



Thermal Radiation and Dufour Effects on MHD Flow of Continuously Moving Vertical Surface with Heat Source

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Abstract

Thermal radiation effects arise as a consequence of enthalpy changes due to phase transitions and chemical reaction. These particles touch various organic materials such as tissue damage may and probably will be done. In general the influence of thermal radiation occur when unstable nuclei of atoms decay and release particles. In this paper we investigate the Dufour effects on MHD viscous flow past a porous vertical surface in the presence of thermal radiation and radiation absorption. Using non-dimensional quantities, the governing partial differential equations are converted into ordinary differential equations and them solved analytically by using regular perturbation technique. The results for various flow parameters on fluid velocity, temperature and concentrations as well as the skin coefficient are presented and discussed with graphically.

Keywords: Dufour number, MHD, Viscous fluid, thermal radiation, porous medium, radiation absorption.

1. INTRODUCTION

The Dufour effect describes the energy (heat) flux created when a chemical system is under a concentration gradient. This could also be important in CVD problems. It is the reciprocal phenomenon to the Soret effect. This effect is found in the energy equation. This is enthalpy flux due to a concentration gradient and appears in the energy equation for a multi component mixture. These effects depend on thermal diffusion which is generally very small but can be sometimes significant when the participation species are of widely differing molecular weights (Anwar, 2008), (Bejan, 1996). The efficient utilization of energy is the primary objective in the design of any thermodynamic system, which can be achieved by entropy generation minimization. Theoretical method of entropy generation has been used to treat external and internal irreversibilities. In thermo solutal convection, irreversibilities are mainly due to heat transfer, mass transfer and fluid flow. Second law analysis in heat transfer and thermal design was described in details by Bejan (Bejan, 1982, 96). Generally speaking, the irreversibility phenomena, which are expressed by entropy generation, are related to heat transfer, mass transfer, viscous dissipation, chemical reactions, magnetic field, etc. (Magherbi *et al.* 2006) numerically studied entropy generation in convective heat and mass transfer through an oriented square cavity for the case of aiding buoyancy forces. Sakiadis [18] studied the growth of the two-dimensional velocity boundary layer over a continuously moving flat plate (Vajravelu, 1988). He studied the exact solutions for hydrodynamic boundary layer flow and heat transfer over a continuous, moving, horizontal flat surface with uniform suction and internal heat generation/absorption. Again, Vajravelu (Vajravelu, 1988) extended the problem to a vertical surface. Girish Kumar [8] Chemical reaction effects on MHD flow of continuously moving vertical surface with heat and mass flux through porous medium.

Thermal radiation is due to the movement of atoms and molecules in a material. As these atoms and molecules are composed of charged particles (protons and electrons) and their movements result in the emission of electromagnetic radiation, which carries energy away from

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the surface. One of the important difference between a nuclear and a conventional high-explosive weapon is the large proportion of the energy of a nuclear explosion which is released in the form of the thermal (or heat) radiation. Because of the enormous amount of energy liberated per unit mass in a nuclear weapon, very high temperature is attained. These are estimated to be several tens of million degrees, compared with a few thousand degrees in the case of convectional explosion. As a consequence of these high temperatures, about 70 to 80 percent of the total energy (excluding the energy of the residual radiation) is released in the form of electromagnetic radiation of short wave length. (Anwar et al. 2008); Mansour (Mansour et al. 1990) & Modest (Modest et al. 1993). Soundalgekar and Takhar (Soundalgekar et al. 1993) considered the radiation free convective flow of an optically thin gray gas past a semi-infinite vertical plate. Hossain et al. (Hossain et al. 1999) studied the radiation interaction on combined forced and free convection across a horizontal cylinder and Chamkha et al. (Chamkha et al. 2011) studied radiation effects on the free convection flow past a semi-finite vertical plate with mass transfer. Prasad et al. (Prasad et al. 2007) reported on the radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate. They observed that when the radiation parameter increases, the velocity and temperature decrease in the boundary layer.

