LRS BIANCHI TYPE-II COSMOLOGICAL MODEL WITH VARYING COSMOLOGICAL CONSTANT Λ

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Abstract

We consider a locally rotationally symmetric Bianchi-II model in the presence of still fluid variable cosmological constantΛ.To solve the Einstein's field equation for LRS Bianchi type II space time has been obtained under the assumption cosmological constant proportional of H where H being Hubble parameter. Some physical and model geometric behavior of the models is discussed.

Keywords: LRS Bianchi type-II, Cosmic strings, perfect fluid

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1. Introduction

Cosmology is the scientific study of large scale properties of the universe as a whole cosmology is study of the motion of crystalline objects. The origin of the universe is greatest cosmological mystery even today. The recent observations that $\Lambda \sim 10^{-55}$ cm^{-2} While the particle physics predication for Λ is greater than this value by a factor of order 10^{120} observations of large scale structure (Tegmark et al. 2004) and cosmic microwave back ground (Benettetal 2003); (spergel et al. 2003a, 2003b) indicate that the universe is highly homogeneous and isotropic on large scales Researchers have proposed a carpological model with cosmological constant of$\Lambda \propto H$. A number of models with different decay laws for the variation of cosmological term investigated during the last two decades Chen & Wu (1990); Pavan (1991). Stiff fluid cosmological models create more interest in the study because for these models, the speed of light is equal to speed of sound and its governing equation have the same characteristics as those of gravitational field. In this paper we study LRS Bianchi type II stiff fluid cosmological model with variable Λ. We obtain solution of the Einstein's field equation assuming the cosmological term proportional to H i.e.,

$\Lambda \propto H$ (Where H is Hubble parameter)

2. Metric and field equation

We consider the LRS Bianchi type II metric in the form

$$ds^2 = -dt^2 + A^2 (dy+ xdz)^2 + B^2 (dx^2+dz^2) \qquad (1)$$

Where A, B are the metric function of cosmic time 't' only.

The energy momentum tensor of a perfect fluid is

$$T_{ij} = (\rho+p)\, v_i v_j + p g_{ij} \qquad (2)$$

Satisfying the equation of state p=p (ρ) in the form us.

$$p=k\rho^m \qquad (3)$$

k, m are polytrophic constant.

Where ρ is the energy density; p is the pressure and v^i is the four velocity vector satisfying.

$$v_i\, v^i = 1 \qquad (4)$$

We take the equation at state p=$\eta\rho$, $0\leq \eta \leq 1$ (5)

Einstein's field equations with time dependent;

Λ and 8 π G = C=1 given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} - \Lambda g_{ij} \quad (6)$$

In co-moving co-ordinate system for line element (1) and energy momentum tensor (2), the field equation (6) yields

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} - \frac{A^2}{4A^4} = \rho - \Lambda \quad (7)$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} - \frac{3A^2}{4B^4} = -p - \Lambda \quad (8)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} - \frac{AB}{AB} + \frac{1}{4}\frac{A^2}{B^4} = -p - \Lambda \quad (9)$$

The spatial volume for the model is given by

$$\xi^3(t) = AB^2$$

$$\xi = (AB^2)^{\frac{1}{3}} \quad (10)$$

Where $\xi = (AB^2)^{\frac{1}{3}}$ is as average scale factor.

The Hubble parameter and deceleration parameter are respectively defined as.

$$H = \frac{\xi_4}{\xi}, \quad q = -\frac{\xi_{44}\xi}{\xi_4^2} \quad (11)$$

Here a suffix '4' indicates an ordinary differentiation with respect to cosmic time t.

The physical quantities of the expansion scalar θ and shear tensor σ^2 are defined as.

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B} \quad (12)$$

$$\sigma^2 = \frac{1}{2}\left(\frac{A_4^2}{A^2} + \frac{2B_4^2}{B^2}\right) - \frac{1}{6}\theta^2 \quad (13)$$

$$= \frac{1}{3}\left(\frac{A_4}{A} - \frac{B_4}{B}\right)^2 \quad (14)$$

3. Solution of the field equation

The field equation (7)−(9) are three equations in five unkown parameter A,B, p, ρ, Λ. There fore, we need extra condition to solve the system completely we assume that expansion scalar is proportional to shear scalar$\theta \propto \sigma$. This condition leads to

$$A = B^m \quad (15)$$

Where 'm' is constant

We also assume that the fluid obeys the stiffs fluid equation state i.e.,

$$p = \rho \quad (16)$$

Omitting ρ from equation (7) & (9) we get

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{3A_4B_4}{AB} + \frac{B_4^2}{B^2} = -2\Lambda \quad (17)$$

