure could also be irregular at the pore scale, regionally space-averaged measurements of those quantities vary swimmingly. The macroscopic equations are normally derived by spatially averaging the microscopic ones over a Representative Elementary Volume (REV) of the porous media. A REV ought to be the tiniest differential volume, which ends up in the useful average properties. It implies that the length scale of this volume should be sufficiently larger than the pore scale (Whitaker, 1999). Also, the size of the system should be significantly larger than the REV's length scale for avoiding the non-homogeneities i.e.

$$p \ll D \ll L$$

Where p is the pore scale or the microscopic length scale, *D* is the macroscopic length scale and *L* is the mega scale or scale of the system as shown by figure-1.

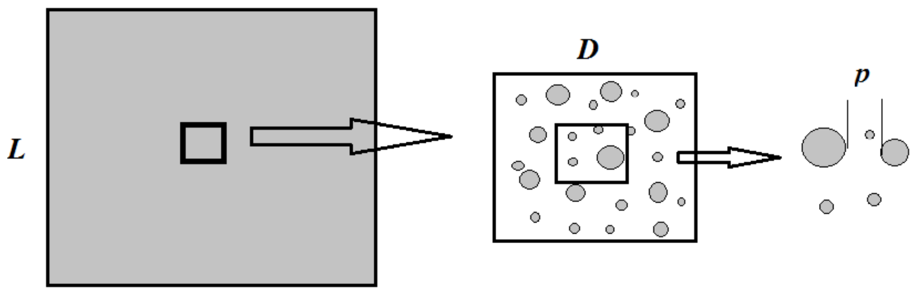

Figure-1. Identification of different length scales.

A schematic illustration of a spherical REV consisting of a hard and fast solid section saturated with a nonstop fluid section and is shown by the figure-2, here the solid section is mounted, i.e., the solid section doesn't amendment every which way if totally different ensembles are thought-about. The degree of the REV is constant i.e., not dependent of the area and it is equal to the addition of the fluid and solid volumes within the REV, i.e.

$$V = V_s + V_f$$

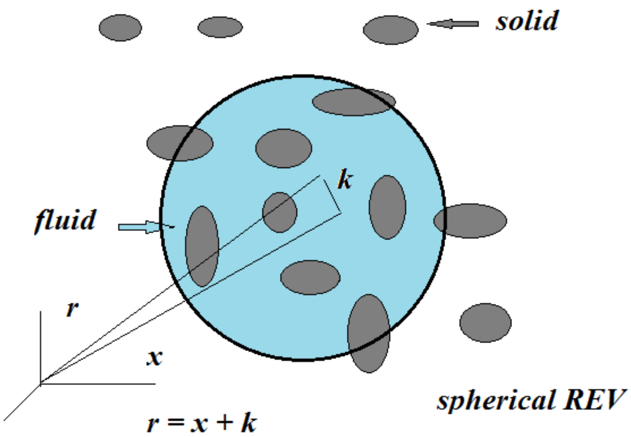

Figure 2. Spherical Representative Elementary Volume (REV).

The spherical representative elementary volume is shown by figure-2.On taking the time fluctuations of the flow properties with spatial deviations, there are generally two methods for deriving and studying the macroscopic equations. The first method based on the time-average operator followed by the volume-averaging initially used by Kuwahara (Kuwahara et al. 1998) in 1998. The second method based on the concept of volume-averaging before time averaging that was used by Lee & Howell in 1987, and the macroscopic transport equations established by these two methods are equivalent (Pedras et al. 1999). This initial method for the flow variables has been extended to the non buoyant heat transfer for the porous media by considering the phenomenon of time variations and spatial deviations was taken by Rocamora & Lemos (Rocamora et al. 2000) in 2000.Later, the researches on the natural convection flow on the porous layer, double-diffusive convection for the turbulent flow and heat transfer in the porous media was given by de Lemos (de Lemos et al. 2004) in 2004. The numerical based analysis for applications of double-decomposition theory to buoyant flow was also reviewed by de Lemos in 2003(de Lemos et al. 2003).