Heat is generated when a gas absorbs in the liquid, due to release of heat of dissolution. Heat is also generated when exothermic chemical reactions take place in the liquid phase. Such heat effects will cause a temperature gradient to develop near the gas-liquid interface. The developed temperature profile will influence (i) the solubility of the dissolving component, (ii) the chemical reaction rate constant, and (iii) the diffusivities of the transferring species in the liquid phase. As the mass transfer rate depends on all these. Makinde (Makinde, 2010), Olanrewaju et al. (Olanrewaju et al. 2012) and Gangadhar et al. (Gangadhar et al. 2012) to expose the effect of chemical reaction on MHD heat and mass transfer over a moving vertical plate in presence of heat source along with convective surface boundary condition (Vafai, 2005). Heat and mass transfer effects on Ice growth mechanisms in pure water and aqueous solutions studied by Michael (Kapembwa et al. 2014). Recently Chenna Kesavaiah and Sudhakaraiyah studied (Chenna, 2014). Effects of heat and mass flux to MHD flow in vertical surface with radiation absorption.

In view of the above studies, the present work tends to study the effect of thermal conductivity on heat and mass transfer flow past an infinite vertical plate taking into account the radiation and Dufour effects in the presence of chemical reaction.

2. FORMULATION OF THE PROBLEM

Consider the steady, two - dimensional laminar, incompressible flow of a chemically reacting, viscous fluid on a continuously moving vertical surface in the presence of a uniform magnetic field and Dufour effect with heat generation, uniform heat and mass flux effects issuing a slot and moving with uniform velocity in a fluid at rest. Let the x -axis be taken along the direction of motion of the surface in the upward direction and y -axis is normal to the surface. The temperature and concentration levels near the surface are raised uniformly. The induced magnetic field, viscous dissipation is assumed to be neglected. Now, under the usual Boussinesq's approximation, the flow field is governed by the following equations.

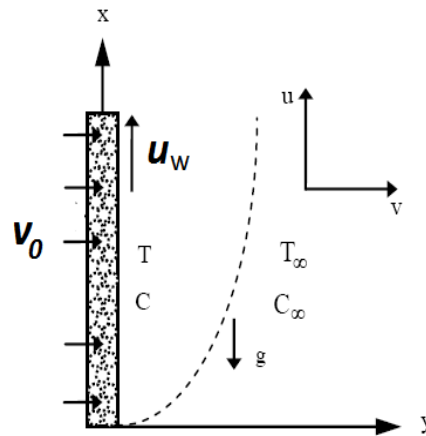


Figure: Flow configuration and coordinate system

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho} - \frac{\nu}{K_p} u \quad (2)$$

Energy equation

$$\rho C_p \left(u \frac{\partial T'}{\partial x} + v \frac{\partial T'}{\partial y} \right) = k \frac{\partial^2 T'}{\partial y^2} - \frac{\partial q_r}{\partial y} - Q_0 (T' - T'_\infty) + Q_l (C' - C'_\infty) + \frac{D_M K_T}{C_s C_p} \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

Diffusion Equation

$$u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} = D \frac{\partial^2 C'}{\partial y^2} - Kr'(C' - C'_\infty) \quad (4)$$

The initial and boundary conditions

$$u = u_w, v = -v_0 \text{ const}, < 0, \frac{\partial T}{\partial y} = -\frac{q}{k}, \frac{\partial C}{\partial y} = -\frac{j''}{k} \text{ at } y = 0 \quad (5)$$

$$u \rightarrow 0, T \rightarrow T'_\infty, C \rightarrow C'_\infty \text{ as } y \rightarrow \infty$$

Where u, v are velocity components in x and y directions respectively. g is the acceleration due to gravity, β is volumetric coefficient of thermal expansion, β^* is the volumetric coefficient of expansion with concentration, T' is the temperature of the fluid, C' is the species concentration, T'_w is the wall temperature, C'_w is the concentration at the plate, T'_∞ is the free stream temperature far away from the plate, C'_∞ is the free stream concentration in fluid far away from the plate, ν is the kinematic viscosity, D is the species diffusion coefficient, Kr is the chemical reaction

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parameter. The term is assumed to be the amount of heat generated or absorbed per unit volume. Q_0 is a constant, which may take on either positive or negative values. When the wall temperature T'_w exceeds the free stream temperature T'_∞ , the source term represents the heat source $Q_0 > 0$ when and heat sink when $Q_0 < 0$. The first term and second term on the right hand side of the momentum equation (2) denote the thermal and concentration buoyancy effects respectively.

