International Journal of Pure and Applied Researches http://ijopaar.com; 2017 Vol. 4(1); pp. 69-73, ISSN: 2455-474X

LRS BIANCHI TYPE – I COSMOLOGICAL MODEL WITH POLYTROPIC EQUATION OF STATE

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Abstract

We consider a rotationally symmetric Bianchi-I Cosmological model in the presence of perfect fluid with polytropic equation of state $p=K\rho^{v}$*, Where K and* v *are constants called as polytropic constant and polytropic index respectively. To solve the Einstein's field equations for LRS Bianchi type I space time has been obtained under the assumption of the scalar expansion* θ *is proportional to the shear scalar* σ^2*. Some physical and model geometric behaviors of the models are discussed.*

Keywords: LRS Bianchi type-I, polytropic equation of state, Perfect fluid.

MSC2010 Classification: 83C05, 83C15.

PACS Number: 98.80.Es,04.20.-q

1. Introduction

Cosmology is the scientific study of large scale properties of the universe as a whole. Cosmology is study of motion of crystalline objects. The recent observation of large scale structure (Tegmark et al., 2004) and Cosmic Microwave background (Benett et al. 2003) (Spergel et al, 2003 a, 2003b,) indicate that the universe is highly homogenous and isotropic on large scales. Since, long scientists have been searching for the cause behind the acceleration of the universe. It has been attributed to the existence of an exotic matter component which contains about 69.99% of the average energy density in the present universe. It is popularly known dark energy. Also the basic problem in modern cosmology is that of dark energy which is supported to drive the accelerated expansian of the universe (Riess et al.1998); (Permutter et al. 1999). Hence, dark energy models of the universe, as we are aware that the expansion of the universe is undergoing time acceleration (Permatter et al. 1997, 1998, 1999); (Allen et al. 2004); (Peebles et al. 2003); (Patmanabhari, 2003) & (Lima, 2004). With different equations of state have been discussed in literature many attempts have been made by researchers to fill the gap between the cosmic observation and the theory by admitting an exotic matter component with an ad hoc equation of state impressing the pressure p of the component as a function of its energy density.

$$p = \phi\ (\rho) \tag{1}$$

Kamenshchik et al. (2001) studied chaplain gas cosmological model using generalization of equation of state as

$$\phi\ (\rho) = p_0 + \eta\rho + \frac{\gamma}{\rho} \tag{2}$$

Bianchi-I universe using dynamical system theory and quadratic equation of state in the form as

$$\phi\ (\rho) = p_0 + \eta\rho + \gamma\rho^2 \tag{3}$$

Chavanis (2012, 2013, 2014, a, b) has published many papers on equation state

$$\phi\ (\rho) = \eta\rho + \gamma\rho^q \tag{4}$$

in Robertson – walker universe including analytical solutions of the Friedmann equation and connection with inflation. Reddy, Adhav Purandare (2015) examined Bianchi type universe with equation of state in the form.

$$\phi(\rho) = -\rho + \gamma\rho^2 \quad (5)$$

Singh and Bishi (2015) considered the same situation in ϕ (R, T) modified theory of gravity.

We are study and focus on LRS Bianchi type –I Cosmological model with polytropic equation of state.

2. Metric and held equation

We consider the LRS Bianchi type-I metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2(dy^2 + dz^2) \quad (6)$$

Where A, B are the metric function of cosmic time 't' only.

The Einstein field equation are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad (7)$$

The conservation equation of the energy momentum tensor is

$$T;_j{}^{ij} = 0 \quad (8)$$

The energy momentum tensor of perfect fluid

$$T_{ij} = (\rho + p)v_iv_j + pg_{ij} \quad (9)$$

Where ρ is the energy density of the cosmic matter and p is its pressure v_i is the four velocity vector such that

$$g_{ij}\, v^i v_j = 1$$

We take the equation of state

$$p = \omega\rho, \qquad 0 \leq \omega \leq 1 \quad (10)$$

We assume an Equation of P = p (ρ) in form as

$$p = K\rho^v \quad (11)$$

Where, K and n are constants called polytropic constants and polytropic index respectively.

