On Mathematical Analysis LRS Bianchi Type II String Cosmological Models with Viscosity Distribution in Modified General Relativity

***R. K. Dubey[1], Shishir Kumar Srivastava[2] & J .N. Mishra[3]**

[1]Department of Mathematics Govt. Model Science College, Rewa (M.P.) India
[2]Department of Mathematics Ganpat Sahai P.G. College, Sultanpur (U.P.) India
[3]Department of Mathematics Ganpat Sahai P.G. College, Sultanpur (U.P.) India

Email: shishir9918825052@gmail.com

Abstract

We consider locally rotationally symmetric Bianchi type II string cosmological model in the presence of viscous fluid. To solve the Einstein's field equations for LRS Bianchi type II space time has been obtained under the assumption $\rho = K_{m_1}\eta$ *, here* ρ *is the energy density,* η *is the string tension density and* K_{m_1} *is a constant. We have also used a condition that, the scalar expansion is proportional to the shear, to get determinate solution in terms of cosmic time t. Some physical and model geometric behavior of the models is discussed.*

Keywords: LRS Bianchi type II, massive string, viscous fluid.

MSC2010 CLASSIFICATIONS: 83C05, 83C15.

PACS Number: 98.80.cq, 04.20.-q

1. Introduction

The string theory is useful concept before the creation of the particle in the universe. The string are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe. The present day configuration of the universe is not contradicted by the large scale network of strings in the early universe. In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe (Kibble, 1980). Cosmic strings play an important role in the study of the early universe. Banerjee et al. investigated a spatially homogeneous and locally rotationally symmetric Bianchi type II cosmological model under the effect of both shear and bulk viscosity (Banerjee et al. 1986). Singh and Agrawal have studied Bianchi type II, VIII and IX models in scalar tensor theory under the assumption of a relationship between the cosmological constant (Λ) and the scalar field (Ψ) (Agarwal et al. 1997). These arise during the phase transition after the big bang explosion as the temperature decreased below some critical temperature predicted by GUT (Zel, dovichet et. al. 1975); (Kibble, 1976); (Vilenkin, 1981).

The pioneering work in the formation of the energy momentum tensor for classical massive strings was done by Litelier who considered the massive string to be formed by geometric strings with particle attached along extension and Leterlier first used this idea in obtaining cosmological solutions in Bianchi type I and Kantowski – Sachs space times (Letelier, 1983). Stachel has studied massive strings (Stachel, 1980). Bali et al. have obtained Bianchi type I, III, V and IX string cosmological models with magnetic field and bulk viscosity in general relativity (Bali et al.

2007). Yadav et al. have studied some Bianchi type I viscous fluid string cosmological models with magnetic field (Yadav et al. 2007). Wang has also discussed LRS Bians chi type I, Kantowski – Sachs and Bianchi type III cosmological models for a cloud of string with bulk viscosity (Wang, 2006). Recently Yadav et al. have obtained the integrality of cosmic string in Bianchi type III space time in presence of bulk viscous fluid by applying a new technique (Yadav et al. 2007).

Taygi et al. has studied Bianchi type II Bulk viscous string cosmological models in general relativity (Tyagi et al. 2010). Pradhan et al. have deal with LRS cosmological models of Bianchi type II representing clouds of geometrical as well as massive strings (Pradhan et al. 2007). In this paper we consider on mathematical analysis LRS Bianchi type II string cosmological models with viscosity distribution in modified general relativity. To obtain cosmological model we assume that$\rho = K_{m_1}\eta$, here ρ is the rest energy density, η is the string tension density.

2. The metric and field equation

We consider the LRS Bianchi type II metric in the from

$$ds^2 = -dt^2 + A^2(dx^2 + dz^2) + B^2(dy^2 - xdz)^2 \qquad (1)$$

Where A, B are the metric function of Cosmic time 't' only.

The energy momentum tensor T_α^β for a cloud of massive string for viscous fluid distribution is given by –

$$T_\alpha^\beta = \rho v_\alpha v^\beta - \eta x_\alpha x^\beta - \mu\theta\left(v_\alpha v^\beta + g_\alpha^\beta\right) \qquad (2)$$

We take the equation of state

$$p = \omega\rho \ , \qquad 0 \leq \omega \leq 1 \qquad (3)$$

In eqn. (2) ρ is the energy density, η is the string tension density, μ is the coefficient of bulk viscority.