In order to write the governing equations and the boundary conditions the following non-dimensional quantities are introduced.

$$Y = \frac{yv_0}{\nu}, U = \frac{u}{u_w}, Pr = \frac{\mu C_p}{k}, Q = \frac{Q_0 \nu}{\rho C_p v_0^2}, Sc = \frac{\nu}{D}, k = \frac{K_p v_0^2}{\nu^2}$$

$$Gr = \frac{\nu g \beta \left(\frac{qv}{kv_0} \right)}{u_w v_0^2}, Gc = \frac{\nu g \beta^* \left(\frac{j'' \nu}{kv_0} \right)}{u_w v_0^2}, T = \frac{T' - T'_\infty}{\left(\frac{qv}{kv_0} \right)}, C = \frac{C' - C'_\infty}{\left(\frac{j'' \nu}{kv_0} \right)} \quad (6)$$

$$Ql = \frac{Ql' j'' \nu}{qv_0^2 \rho C_p}, R = \frac{4qvI}{kv_0}, Kr = \frac{Kr' \nu}{v_0^2}, M = \frac{\sigma B_0^2 \nu}{\rho}$$

In view of (6) the equations (2) – (4) are reduced to the following non-dimensional form

$$\frac{d^2 U}{dY^2} + \frac{dU}{dY} - \left(M + \frac{1}{k} \right) U = -GrT - GrC \quad (7)$$

$$\frac{d^2 T}{dY^2} + Pr \frac{dT}{dY} - (R + Q) Pr T = -Ql Pr C - Du Pr \frac{d^2 C}{dY^2} \quad (8)$$

$$\frac{d^2 C}{dY^2} + Sc \frac{dC}{dY} - Kr Sc C = 0 \quad (9)$$

The corresponding initial and boundary conditions in non-dimensional form are

$$U = 1, \frac{\partial T}{\partial Y} = -1, \frac{\partial C}{\partial Y} = -1 \text{ at } Y = 0 \quad (10)$$

$$U \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } Y \rightarrow \infty$$

The radiative heat flux q_r is given by equation (5) in the spirit of Cogly (Cogly et al. 2014)

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)I$$

(11) where $I = \int_0^\infty K_{\lambda w} \frac{\partial e_{b\lambda}}{\partial T} d\lambda$, $K_{\lambda w}$ – is the absorption coefficient at the wall and $e_{b\lambda}$ – is

Planck's function, I is absorption coefficient

Where Gr is the thermal Grashof number, Gc is the solutal Grashof number, Pr is the fluid Prandtl number, Sc is the Schmidt number and Kr is the chemical reaction parameter, Q is the heat generation/absorption parameter and Q_l is the radiation absorption parameter.

3. METHOD OF SOLUTION

The study of ordinary differential equations (7), (8) and (9) along with their initial and boundary conditions (10) have been solved by using the method of ordinary linear differential equations with constant coefficients. We get the following analytical solutions for the velocity, temperature and concentration

$$U = L_1 e^{m_2 y} + L_2 e^{m_2 y} + L_3 e^{m_4 y} + L_4 e^{m_2 y} + L_5 e^{m_6 y}$$

$$T = B_1 e^{m_2 y} + B_2 e^{m_2 y} + B_3 e^{m_4 y}$$

$$C = A_1 e^{m_2 y}$$

Skin friction

$$\tau = \left(\frac{\partial U}{\partial y} \right)_{y=0} = L_1 m_2 + L_2 m_2 + L_3 m_4 + L_4 m_2 + L_5 m_6$$