For line element (6) and energy momentum tensor (9) co-moving system of coordinates, the field equation (7) yields.

$$\frac{2A_4B_4}{AB} + \frac{B_4^2}{B^2} = \rho \quad (12)$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = -p \quad (13)$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4B_4}{AB} = -p \quad (14)$$

Here a suffix '4' indicates an ordinary differentiation with respect to cosmic time't'

The energy conservation equation $T;_j{}^{ij} = 0$ leads to the following simple expression

$$\rho_4 + \left(\frac{A_4}{A} + \frac{2B_4}{B}\right)(p + \rho) = 0 \quad (15)$$

The spatial volume V and the average scale factor ξ(t) is given by

$$V = \xi^3(t) = AB^2 \quad (16)$$

$$\xi = (AB^2)^{\frac{1}{3}} \tag{17}$$

The Hubble parameter defined as

$$H = \frac{\xi_4}{\xi} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) \tag{18}$$

The physical quantities of the expansion scalar θ and shear tensor σ^2 are defined as

$$\theta = 3H$$

$$= \frac{3\xi_4}{\xi}$$

$$\theta = 3\,\frac{1}{3}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right)$$

$$\theta = \frac{A_4}{A} + \frac{2B_4}{B} \tag{19}$$

$$\sigma^2 = \sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\sum_{i=1}^{3} H_i^2 - 3H^2\right) \tag{20}$$

The average anisotropy parameter A_m is given by.

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2 \tag{21}$$

Where $\Delta H_i = H_i - H, (i = 1,2,3)$

Represent the directional Hubble parameter in x, y, z directions respectively.

And $A_m = 0$ Corresponds to isotropic expansion.

We assume that expansion scalar is proportional to shear scalar $\theta \propto \sigma$ this condition leads to

$$A = B^n \tag{22}$$

Where $n \neq 0$ is constant

Also the field equation can be written as

$$(2n+1)\frac{B_4^2}{B^2} = \rho \tag{23}$$

$$\frac{2B_{44}}{B} + \frac{B_4^2}{B^2} = -p \tag{24}$$

$$(n+1)\frac{B_{44}}{B} + n^2\frac{B_4^2}{B^2} = -p \tag{25}$$

Subtracting equation (24) from equation (25) we get

$$\left(\frac{B_{44}}{B} - \frac{B_4^2}{B^2}\right) + (n+2)\frac{B_4^2}{B^2} = 0$$

$$\frac{B_{44}B}{B_4^2} + (n+1) = 0$$

$$1 - \frac{d}{dt}\left(\frac{B}{B_4}\right) + n + 1 = 0$$

$$\frac{d}{dt}\left(\frac{B}{B_4}\right) = (n+2) \tag{26}$$

Integrating we get

$$B(t) = b_0[(n+2)t + k_0]^{\frac{1}{n+2}} \tag{27}$$

Where b_0 and k_0 are constants of integration.

From equation (22) we get

$$A(t) = {b_0}^n \left[(n+2)t + k_0\right]^{\frac{n}{n+2}} \tag{28}$$

We put $b_0 = 1$, $k_0 = 0$, the metric (6) can be written as

$$ds^2 = dt^2 - \left[(n+2)t\right]^{\frac{n}{n+2}} dx^2 - \left[(n+2)t\right]^{\frac{1}{n+2}} (dy^2 + dz^2) \tag{29}$$

3. Discussion Remarks

From equation (23) we get the energy density

$$\rho = \frac{(2n+1)}{(n+2)^2 t^2}$$

also from equation (11) we obtain the pressure as

$$p = K\left[\frac{2n+1}{(n+2)^2 t^2}\right]^v$$

The mean Hubble parameter in the model is

$$H = \frac{1}{3t}$$

The special volume is

$$V = (n+2)t$$

The scalar of expansion and shear scalar is

$$\theta = \frac{1}{t}$$

$$\sigma^2 = \frac{(n-1)^2}{(n+2)^2 t^2}$$

$$A_m = \frac{2(n-1)^2}{(n+2)^2}$$

The deceleration parameter

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = 2$$

for n=1 we get A=B and $A_m = 0$ and $\sigma^2 = 0$

4. Conclusion

In this paper we have studied the LRS Bianchi type-I cosmological model with polytropic equation of state. The physical and kinematical parameters which play a key role in the discussion of cosmological models are obtained. We see that the spatial volume of the universe increases with time. The energy density, the pressure the average Hubble parameter, scalar expansion and shear scalar are infinite at $t \to 0$ and approaches Zero as t $\longrightarrow \infty$.

5. Observation

1- $\frac{\sigma}{\theta}$ =Constant, therefore model does not approach isotropy for large value of t.

2- When $t \longrightarrow \infty$ the scalar of expansion $\theta \longrightarrow 0$. Also at $t \longrightarrow 0$ shear scalar $\sigma \longrightarrow \infty$ and $t \longrightarrow \infty$ shear scalar σ tends to zero.

3- As t leads to infinite, the scale factor ξ become infinitely large where as the parameter (H, σ^2, ρ) converges to zero.

4- Average anisotropy parameter and the deceleration parameter shows early inflation and late time acceleration which is the scenario of modern cosmology (Riess et al. 1998 and perlmutter et al. 1999)

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