Analysis of Fitted Bearing of Fluid Mechanics with Respect to the Effect of Rotation

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Abstract

With Reynolds' standard extended calculations, the second rotating hydrodynamic lubrication theory was developed in accordance with the principles containing the first and second forces of the rotation number. In the present paper, there is a load analysis of the minimum number of rotations and the maximum number of rotations. Load comparisons are made with the help of expressions and calculated tables included in the second order hydrodynamic lubrication rotation. Analysis of load capacity statistics and tables shows that load capacity increases with increasing rotation. With a high rotation number the pressure rises much faster than with a low rotation number. Appropriate tables and graphs confirm this important investigation in the current paper.

Keywords: Continuity, rotation number, Reynolds figure, film thickness.

1. Introduction

Osborne Reynolds himself later received an improved version of the Reynolds Equation known as: "Generalized Reynolds Equation", based on density, viscosity, film thickness, high speed and flexibility. The concept of rotation was discussed by Banerjee et al. [1], [2] and [5] in 1981 that the rotation of the liquid film found throughout the film gives a new impetus to the lubrication problems of the liquid machine. The origin of the rotation can be traced to certain common assumptions related to the fluctuations of the surrounding fluid. Rotation causes part of the fluctuations in the liquid film cycle and the effects from it are prominent, up to a large Taylor Number, causing the pipes to flex as the film turns.

A newer version of the “Generalized Reynolds Equation” is called “Extended Generalized Reynolds Equation” provided by Banerjee et al. [1], [2] and [5], taking into account the effects of the same rotation on the axis lying across the liquid film and depending on the rotation number M, i.e. the square root of Taylor's normal number. A common practice of ancient hydrodynamic lubrication is known as the "Rotatory Theory of Hydrodynamic Lubrication" provided by Banerjee et al. [1], [2] and [5]. The “First Order Rotatory Theory of Hydrodynamic Lubrication” and the “Second Order Rotatory Theory of Hydrodynamic Lubrication” were issued in accordance with principles containing powers reaching up to the first M and the second M respectively, ignoring the higher power M, provided by. Banerjee et al. . [1], [2], [3] and [5]. Journal-sized bears are known as inserted bearings or non-clearance bearings. In these bearings the radial clearance is zero.

2. Governing Equations and Boundary Conditions

The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* and by retaining the terms containing up to second powers of *M* and neglecting higher powers of *M*, can be written as:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+$$

$$\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$

$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right]$$
$$-\rho W^* \qquad (1)$$

Where *x, y* and *z* are coordinates, *P* is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in *z*-direction.

For the case of pure sliding $W^*=0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \qquad (2)$$

Let we assume the bearing to be infinitely long in *x*-direction, which implies that the variation of pressure in *y*-direction is very small as compared to the variation of pressure in *x*-direction i.e., $\frac{\partial P}{\partial x}\gg\frac{\partial P}{\partial y}$, then the equation (3) will be

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right]-\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right](3)$$

Taking *h, U, P* has given

$h = h(x),\ U = -U,\ P = P(x)$ (4)

The film thickness in angular coordinates is given as:

$h = e_0 \cos\theta$ (5)

Here e_0 is the eccentricity.

By rotating the angular coordinate 90°, in the direction of motion, we have

$h= e_0 \sin\theta,\ x = R\theta$ (6)

In view of above conditions, the equation (4) [7], [8], [9], [10], [11], [12] and [14], can be written as:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]=-\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\left\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\right\}\right] \qquad (7)$$

3. Discussions

For the determination of pressure distribution excluding the negative regions, the boundary conditions are as follows:

$P = K\frac{dP}{d\theta} = 0$, at $\theta=\theta_2$, K=constant (8)

Where θ_1 and θ_2 are connected by the condition that

P=0 at $\theta=\theta_1$. (9)

On integrating and using the boundary conditions (9), (10) the equation of pressure after neglecting the higher powers of M and retaining the terms up to M^2 [13], is given as:

$$P = \frac{3\mu UR}{{e_0}^2}\left[F_1(\theta_1) - F_1(\theta)\right] + \frac{M^2\rho^2{e_0}^2\,RU}{280\mu}\left[F_2(\theta_1) - F_2(\theta)\right] \quad (10)$$

Where $F_1(\theta)$ and $F_2(\theta)$ are given by expressions:

$$F_1(\theta) = 2cot\theta - sin\theta_2\left(cosec\theta\,cot\theta - logtan\frac{\theta}{2}\right) \quad (11)$$

$$F_2(\theta) = 17\left(\frac{sin2\theta}{4} - \frac{\theta}{2} - sin\theta_2\,cos\theta\right) - 7\left\{\begin{matrix} sin^5\theta_2\left(logtan\frac{\theta}{2} - cosec\theta\,cot\theta\right) \\ -\theta + \frac{sin2\theta}{2} \end{matrix}\right\} \quad (12)$$