Nusselt number

$$Nu = \left(\frac{\partial T}{\partial y} \right)_{y=0} = B_1 m_2 + B_2 m_2 + B_3 m_4$$

Sherwood number

$$Sh = \left(\frac{\partial C}{\partial y} \right)_{y=0} = A_1 m_2$$

APPENDIX

$$\beta = \left(M + \frac{1}{K} \right), m_2 = - \left(\frac{Sc + \sqrt{Sc^2 + 4KrSc}}{2} \right), m_4 = - \left(\frac{Pr + \sqrt{Pr^2 + 4(Q+R)Pr}}{2} \right)$$

$$m_6 = - \left(\frac{1 + \sqrt{1 + 4\beta}}{2} \right), A_1 = \frac{1}{m_2},$$

$$B_1 = - \left(\frac{Q_l Pr A_1}{m_2^2 + Pr m_2 - (Q+R)Pr} \right), B_2 = - \left(\frac{Du Pr A_1 m_2^2}{m_2^2 + Pr m_2 - (Q+R)Pr} \right)$$

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$$B_3 = -\left(\frac{1 + m_2 B_1 + m_2 B_2}{m_4}\right),$$
$$L_1 = -\left(\frac{GrB_1}{m_2^2 + m_2 - \beta}\right), L_2 = -\left(\frac{GrB_2}{m_4^2 + m_4 - \beta}\right), L_3 = -\left(\frac{GrB_3}{m_4^2 + m_4 - \beta}\right)$$
$$L_4 = -\left(\frac{GcA_1}{m_2^2 + m_2 - \beta}\right), L_5 = (1 - L_1 - L_2 - L_3 - L_4)$$

4. RESULTS AND DISCUSSION

A representative set of numerical results is shown graphically in figures to illustrate the influence of physical parameter viz., Chemical reaction parameter (Kr), Magnetic parameter (M), Prandtl number (Pr), Schmidt number (Sc), Permeability parameter (k), thermal Grashof number (Gr), solutal Grashof number (Gc), Radiation parameter (R), Heat generation parameter (Q) on velocity, temperature and concentration profiles, on two-dimensional, incompressible and chemically reacting flow of a viscous fluid on a continuously moving vertical plate in the presence of magnetic field with heat generation. The thermal Grashof number (Gr) and solutal Grashof number (Gc) represents here the effects of free convection currents, and receives positive. The thermal Grashof number (Gr) and solutal Grashof number (Gc) represents here the effects of free convection currents, and receives positive, zero or negative values. The case ($Gr < 0; Gc < 0$) corresponds physically to an externally heated surface as the free convection currents are moving towards the surface. The case ($Gr > 0; Gc > 0$) corresponds to an externally cooled surface and the case ($Gr = 0; Gc = 0$) corresponds to absence of the free convection currents.

Figures (1) and (2) shows the effects of permeability parameter (k) and Dufour effects (Du). We observe that when $Gr > 0; Gc > 0$ the velocity increases and when $Gr < 0; Gc < 0$ leads to fall the velocity. The reverse effects observed for chemical reaction parameter (Kr) and magnetic parameter (M) in figures (3) and (4). It is clearly seen in figure (4) that the effect of increasing the magnetic field strength on the momentum boundary layer thickness is illustrated. Increasing this parameter leads to a decrease in the velocity which confirmed the fact that the magnetic field results in a damping effect on the velocity by creating a drag force that opposes the field motion. Figures (5) and (6) show the effects of Prandtl number (Pr) and heat source parameter (Q). We observe that when $Gr > 0; Gc > 0$ and $Gr < 0; Gc < 0$ lead to fall the velocity in both the parameters. In figure (7) it is clearly seen that an increase in the value of the Radiation absorption (Q_1) leads to an increase in the velocity profile when $Gr > 0; Gc > 0$ and decrease when $Gr < 0; Gc < 0$. In figure (8) it is clearly seen that an increase in the value of the Schmidt number (Sc) leads to a decrease in the velocity profile when $Gr > 0; Gc > 0$ and increase when $Gr < 0; Gc < 0$. This means that the larger the value of Sc , the thinner the

momentum boundary layer size, hence decrease in the velocity. It is clearly seen in figure (9) the effect of Radiation parameter (R) on the temperature profiles. The thermal boundary layer thickness is found to decrease upon increasing the radiation parameter in both $Gr > 0; Gc > 0$ and $Gr < 0; Gc < 0$.

