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Extended Generalized Reynolds Equation in Fluid Mechanics

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Abstract

In the theory of hydrodynamic lubrication, two dimensional classical theories were first given by Osborne Reynolds. In 1886, in the wake of a classical Beauchamp Tower's experiment given by Reynolds, he formulated an important differential equation, which was known as: Reynolds Equation given by Reynolds in 1886. Later Osborne Reynolds himself derived an improved version of Reynolds Equation known as Generalized Reynolds Equation, which depends on density, viscosity, film thickness, surface and transverse velocities. The concept of rotation was discussed by Banerjee et al. in 1981 that the rotation of the fluid film which lies across the film gives some new results in lubrication problems of fluid mechanics. The equations for motion of first order rotatory theory and second order rotatory are derived, which have given very important and useful results for journal and thrust bearings.

Keywords: Lubrication theory, Reynolds equation, Rotation number.

1. Introduction

The Reynolds Equation may be a partial differential equation governing the pressure distribution of thin viscous fluid films in Lubrication theory. It shouldn't be confused with Reynolds' alternative namesakes, Reynolds number and Reynolds-averaged Navier–Stokes equations. It absolutely was first derived by Reynolds in 1886. The classical Reynolds Equation may be wont to describe the pressure distribution in nearly any form of fluid film bearing; an impact kind within which the bounding bodies are absolutely separated by a thin layer of liquid or gas. In general, Reynolds equation has got to be resolved by the methods of numerical analysis similar to finite distinction, or finite component. In some simplified cases, however, analytical or approximate solutions may be obtained [13, 17].

The Reynolds number (Re) is a dimensionless quantity in hydrodynamics wont to facilitate predict flow patterns in numerous fluid flow things. At low Reynolds numbers, flows tend to be dominated by stratified (sheet-like) flow, whereas at high Reynolds numbers turbulence results from variations within the fluid's speed and direction, which can generally meet or perhaps move counter to the general direction of the flow (eddy currents). These eddy currents begin to churn the flow, expenditure energy within the method, which for liquids will increase the possibilities of cavitations. The Reynolds number has wide applications, starting from liquid flow in a very narrow passage of air over a craft wing. It's wont to predict the transition from stratified to flow, and is employed within the scaling of comparable however different-sized flow things, similar to between associate craft model in an exceedingly construction and therefore the full size version. The predictions of the onset of turbulence and therefore the ability to calculate scaling effects may be wont to facilitate predict fluid behavior on a bigger scale, similar to in local or global air or water movement and thereby the associated meteorological and climatologically effects [14, 18].

The thought was introduced by Sir George Stokes in 1851, however the Reynolds number was named by Arnold Sommerfeld in 1908 once Reynolds (1842–1912), who popularized its use in 1883. The Reynolds number is that the quantitative relation of mechanical phenomenon forces to viscous forces among a fluid that is subjected to relative internal movement because of completely different fluid velocities, within which is thought as a physical phenomenon within the case of a bounding surface similar to the inside of a pipe. The same impact is made by the introduction of a stream of upper speed fluid, similar to the recent gases from a flame in air. This relative movement generates fluid friction, which may be a think about developing flow. Counteracting this impact is that the body of the fluid, that because it will increase, increasingly inhibits turbulence, as additional K.E. is absorbed by an additional viscous fluid. The Reynolds variety quantifies the relative importance of those two sorts of forces for given flow conditions, and may be a guide to once flow can occur in an exceedingly specific state of affairs [18, 19].

This ability to predict the onset of flow is a crucial style tool for instrumentality similar to piping systems or craft wings, however the Reynolds number is additionally utilized in scaling of fluid dynamics issues, and is employed to work out dynamic similitude between two completely different cases of fluid flow, similar to between a model craft, and its full size version. Such scaling isn't linear and therefore the application of Reynolds number to each thing permits scaling factors to be developed [18, 19].

For the case of rigid sphere on flat pure mathematics, steady-state case and half-Sommerfeld cavitation stipulation, the 2-D Reynolds equation may be resolved analytically. This answer was projected by Kapitza. Half-Sommerfeld stipulation was shown to be inaccurate and this answer has got to be used with care [18, 19].

In case of 1-D Reynolds equation many analytical or semi-analytical solutions are accessible. In 1916 Martin obtained a closed type answer for a minimum film thickness and pressure for a rigid cylinder and geometry. This answer isn't correct for the cases once the elastic deformation of the surfaces contributes significantly to the film thickness. In 1949, Grubin obtained associate approximate answer for therefore known as elasto-hydrodynamic lubrication (EHL) line contact downside, wherever he combined each elastic deformation and material fluid mechanics flow. During this answer it absolutely was assumed that the pressure profile follows Hertz answer. The model is so correct at high masses, once the fluid mechanics pressure tends to be on the point of the Hertz contact pressure [15-20].