The load capacity for porous bearing is given by

$$W = \sqrt{W_x^2 + W_y^2} \quad (13)$$

Here W_x and W_y are the components of the load capacity in x-direction and y-direction respectively.

$$W_x = \int_{\theta_1}^{\theta_2} LRP\,sin\theta\,d\theta \quad (14)$$

$$W_y = \int_{\theta_1}^{\theta_2} LRP\,cos\theta\,d\theta \quad (15)$$

The W_x and W_y in the increasing values of M, [13] are given by

$$W_x = \frac{3\mu ULR^2}{{e_0}^2}\left[F_1(\theta_1)cos\theta_1 - F_1(\theta_1)cos\theta_2 + G_1(\theta_1) - G_1(\theta_2)\right] + \frac{M^2\rho^2{e_0}^2R^2\,UL}{280\mu}\left[F_2(\theta_1)cos\theta_1 - F_2(\theta_1)cos\theta_2 + G_2(\theta_1) - G_2(\theta_2)\right] \quad (16)$$

$$W_y = \frac{3\mu ULR^2}{{e_0}^2}\left[F_1(\theta_1)sin\theta_2 - F_1(\theta_1)sin\theta_1 + H_1(\theta_1) - H_1(\theta_2)\right] + \frac{M^2\rho^2{e_0}^2R^2\,UL}{280\mu}\left[F_2(\theta_1)sin\theta_2 - F_2(\theta_1)sin\theta_1 + H_2(\theta_1) - H_2(\theta_2)\right] \quad (17)$$

Where $G_1(\theta)$, $G_2(\theta)$, $H_1(\theta)$ and $H_2(\theta)$ are given by the expressions:

$$G_1(\theta) = -2sin\theta + sin\theta_2\,log\,sin\theta - sin\theta_2\left(log\,sin^2\frac{\theta}{2} - 2cos^2\frac{\theta}{2}\,log\,tan\frac{\theta}{2}\right) \quad (18)$$

$$G_2(\theta) = \frac{3}{2}(sin\theta - \theta\, cos\theta) - \frac{1}{2}sin^3\theta - \frac{17}{4}sin\theta_2\, cos2\theta + 7sin^5\,\theta_2\left(log\, sin^2\frac{\theta}{2} - 2cos^2\frac{\theta}{2}\, log\, tan\frac{\theta}{2} - log\, sin\theta\right) \quad (19)$$

$$H_1(\theta) = -2\left(cos\theta + log\, tan\frac{\theta}{2}\right) - sin\theta_2\left(cot\theta + sin\theta\, log\, tan\frac{\theta}{2}\right) \quad (20)$$

$$H_2(\theta) = \frac{3}{2}(cos\theta + \theta\, sin\theta) + \frac{1}{2}cos^3\theta + \frac{17}{2}\, sin\theta_2\left(\theta + \frac{sin2\theta}{2}\right) + 7sin^5\,\theta_2\left(sin\theta\, log\, tan\frac{\theta}{2} + cot\theta\right) \quad (21)$$

4. Numerical Analysis

Let us take the values of mathematical terms as follows:

$\rho = 0.9$,

$\mu = 0.0002$,

$e_0 = 0.3$,

$R = 3.35$,

$U = 500$,

$\theta_1 = 20^0$,

$\theta = 50^0$,

$\theta_2 = 160^0$.

The calculated values of pressure and load capacity are given by the table-1.

Table-1 (The variation of load capacity with respect to Low Rotation Number *M*)

S. No.	M	W
1.	0.1	45449.4097
2.	0.2	296635.8569
3.	0.3	427591.0792
4.	0.4	761970.4658
5.	0.5	1191886.946

Table-2 (The variation of load capacity with respect to High Rotation Number *M*)

S. No.	M	W X 10^5
1.	1	47.74526387
2.	2	191.0508451
3.	3	429.8934814
4.	4	764.2731723
5.	5	1194.189917

5. Conclusions

The obtained pressure equation is given in equation (10). Pressure comparisons were made with the help of geometric calculations, expressions, calculated tables and graphs of bearings included in the rotatory theory of second order hydrodynamic lubrication. Analysis of pressure figures and tables shows that pressures increase significantly with increasing number of rotations. With a high rotation number the pressure rises much faster than with a low rotation number. Appropriate tables confirm this important investigation into the current paper.

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