Figure (10) gives the effects of Dufour number (Du) on temperature profiles. Increasing these parameter leads to an increase in the thermal boundary layer thickness. Figure (11) shows the solution of temperature profiles across the boundary layer thickness for different values of chemical reaction parameter (Kr). Increase in this parameter leads to a decrease in the temperature profile. It is clearly seen in figure (12) the effect of Prandtl number (Pr) on the temperature profiles. The thermal boundary layer thickness is found to decrease upon increasing the Prandtl number. It is also observed from the figure (13) when increase in heat source parameter (Q) leads to decrease in temperature profiles. From the figure (14) when increase in radiation absorption parameter (Q_l) increases in temperature profiles. The increase of Radiation parameter (R) and Schmidt number (Sc) decreases the temperature profiles it is seen in figure (15) and (16); it is observed that an increasing the results is decreasing in both the parameter.

The influences of various embedded thermophysical parameters on the fluid concentration had been illustrated in figures (17) and (18). Figure (17) reflects that with increase in chemical reaction parameter (Kr) the fluid concentration decreases. Figure (18) is the graph of concentration profile against y for different values of Schmidt number (Sc). It is not worthy that increase in this parameter decreases the concentration boundary layer thickens.

The dependence of the skin friction, for different values of Schmidt number (Sc) shown is shown in figure (19). It is clear that the skin friction decreases when $Gr > 0; Gc > 0$ and increases $Gr < 0; Gc < 0$ with the increase of Sc .

5. CONCLUSION

In this paper we have studied analytically the effect of thermal radiation, chemical reaction with heat and mass transfer flow past an infinite vertical plate with Dufour effects. The non-dimensional governing equations are solved with the help of perturbation technique. The conclusions of the study are as follows:

- The velocity decreases with an increase in the Prandtl number in both cases.
- The velocity as well as the temperature increases with an increase in Dufour number.
- An increase in the Schmidt number leads to decrease in the concentration.

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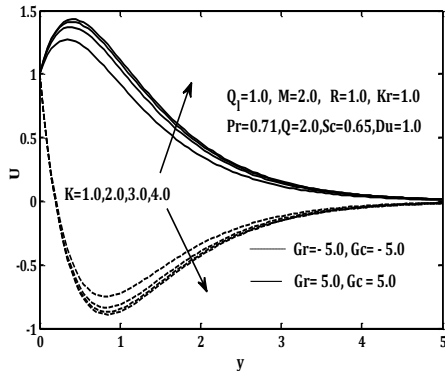