In general the bearings [12, 13] may be divided in to four categories:

(1) Rolling component bearings for example; ball, cylindrical, spherical or tapered roller and needle etc.

(2) Dry bearings for example; plastic bushings, coated metal bushings etc.

(3) Semi-lubricated bearings for example; oil-impregnated bronze bushings etc.

(4) Fluid film bearings for example; shaft bearings etc.

Except from some radial-configuration craft engines, most piston engines use fluid film bearings [7]. This is often true for the shaft and generally within the rotating shaft, though typically the later runs directly within the engine structure. Here we've got to debate the operating of the fluid film operating and to demonstrate however engine designers are reducing friction losses through bearing technology [12]. The fluid film bearings operate by generating, as a by-product of the relative motion between the shaft and therefore the bearing, an awfully thin film of material at a sufficiently air mass to match the applied load, as long as that load is among the bearing capability. Fluid film bearings represent a type of scientific method, by virtue of providing terribly massive load carrying capabilities in an exceedingly compact, light-weight implementation, and in contrast to the opposite categories, in most cases may be designed for infinite life. The fluid film bearings operate in any of the three modes:

(a) Fully-hydrodynamic

(b) Boundary

(c) Mixed.

In absolutely fluid mechanics or "full-film" [7, 13] lubrication, the moving surface of the journal is totally separated from the bearing surface by an awfully thin film of material. The applied load causes the line of the journal to be displaced from the line of the bearing. This eccentricity creates a circular "wedge" within the clearance house.

The material, by virtue of its body, clings to the surface of the rotating journal, and is drawn into the wedge, making an awfully air mass, that acts to separate the journal from the bearing to support the applied load.

The bearing eccentricity is expressed because the line displacement divided by the radial clearance. The bearing eccentricity will increase with applied load and reduces with bigger journal speed and body. The fluid mechanics pressure has no relationship in any respect to the engine pressure level, except that if there's deficient engine pressure level to deliver the specified copious volume of oil into the bearing, the fluid mechanics pressure mechanism can fail and therefore the bearing and journal are going to be destroyed. The pressure distribution within the fluid mechanics region of a fluid film bearing will increase from quite low within the massive clearance zone to its most at the purpose of minimum film thickness for the incompressible fluid like oil is force into the connection "wedge" [4, 5] zone of the bearing. However, this radial profile doesn't exist homogeneously across the axial length of the bearing. If the bearing has sufficient breadth, the profile can have a virtually flat across the hard-hitting region.

The second mode of bearing operation is boundary lubrication. In boundary lubrication, the "peaks" of the inclined surfaces i.e., journal and bearing, are touching one another, however there's additionally an especially thin film of the material solely a number of molecules thick that is found within the surface "valleys". That thin film tends to scale back the friction from what it'd be if the surfaces were utterly dry.

The mixed mode may be a region of transition between boundary and full-film lubrication. The surface peaks on the journal and bearing surfaces part penetrate the fluid film and a few surface contacts happens, however the fluid mechanics pressure is beginning to increase.

When motion starts, the journal tries to jump on the wall of the bearing because of the metal-to-metal friction between the two surfaces. If there's associate adequate provider of material, the motion of the journal starts to pull the material into the wedge space and fluid mechanics lubrication begins to occur beside the boundary lubrication.

If we tend to assume that the load and body stay comparatively constant throughout this startup amount, then as revolution per minute will increase, the fluid mechanics operation strengthens till it’s absolutely developed and it moves the journal into its steady state orientation. The direction of the eccentricity and therefore the minimum film thickness, don't occur in line with the load vector and are angularly displaced from the load.

There are another type of fluid-film lubrication, which has the squeeze-film lubrication [12] i.e., the piston engine etc. Squeeze-film action relies on the actual fact that a given quantity of your time is needed to squeeze the material out of an impact axially, thereby adding to the fluid mechanics pressure, and so to the load capability. Since there's very little or no vital rotating action within the wrist-pin bores, squeeze-film fluid mechanics lubrication is that the prevailing mechanism that separates wrist joint pins from their bores within the rods and pistons.

In general the two sorts of cavitations are type within the bearing.

(a) Gassy cavitations: this is often related to air and gases mixed with material. If the pressure of material falls below the air pressure then the gases embark to create the cavitations.