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2. Governing Equations

The macroscopic instantaneous transfer equations for incompressible fluid flow having the constant properties are given as:

$$\nabla . \bar{v} = 0 \tag{2.1}$$

$$\rho\, \nabla . (\bar{v} . \bar{v}) = -\nabla P + \mu \nabla^2 \bar{v} + \rho\, \bar{g} \tag{2.2}$$

$$(\rho\, C_P)\, \nabla . (\bar{v}\, T) = \nabla . (\lambda\, \nabla\, T) \tag{2.3}$$

Where $\bar{v}$ is the velocity vector, P is the pressure, μ is the viscosity of the fluid, ρ is the density of the fluid, $\bar{g}$ is the acceleration vector due to gravity, C_P is the specific heat, T is the temperature and λ is the thermal conductivity of the fluid. The mass fraction distribution related to chemical species e is governed by the transport equation given as:

$$\nabla . (\rho\, \bar{v}\, m_e + \bar{J}_e) = \rho\, R_e \tag{2.4}$$

Where m_e is the mass fraction of component e, $\bar{v}$ is the mass-averaged velocity of the fluid mixture, so we have

$$\bar{v} = \sum_e m_e\, \bar{v}_e \tag{2.5}$$

Where $\bar{v}_e$ is the velocity of species e. The mass diffusion flux $\bar{J}_e$ is due to velocity slip of the species e and is given as:

$$\bar{J}_e = \rho_e\, (\bar{v}_e - \bar{v}) = -\rho\, D_e\, \nabla\, m_e \tag{2.6}$$

Here D_e is the diffusion coefficient of species e for the mixture. The equation (2.6) is also known as the Fick's law. The R_e represents the generation rate of species per unit mass.

If the density ρ varies with the temperature T for the natural convection flow, the remaining density based on the Boussinesq concept will be given as:

$$\rho_T \cong \rho\, [1 - \beta(T - T_r)] \tag{2.7}$$

Here T_r is the temperature at reference value and β is the thermal expansion coefficient and is defined as:

$$\beta = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T}\right)_P \tag{2.8}$$

By using the equation (2.2) and (2.7), we get

$$\rho\, \nabla . (\bar{v}\, \bar{v}) = -(\nabla\, P)^* + \mu\, \nabla^2\, \bar{v} - \rho\, \bar{g} \beta\, (T - T_r) \tag{2.9}$$

Here $(\nabla\, P)^* = \nabla\, P - \rho\, \bar{g}$, represents the modified pressure gradient.

From equation (2.3), we have the equation for fluid as:

$$\left(\rho\, C_p\right)_F \nabla . (\bar{v}\, T_F) = \nabla . (\lambda_F\, \nabla\, T_F) + S_F \tag{2.10}$$

Also from equation (2.3), we have the equation for solid or porous matrix as:

$$\nabla . \left(\lambda_p\, \nabla\, T_p\right) + S_p = 0 \tag{2.11}$$

Here the suffix F and p are used for fluid and porous matrix respectively. The factor S_F or S_p vanishes in the absence of heat generation. The volume-averaging in the porous medium was given by Slattery in 1967, Whitaker in 1999 (Slattery et al. 1967), (Whitaker, 1999) and Gray *et al.* in 1977 (Gray et al.

1977). It makes the concept of *REV* (representative elementary volume) and by using the concept, the equations are integrated.

2.1 Volume and Time Average Operators

The volume average of the general property term φ over REV for the porous medium was given by Gray (Gray et al. 1977) in 1977 and is written as:

$$[\varphi]_V = \frac{1}{\delta V}\int \varphi \, dV \tag{2.12}$$

Here $[\varphi]_V$ is taken for any point surrounded by REV of size δV. The average is given as:

$$[\varphi_F]_V = \phi \, [\varphi_F]_i \tag{2.13}$$

Here the suffix 'i' is used for the intrinsic average and ϕ is the porosity of the medium and is defined as:

$$\phi = \frac{\delta V_F}{\delta V}$$

$$\varphi = [\varphi]_i + \varphi_i \tag{2.14}$$

in addition to the condition that

$$[\varphi_i]_i = 0 \tag{2.15}$$

Here φ_i is the spatial deviation of φ for the intrinsic average φ_i. To derive the flow equations, we have to know the relation between the volume average of derivatives and derivatives of volume average. The relation between these two was presented by Slattery & Gray (Gray et al. 1977) in 1977. So we have

$$[\nabla\varphi]_V = \nabla\{\phi(\varphi)_i\} + \frac{1}{\delta V}\left[\iint \hat{n}\ \varphi \, ds\right]_{\alpha_i} \tag{2.16}$$

$$[\nabla.\varphi]_V = \nabla.\{\phi(\varphi)_i\} + \frac{1}{\delta V}\left[\iint \hat{n}\,.\varphi \, ds\right]_{\alpha_i} \tag{2.17}$$

$$\left[\frac{\partial\varphi}{\partial t}\right]_V = \frac{\partial}{\partial t}\{\phi(\varphi)_i\} - \frac{1}{\delta V}\left[\iint \hat{n}\,.(\bar{v_i}\,\varphi) \, ds\right]_{\alpha_i} \tag{2.18}$$