Figure (1): Velocity profiles for different values of K

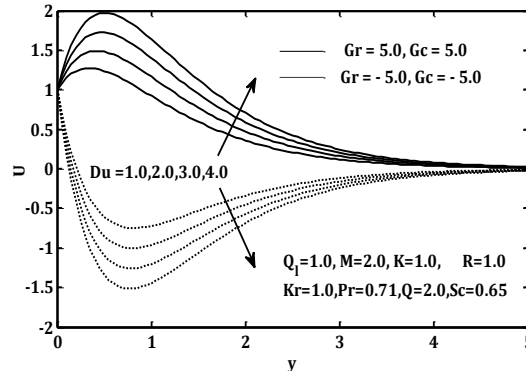


Figure (2): Velocity profiles for different values of Du

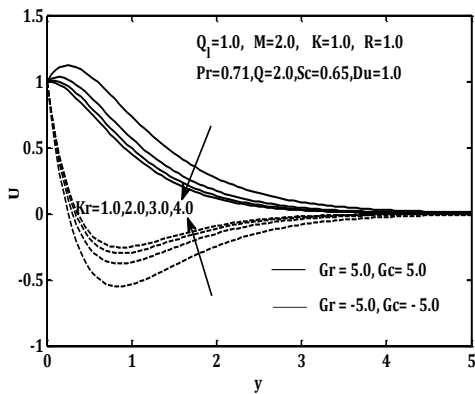


Figure (3): Velocity profiles for different values of Kr

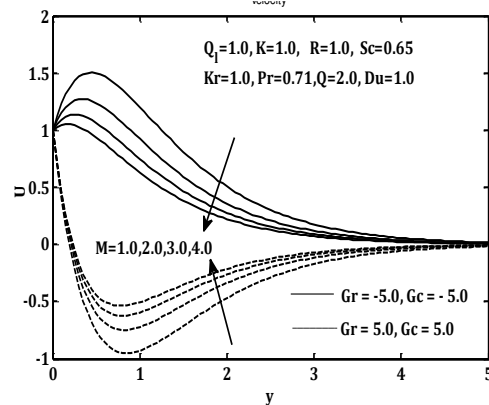


Figure (4): Velocity profiles for different values of M

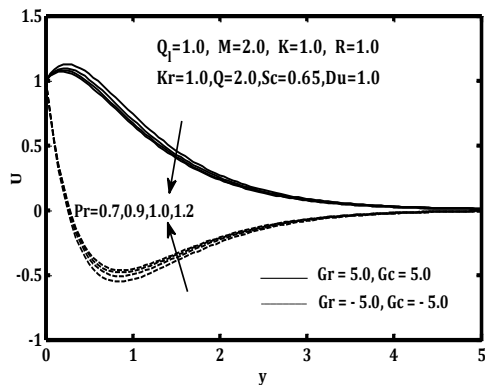


Figure (5): Velocity profiles for different values of Pr

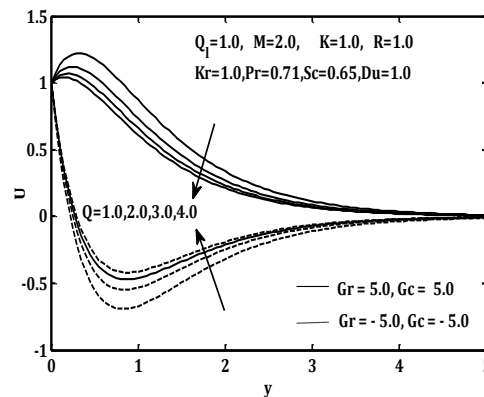


Figure (6): Velocity profiles for different values of Q

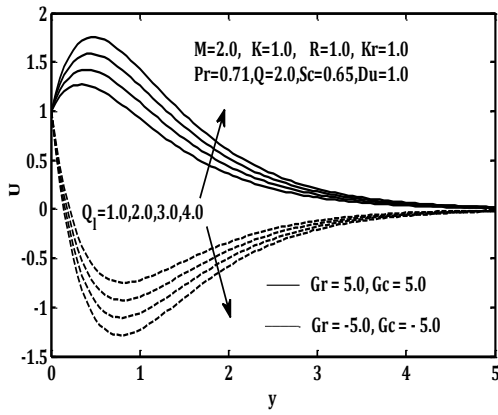


Figure (7): Velocity profiles for different values of Q_1

