# Mathematical Formulation on the Multi-phase Flow through Composite Stenosis

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**Abstract**

*The present article provides a concept concerning the mathematical model describing flow of blood through a Composite Stenosis Artery. Stenosis or sclerosis is abnormal and strange condition of the obstruction of flow of blood across the semi lunar valve resulting in serious consequences. The matter of stricture is increasing at dreaded rate within the developing and also the underdeveloped countries. During this paper there's analysis of the mathematical laws and equations of blood flow through a composite stenosis in an artery and a few vital results are found. The result shows that the speed of the blood varies reciprocally with the radius of artery. The pressure exerted by the blood varies directly with its rate.*

**Keywords:** Formulation, Multi-phase flow, Stenosis.

**1. Introduction**:

Due to the rising complications within the human and animal body just like the drawback of stricture etc, a brand new branch of science named biomechanics has been developed. So as to debate blood flow through a composite stricture in associate artery, we tend to should bear in mind concerning the properties of blood like its composition, blood consistency etc. Since biomechanics involves an excellent facilitate within the discussion it'd be higher to understand concerning Biomechanics 1st. (Caro, C. G. et al., 1978), (Charm, S. E. and Kurland, G. S., 1965).

The word Biomechanics has been derived by combining 2 Greek words bios (meaning- life) and mechanike (meaning – mechanics). Biomechanics could be a government department of science that's closely associated with engineering in addition as biology. During this branch the ways of applied science helps to research the Biological systems. By the Biological systems we tend to mean human, animal, plants, fungi and cells. Hence during this science we tend to study the perform and structure of human, animals, plants etc. as a result of this combines the sector of applied mechanics to the sector of biology i.e. , in biomechanics, ancient engineering techniques area unit applied to unravel and illustrate the issues associated with biology. The branch of science is predicated on the conception that man-built systems area unit a lot of easier than the biological systems, therefore easy numerical methodology will be accustomed study each Biological drawback. Thus mathematical modeling, process simulations and experimental measurements will be accustomed solve issues associated with humans and animal body. Several branches of applied mechanics play a vital role within the study of biological systems. A number of the foremost of times used branches of mechanics are mechanism analysis, structural analysis, Kinematics, Dynamics etc. In recent years scope and applications of biomechanics has greatly augmented (Ahmed, P. S. and Giddens, D. P., 1983), (Bandyopadhyay, S. and Layek, G. C., 2012), (Ku, D. N., 1997).

The credit for the event of Biomechanics really goes to the philosopher who was the primary person to create the association between physics and living sciences. The fifteen century witnessed the trendy development of biomechanics. One in all the most contributors during this field was William Harvey (1578-1657) who proven that blood should travel in closed manner within the vascular system, though at that point there was no construct of the existence of blood vessels and even the magnifier wasn't fictitious. Astronomer proposes his theory that declared that the direction of flow of blood out of the verticals is in one direction solely. By calculations he proven that capability of the guts was 2 ounces per beat. Italian stargazer Giovanni Alfonso Borelli (1608-1679) explained the action of muscles through the mechanical ideas (Huckaba, C. E. and Hahu A. W., 1968), (Cokelet, G. R., 1972), (Deshpande, M. D. et al., 1976).

**2. Formulation of the Problem**

The flow of blood through blood capillaries are often thought of to be a multi-phase flow owing to its composition i.e., the liquid half, plasma and also the cellular parts that are, WBCs, RBCs and blood platelets. Currently to represent the matter of stenosis in type of equations we tend to shall take into thought the blood flow through a stenosis in artery of circular cross section.

$$\frac{R(z)}{R_0} = 1 - 2\frac{2\delta}{R_0 L_0}(Z-d) \quad ; \quad d \le d + L_0/2\,, \tag{1}$$

$$=1 - \frac{\delta}{2R_0}\left[1 + cos\frac{2\pi}{L_0}\,(z - d - L_0/2\right]; \; d + L_0/2 \le z \le d + L_0 \tag{2}$$

$$=1 \quad ; \text{otherwise}, \tag{3}$$

In the above equation, the radius of artery with stenosis is given by $R \cong R(z)$ and without stenosis is given by $R_0$, the length and location of stenosis are represented by $L_0$ and d respectively, also $\delta$ represents the maximum projection in lumen located at $z=d+L_0/2$.