If the particle density of the configuration is denoted by ρ_p then

we get –

$$\rho = \rho_p + \eta \qquad (4)$$

v^α the four velocity for the cloud of particles and x^α the four vector which repsrents the strings direction. Thus we get

$$v_\alpha v^\alpha = -1 \ = -x_\alpha x^\alpha \qquad (5)$$

$$v_\alpha x^\alpha = 0 \qquad (6)$$

In commoving co-ordinate system we get –

$$v^\alpha = (0, \quad 0, \quad 0, 1) \qquad (7)$$

$$x^\alpha = (A^{-1}, \quad 0, 0, 0) \qquad (8)$$

The Einstein's field equations a system of string are given by –

$$R_\alpha^\beta - \frac{1}{2} R\, g_\alpha^\beta = T_\alpha^\beta \qquad (9)$$

The Einstein's field equations (9) for metric (1) leads to

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{B^2}{4A^4} = \eta + \mu\theta \tag{10}$$

$$\frac{2A_{44}}{A} + \frac{A_4^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = \mu\theta \tag{11}$$

$$\frac{A_4^2}{A} + \frac{2A_4 B_4}{AB} - \frac{B^2}{4A^4} = \rho \tag{12}$$

For the complete dermination, of the set, we assume that

$$\rho = K_{m_1}\eta \tag{13}$$

From equation (10), (12) and (13) we get

$$K_{m_1}\frac{A_{44}}{A} - K_{m_1}\frac{B_{44}}{B} + \left(2 - K_{m_1}\right)\frac{A_4 B_4}{AB} + \frac{A_4^2}{A}\left(K_{m_1} + 1\right) = \frac{B^2}{4A^4}\left(4K_{m_1} + 1\right) \tag{14}$$

Using equation (14) $B = hA^m$, we get

$$\frac{A_{44}}{A} + \delta\frac{A_4^2}{A^2} = \frac{M}{A^{4-2m}} \tag{15}$$

where

$$\delta = \left\{\frac{K_{m_1}(1 - m^2) + (2m + 1)}{K_{m_1}(1 - m)}\right\} \tag{16}$$

$$M = \frac{h\left(4K_{m_1} + 1\right)}{4K_{m_1}(1 - m)} \tag{17}$$

equation (16) reduce to

$$\frac{d}{dA}\left[A^{2\delta}A_4^2\right] = \frac{2M}{A^{3-2m-2\delta}} \tag{18}$$

on intergrating we get –

$$dt = \left[\frac{h^2\left(4K_{m_1} + 1\right)}{4A^{2-2m}\left[2mK_{m_1}(1 - m) + (2m + 1)\right]} + \frac{L}{A^{2\left\{(1+m)+\frac{(2m+1)}{K_{m_1}(1-m)}\right\}}}\right]^{-\frac{1}{2}} dA \tag{19}$$

Where L, the constant of intergration using proper transformation an equation (9) the above metric (1) can be written as –

$$ds^2 = \left[\frac{h^2(4K_{m_1}+1)}{4\tau^{2-2m}\left[2mK_{m_1}(1-m)+(2m+1)\right]} + \frac{L}{A^{2\left\{(1+m)+\frac{2m+1}{K_{m_1}(1-m)}\right\}}}\right] d\tau^2 + \tau^2(dx^2 + dz^2) + h^2\tau^{2m}(dy - x dz)^2 \tag{20}$$

3. Some physical and geometrical properties

The rest energy density ρ, the string tension density η, the particle density ρ_p for model (20) are given by.

$$\rho = \frac{h^2}{4\tau^{4-2m}}\left[\frac{(2m+1)(4K_{m_1}+1)}{\{2mK_{m_1}(1-m)+(2m-1)^2\}}\right] + \frac{L}{\tau^{2\left\{\frac{m+2}{K_{m_1}(1-m)}\right\}}} \quad (21)$$

$$\eta = \frac{\rho}{K_{m_1}} \quad (22)$$

$$\rho_p = \frac{(n_1-1)}{n_1}\rho \quad (23)$$

Where ρ, is given by equ (21) The scalar of expansion θ, coefficient of bulk viscosity μ, the spatial volume $s(t)^3$ and the shear σ for the

$$\theta = (m+2)\left\{\frac{h^2(4K_{m_1}+1)}{4\left\{2mK_{m_1}(1-m)+(2m+1)\tau^{4-2m}\right\}} + \frac{L}{\tau^{2\left\{(m+2)+\frac{2m+1}{K_{m_1}(1-m)}\right\}}}\right\}^{\frac{1}{2}}$$

(24)

$$\sigma = \frac{1-m}{\sqrt{3}}\left\{\frac{h^2(4K_{m_1}+1)}{4\left\{2mK_{m_1}(1-m)+(2m+1)\tau^{4-2m}\right\}} + \frac{L}{\tau^{2\left\{(m+2)+\frac{2m+1}{K_{m_1}(1-m)}\right\}}}\right\}^{\frac{1}{2}}$$

(25)

The spatial volume $S^3(t)$ is μ are given by

$$S^3(t) = h\tau^{m+2} \quad (26)$$

$$\mu = \left[\frac{h^2\{K_{m_1}(5+3m)+2\}}{4\tau^{4-2m}} + \left\{\frac{2(m^2-2m-1)-K_{m_1}(m+1)}{K_{m_1}(1-m)}\right\}\left\{\frac{h^2(4K_{m_1}+1)}{4\left\{2mK_{m_1}(1-m)+(2m+1)\tau^{4-2m}\right\}}\right\} + \frac{L}{\tau^{2\left\{(m+2)+\frac{2m+1}{K_{m_1}(1-m)}\right\}}}\right] + \frac{1}{m+2}\left[\frac{h^2(4K_{m_1}+1)}{4\left\{2mK_{m_1}(1-m)+(2m+1)\tau^{4-2m}\right\}} + \frac{L}{\tau^{2\left\{(m+2)+\frac{2m+1}{K_{m_1}(1-m)}\right\}}}\right]^{-\frac{1}{2}}$$