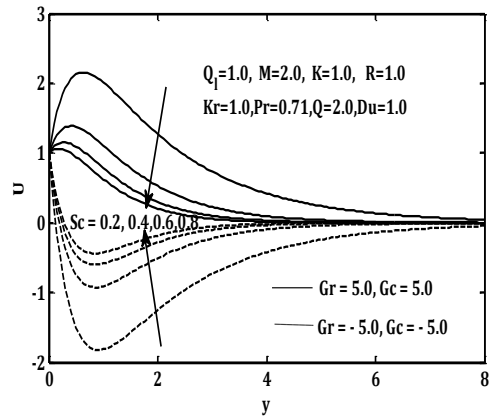


Figure (8): Velocity profiles for different values of Sc

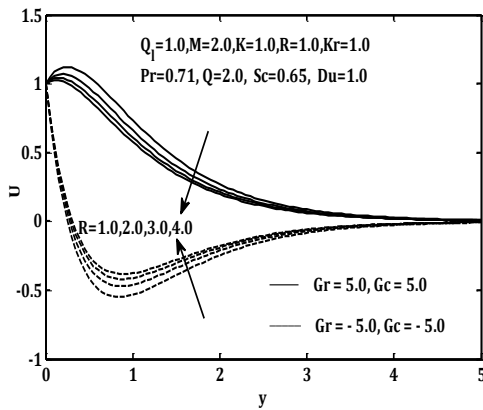


Figure (9): Velocity profiles for different values of R

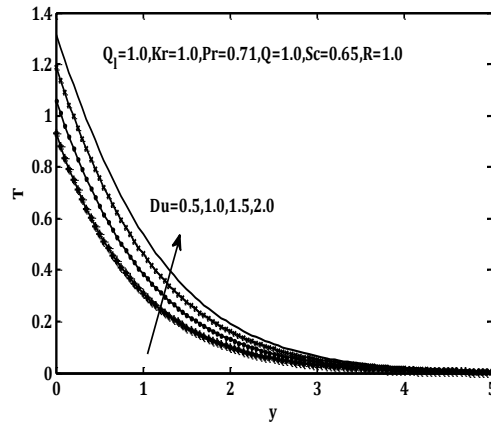


Figure (10): Temperature profiles for different values of Du

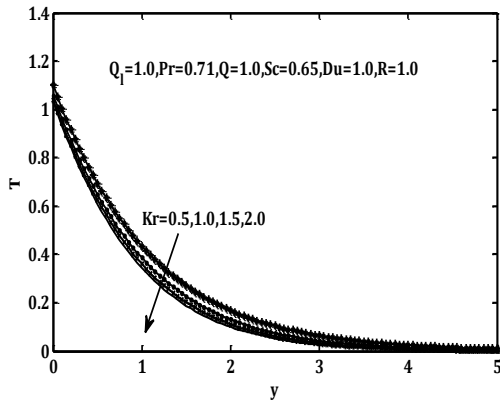


Figure (11): Temperature profiles for different values of Kr

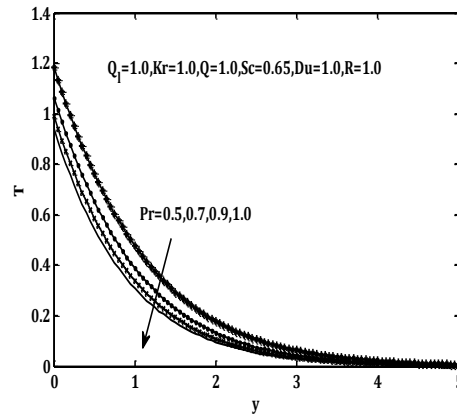


Figure (12): Temperature profiles for different values of Pr

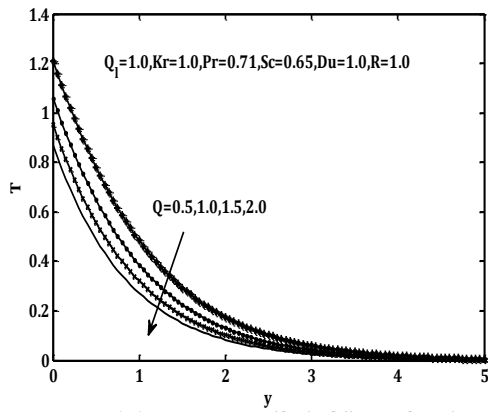


Figure (13): Temperature profiles for different values of Q