From equation (15) taking m=1

$$A = B$$

We put A = B in equation (17) we get

$$\frac{2B_{44}}{B} + \frac{3B_4^2}{B^2} + \frac{B_4^2}{B^2} = -2\Lambda \tag{18}$$

$$\frac{2B_{44}}{B} + \frac{4B_4^2}{B^2} = -2\Lambda \tag{19}$$

$$\frac{B_{44}}{B} + \frac{2B_4^2}{B^2} = -\Lambda \tag{20}$$

From equation (10);

$$\frac{\xi_{44}}{\xi} + \frac{4\xi_4^2}{\xi^2} = -\Lambda \tag{21}$$

We assume that the cosmological constant Λ proportional to the Hubble parameter

$$\Lambda \propto H$$

$$\Lambda = kH$$

$$\Lambda = k\frac{\xi_4}{\xi} \tag{22}$$

From equation (18) we get.

$$\frac{\xi_{44}}{\xi} + \frac{4\xi_4}{\xi^2} = -k \frac{\xi_4}{\xi}$$

$$\frac{\xi_{44}}{\xi} + \frac{4\xi_4}{\xi^2} + k\frac{\xi_4}{\xi} = 0 \tag{23}$$

$$\left(\frac{\xi_{44}}{\xi_4} - \frac{\xi_4}{\xi}\right) + \frac{3\xi_4}{\xi} + k = 0 \tag{24}$$

$$\frac{\frac{d}{dt}\left(\frac{\xi_4}{\xi}\right)}{\frac{\xi_4}{\xi}} + \frac{3\xi_4}{\xi} + k = 0 \tag{25}$$

Integrating we get

$$\log\left(\frac{\xi_4}{\xi}\right) + 3\log\xi + kt = \log k_1$$

$$\log\frac{\xi_4}{\xi} + \log\xi^3 - \log k_1 = -kt$$

$$\xi = \left(-\frac{3k_1}{k}e^{kt} + 3k_2\right)^{\frac{1}{3}} \tag{26}$$

From equation (11);

$$H = k_1 e^{-kt}\left(-\frac{3k_1}{k}e^{kt} + 3k_2\right)^{-1}$$

From equation (22), we get

$$\Lambda = k\,k_1 e^{-kt}\left(-\frac{3k_1}{k}e^{kt} + 3k_2\right)^{-1}$$

$$A = \left(-\frac{3k_1}{k}e^{kt} + 3k_2\right)^{\frac{1}{3}}$$

$$B = \left(-\frac{3k_1}{k}e^{kt} + 3k_2\right)^{\frac{1}{3}}$$

From equation (7), we get

$$\rho = 3k_1^2 e^{-2k}\left(-\frac{3k_1}{k}e^{kt}+3k_2\right)^{-2} - \frac{1}{4}\left(-\frac{3k_1}{k}e^{kt}+3k_2\right)^{-\frac{2}{3}} + kk_1e^{-kt}\left(-\frac{3k_1}{k}e^{kt}+3k_2\right)^{-1}$$

$$\theta = 3k_1\left(-\frac{3k_1}{k}e^{kt}+3k_2\right)^{-1}$$

$$\sigma^2 = -\frac{\sqrt{3k_1}}{3}\left(-\frac{3k_1}{k}e^{kt}+3k_2\right)^{-1}$$

4. **Observations**

1- We see that t=0 the parameter $(H, \Lambda, \rho, \sigma^2)$ are all constant and as t increases exponentially where as the parameters $(H, \Lambda, \rho, \sigma^2)$ decreases.

2- $\frac{\sigma}{\theta}$ =Constant, therefore model does not approach isotropy for large value of t.

3- When $t\longrightarrow \infty$ the scalar of expansion $\theta \longrightarrow 0$. also at $t\longrightarrow 0$ shear scalar σ tends to infinity and $t\longrightarrow \infty$ shear scalar σ tends to zero.

4- As t tends to infinite, the scale factor ξ be were infinity large whereas the parameter $(H, \Lambda, \rho, \sigma^2)$ converges to zero.

This shows that the universe in the model has no initially singularity.

5. Conclusion

We have obtained exact solutions of the LRS Bianchi type II stiff fluid cosmological model with varying Λterm. The field equations have been solved exactly by using $\Lambda \propto H$. H is Hubble parameter. It is integrating the proposed law provides an alternative approach to obtain exact solution of Einstein's field equations.

Recent observational data strongly suggest that this acceleration as well as the cosmological $\Lambda \propto \frac{1}{t^2}$ follows the model in refs. In the present model we obtain$\Lambda\sim H^2$, $\Lambda\sim\frac{\xi_{44}}{\xi}$ & $\Lambda \propto \frac{1}{t^2}$ in accordance with the main dynamical law for the decay of Λ.

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