(27)

$$\frac{\sigma}{\theta} = \frac{(1-m)}{\sqrt{3}(m+2)} = \text{constant} \quad (28)$$

4. Discussion

(1) The energy conditions$\rho \geq 0$ leads to

$$\tau^{\left\{\frac{2m+1}{K_{m_1}(1-m)}-4m\right\}} \geq \frac{h^2}{L}\left[1-\frac{(2m+1)(4K_{m_1}+1)}{\{2mK_{m_1}(1-m)+(2m+1)\}}\right]$$

The model starts expanding with a big bang at $\tau = 0$.

(2) The expansion is the model decreases slowly and it stops when $m = -2$ or when $\tau \to \infty$ when $\tau \to 0$ then $\rho \to \infty$ which shows that there is massive mass at $\tau \to 0$.

The spatial volume $S^3(t)$ increases as τ increase provided (m+2)>0.

(3) $\frac{\sigma}{\theta} =$ constant, therefore model does not approach isotropy for large value of τ.

(4) Equation (21) and (23) show that when $n_1 > 1$, the particle density $\rho_p \geq 0$ and the string tension density $\eta \geq 0$, however, $\rho_p > 0$ and $\eta < 0$, however, when $n_1 < 0$. Further, when $n_1 > 2$ or $n_1 < 0$, we have $\rho_p > |\eta|$, therefore in this case the massive strings dominate the universe in the process of evolution.

However, when $1 < n_1 < 2$; we get $\rho_p < |\eta|$, and in this case the strings dominate over the particles.

5. Special Case

When $n_1 = 1$ then $\rho = \eta$, then the line element (1) can be written as

$$ds^2 = -\left[\frac{5h^2}{4(4m-4m^2+1)} + \frac{L}{\tau^{2(2+2m-m^2)}}\right] d\tau^2 + \tau^2(dx^2 + dz^2) + h^2\tau^{2m}(dy - xdz)^2 \qquad (29)$$

The rest energy density ρ, the scalar of expansion θ, the shear σ , the particle density ρ_p and the coefficient of bulk viscosity μ for the model (29) are given by.

$$\rho = \frac{\mathrm{h}^2}{4\tau^{4-2\mathrm{m}}}\left[\frac{2m^2+3m+2}{1+4m-2m^2}\right] + \left[\frac{L}{\tau^{2\frac{(3+m-m^2)}{1-m}}}\right] \qquad (30)$$

$$\theta = (m+2)\left[\frac{5h^2}{4(4m-2m^2+1)\tau^{4-2m}} + \frac{L}{\tau^{2\frac{(3+m-m^2)}{1-m}}}\right]^{\frac{1}{2}} \qquad (31)$$

$$\sigma = \left(\frac{1-m}{\sqrt{3}}\right)\left[\frac{5h^2}{4(4m-2m^2+1)\tau^{4-2m}} + \frac{L}{\tau^{2\frac{(3+m-m^2)}{1-m}}}\right]^{\frac{1}{2}} \qquad (32)$$

$$\rho_p = 0 \qquad (33)$$

$$\mu = \left[\frac{h^2\,(7+3m)}{4\tau^{4-2m}} + \frac{(2m^2-5m-3)}{(1-m)}\left\{\frac{5h^2}{4(1+4m-2m^2)\tau^{4-2m}} + \frac{L}{\tau^{2\frac{(3+m-m^2)}{1-m}}}\right\}\right]\frac{x}{m+2}\left[\frac{5h^2}{4(1+4m-2m^2)\tau^{4-2m}} + \frac{L}{\tau^{2\frac{(3+m-m^2)}{1-m}}}\right]^{\frac{1}{2}} \qquad (34)$$

The energy condition for model (29) leads to

$$\tau^{\frac{2(2m^2-4m-1)}{1-m}} \geq \frac{h^2(m^2+3m+2)}{2l\,(2m^2-4m)}$$

From equation (33) and (30); we get

$\frac{\rho_p}{|\eta|} = 0$, in this case the strings over the particles.

6. Observation

As $\tau \to 0$, the scalar of expansion θ tends to infinitely large and when $\tau \to \infty$, the scalar of expansion $\theta \to 0$. Also at $\tau \to 0$, shear scalar σ tends to infinity and when $\tau \to \infty$, shear scalar σ tends to zero. The energy density $\rho \to \infty$ when $\tau \to 0$ and $\rho \to 0$ when $\tau \to \infty$, therefore model describes a shearing, non rotating continuously expanding universe with a big- bang start $\lim_{\tau\to\infty}\frac{\sigma}{\theta} \neq 0$ so model does not isotropize for large value of τ.

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