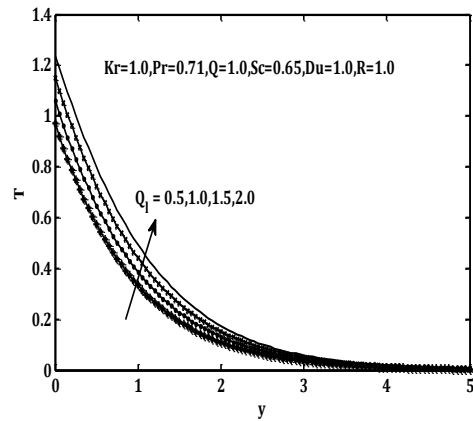


Figure (14): Temperature profiles for different values of Q₁

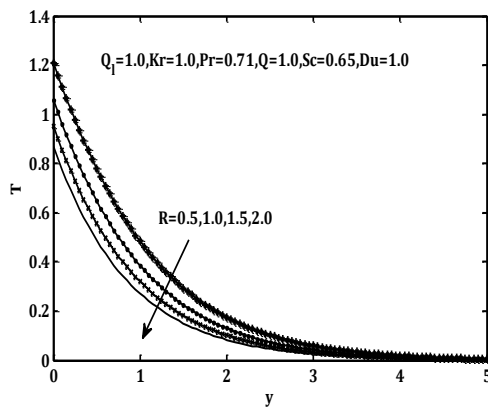


Figure (15): Temperature profiles for different values of R

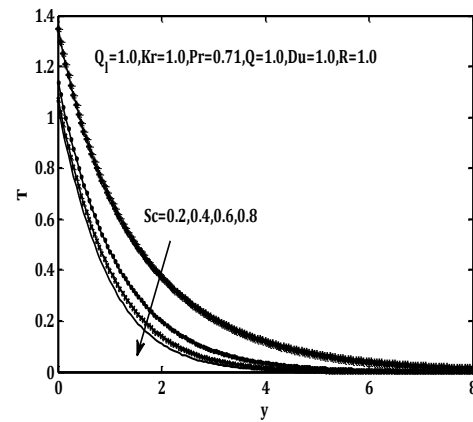


Figure (16): Temperature profiles for different values of Sc

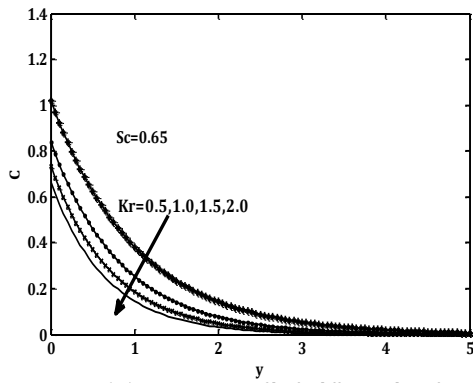


Figure (17): Concentration profiles for different values of Kr

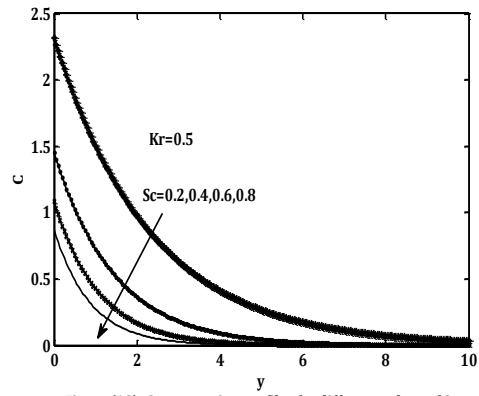


Figure (18): Concentration profiles for different values of Sc

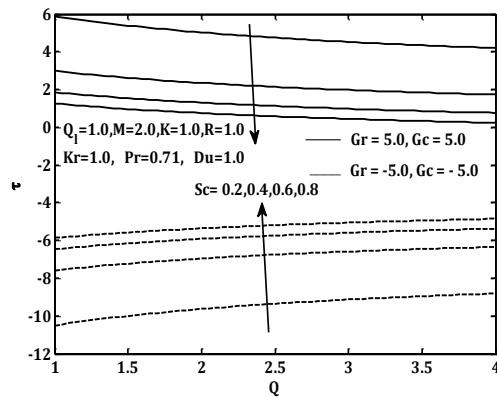


Figure (19): Skin friction for different values of Sc Versus Q