(b) Vapors cavitations: this is often shaped once the load applied to the bearing fluctuates at the high frequency. The pressure of fluid falls quickly and causes the cavitations because of quick evaporation.

Now the fluid pressure creates the supporting force that separates the journal from the surface of the bearing. The fluid mechanics force of friction and force of fluid pressure counterbalance the external load. Therefore the position of journal may be determined by these forces. Within the fluid mechanics regime, the journal climbs within the movement direction. If the operating of journal is within the boundary and mixed lubrication then the fluid mechanics pressure ends and therefore the journal climbs within the opposite to the movement direction.

2. Second Order Rotatory Theory of Hydrodynamic Lubrication

The dimensional classical theory theory [4], [12] of fluid mechanics lubrication was the 1st given by Reynolds [13]. In 1886, after an earlier examination of the Beauchamp Tower [14], he made an equation known as: Reynolds Equation [13]. The composition and basic structure of the liquid film were analyzed by the experiments taking into account other factors:

[a] The thickness of the film is very small compared to the axial and longitudinal thickness of the liquid film.

[b] If the load layer is to transfer pressure between the shaft and therefore the load, the layer must have a flexible thickness.

Osborne Reynolds itself is based on the “Generalized Reynolds Equation” [7], [12], based on body weight, film thickness, high speed and transversal velocities. The figure obtained by Reynolds initially contained a non-pressurized liquid, so it was fully developed to speak enough to incorporate the effects of flexible and dynamic loading and was similar to the Generalized Reynolds Equation. Therefore the final version of the Generalized Reynolds Equation [12], [14] is given:

In the second phase of the hydrodynamic lubrication theory "Extended Generalized Reynolds Equation" [7] is provided by:

$$\frac{\partial}{\partial x}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$
$$+\frac{\partial}{\partial y}\left[-\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}-\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial y}\right]$$
$$-\frac{\partial}{\partial y}\left[-\frac{h}{M}+\sqrt{\frac{2\mu}{M\rho}}\,\frac{1}{M}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}}+\sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}}+\cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\frac{\partial P}{\partial x}\right]$$

$$= -\frac{U}{2}\frac{\partial}{\partial x}\left[\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} + \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} + \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\frac{U}{2}\frac{\partial}{\partial y}\left[-\rho\sqrt{\frac{2\mu}{M\rho}}\left(\frac{\sinh h\sqrt{\frac{M\rho}{2\mu}} - \sin h\sqrt{\frac{M\rho}{2\mu}}}{\cosh h\sqrt{\frac{M\rho}{2\mu}} - \cos h\sqrt{\frac{M\rho}{2\mu}}}\right)\right]$$
$$-\rho W^* \qquad (1)$$

Where *x, y* and *z* are coordinates, *P* is the pressure, ρ is the fluid density, μ is the viscosity and W^* is fluid velocity in *z*-direction. The Extended Generalized Reynolds Equation in view of second order rotatory theory of hydrodynamic lubrication, in ascending powers of rotation number *M* and by retaining the terms containing up to second powers of *M* and neglecting higher powers of *M*, can be written as:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]+$$
$$\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$= -\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right] \quad -\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\}\right]$$
$$-\rho W^* \qquad (2)$$

For the case of pure sliding $W^* = 0$, so we have the equation as given:

$$\frac{\partial}{\partial x}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial x}\right]+\frac{\partial}{\partial y}\left[-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\rho\frac{\partial P}{\partial y}\right]$$
$$+\frac{\partial}{\partial x}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial y}\right]-\frac{\partial}{\partial y}\left[-\frac{M\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\frac{\partial P}{\partial x}\right]$$
$$= -\frac{\partial}{\partial x}\left[\frac{\rho U}{2}\{h-\frac{M^2\rho^2h^5}{120\mu^2}\left(1-\frac{31M^2\rho^2h^4}{3024\mu^2}\right)\}\right] \quad -\frac{\partial}{\partial y}\left[\frac{M\rho^2U}{2}\left\{-\frac{h^3}{12\mu}\left(1-\frac{17M^2\rho^2h^4}{1680\mu^2}\right)\right\}\right] \qquad (3)$$

3. Conclusions

The fluid mechanics force of friction and force of fluid pressure counterbalance the external load. Therefore the position of journal may be determined by these forces. Within the fluid mechanics regime, the journal climbs within the movement direction. If the operating of journal is within the boundary and mixed lubrication then the fluid mechanics pressure ends and therefore the journal climbs within the opposite to the movement direction.

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