Here α_i, $\bar{v_i}$ and $\hat{n}$ are interfacial area, velocity and unit vector normal to α_i respectively. If the porous substrate is fixed then $\bar{v_i} = 0$. But if the medium is rigid and heterogeneous then δV_F depends on the space and doesn't depend on time as taken by Gray *et al.* [3]. The time average of φ is given as:

$$\bar{\varphi} = \frac{1}{\delta t}\int_t^{t+\delta t} \varphi \, dt \tag{2.19}$$

Here δt is very small time interval as compared to $\bar{\varphi}$ but sufficient to calculate the turbulent fluctuations of φ. Now the time decomposition will be taken as:

$$\varphi = \bar{\varphi} + \varphi' \tag{2.20}$$

with the condition that

$$\overline{\varphi'} = 0 \tag{2.21}$$

Here φ' is the time fluctuation of φ with respect to $\bar{\varphi}$.

3. Time-Averaged Transport Equation

Let us consider the following:

$$v = \bar{v} + v_1, T = \bar{T} + T_1, P = \bar{P} + P_1 \tag{3.1}$$

The equations (2.1), (2.2) and (2.9) will be

$$\nabla . \bar{v} = 0 \tag{3.2}$$

$$\rho \nabla . (\bar{v}\, \bar{v}) = -(\nabla \bar{P})^* + \mu\, \nabla^2 \bar{v} + \nabla . (-\rho \overline{v_1 v_1}) - \rho\, \bar{g}\, \beta\, (\bar{T} - T_r) \tag{3.3}$$

$$(\rho\, C_p)\, \nabla . (\bar{v}\, \bar{T}) = \nabla . (K_e\, \nabla\, \bar{T}) + \nabla . \left(-\rho\, C_p\, \overline{(v_1 T_1)}\right) \tag{3.4}$$

Taking,

$$\frac{\{\nabla\, \bar{v} + (\nabla \bar{v})_T\}}{2} = \overline{D_m} = \text{mean deformation tensor} \tag{3.5}$$

$$\frac{(\bar{v}_1 . \bar{v}_1\,)}{2} = K_e = \text{turbulent kinetic energy per unit mass} \tag{3.6}$$

Now from the eddy-diffusivity concept, we have

$$-\rho\, \overline{(v_1 v_1)} = \mu_t\, 2\overline{D_m} - \frac{2}{3}\, \rho K_e \hat{A} \tag{3.7}$$

Here μ_t, $\hat{A}$ are the turbulent viscosity and unity tensor respectively.

Again by using the eddy-diffusivity concept for the turbulent heat flux, we have

$$-\rho\, C_p\, \overline{(v_1 T_1)} = C_p\, \frac{\mu_t}{\sigma_t}\, \nabla\, \bar{T} \tag{3.8}$$

Here σ_t is the turbulent Prandlt number. The transport equation for turbulent kinetic energy will be founded by taking the multiplication of the difference between the instantaneous and the time-averaged momentum equations by v_1. Again, using the time-average operator, the equation takes the form:

$$\rho\, \nabla . (\bar{v}\, K_e) = -\rho\, \nabla . \left\{v_1 \frac{P_1}{\rho} + u\right\} + \mu \nabla^2\, K_e + P_K + Q_K - \rho\, e_1 \tag{3.9}$$

Here

$$P_K = -\rho\, \overline{(v_1 v_1)}$$

$\nabla\, \bar{v}$ = generation rate of K_e due to the mean velocity gradient

$$Q_K = -\rho\, \beta\, \bar{g} . \overline{(v_1 T_1)} \tag{3.10}$$

e_1 = dissipation rate of K_e

The term Q_K is the buoyancy generation rate of K_e.

$$u = \frac{v_1 . v_1}{2} \tag{3.11}$$

4. Conclusion

The present paper gives some new technique for the analysis of flow within the porous media by victimization the time-averaged transport equation. This may well be higher once learning transport over extremely semi permeable media wherever the flow happens within the fluid part. The analysis offers opportunities for environmental and engineering flows from these derivations.

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