**3. Mathematical Solution**

As the blood is the mixture of erythrocytes (red cells) and plasma therefore the blood flow through a stenosis in an artery is supposed to be a two–phase. The equation that describes the two phase flow of model of blood is given by (Srivastava et al., 2007, 09, 12).

$$(1-C)\rho_f\left\{u_f\frac{\partial U_f}{\partial z}+v_r\frac{\partial u_f}{\partial r}\right\}$$
$$=-(1-C)\frac{\partial p}{\partial z}+(1-C)\mu_s(C)(\nabla^2-\frac{1}{r^2}v_f+CS\left(v_p-v_f\right) \quad (4)$$

$$\frac{\partial}{\partial r}\left[(1-C)v_f\right]+(1-C)\frac{v_f}{r}+\frac{\partial}{\partial z}\left[(1-C)u_f\right]=0 \quad (5)$$

$$\frac{\partial}{\partial r}\left[Cv_p\right]+\frac{Cv_p}{r}\frac{\partial\left[Cu_p\right]}{\partial z}=0 \quad (6)$$

Here "r" denotes the radial coordinate directed perpendicular to axis of the tube, ($u_f$, $v_f$) and ($u_p$, $v_p$) denotes the axial and radial components of the fluid and particle velocities respectively, C denotes the volume fraction density of the particles, p denotes the pressure, $\mu_s(C)\approx\mu_s$ is the mixture viscosity , (Ku, D. N., 1997), (Mishra, S. and Siddiqui, S. U., 2012); S denotes the drag coefficient of interaction for the force exerted by one phase on the other, $\rho_r$ and $\rho_p$ show the actual density of the material constituting the fluid, i.e. the plasma and the particle i.e., erythrocyte phases respectively, fluid phase is given by (1-C)$\rho_f$ and the particle phase densities is given by C$\rho_p$. Also here $\nabla^2 = \partial^2/\partial^2 + (1/r)$ $(\partial/\partial r) +\partial^2/\partial z^2$

is a two-dimensional Laplacian operator. In the above analysis the subscripts *f* represents the quantities associated with plasma and *p* represents the quantities associated with erythrocyte phases (Medhavi, A. et al., 2012), (Mekheimer et al., 2011). The expressions for the drag coefficient of interaction, S and the viscosity of the suspension, $\mu_s$ is given as

$$S=\frac{9}{2}\frac{\mu_0}{a_0^2}\frac{4+3\left[8C-3C^2\right]^{\frac{1}{2}}+3C}{(2-3C)^2},\mu_s(C)=\frac{\mu_0}{1-mC} \quad (7)$$

$$m = 0.070\, e^{[2.49C+ (1107/T) \exp (-1.69C)]} \quad (8)$$

Here $\mu_0$ is constant plasma viscosity and $a_0$ is the radius of a red cell. In this discussion Temp T is measured in Kelvin scale.

For a mild stenosis (i.e. stenosis for which $\partial/R_0 << 1$) the equations that govern the laminar, steady, one-dimensional flow of blood in and artery are as follows (Srivastava, V. P. et al., 2007, 12).

$$(1-C)\frac{dp}{dz}=(1-C)\frac{\mu_s}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\mu_f+CS\left(u_p-u_f\right) \quad (9)$$

$$C\frac{dp}{dz}=CS\left(u_p-u_f\right) \quad (10)$$

The boundary conditions for the stenosis problem are as follows

$$\frac{\partial\mu_f}{\partial r}=0\ at\ r=0;\ \mu_f=0\ at\quad r=R(z) \quad (11)$$

The solution of the differential equation under the boundary conditions is given as:

$$u_f = -\frac{R_0^2}{4(1-C)\mu_s}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2\right\} \tag{12}$$

$$u_p = -\frac{R_0^2}{4(1-C)\mu_s}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2 + \frac{4(1-C)\mu_s}{SR_0^2}\right\} \tag{13}$$

The volumetric flow rate Q is given by

$$Q = -\frac{\pi R_0^4}{8(1-C)\mu_s}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^4 - \beta\left(\frac{R}{R_0}\right)^2\right\} \tag{14}$$

$$\frac{dp}{dz} = -\frac{8(1-C)\mu_s Q}{\mu R_0^4}\ \emptyset(z) \tag{15}$$

With $\beta = 8C(1-C)\mu_0/SR_0^2$, a non-dimensional suspension parameter, and

$$\emptyset(z) = 1/F(z)F(z) = (R/R_0)^4 + \beta(R/R_0)^2$$

The pressure drop, $\Delta p(= p\ at\ z = -L, -p\ at\ z = L)$ across the stenosis in the tube of length, L is obtained as

$$\Delta p = \int_{-L}^{L}\left(-\frac{dp}{dz}\right)dz = \frac{8(1-C)\mu_s Q}{\pi R_0^4}\Psi \tag{16}$$

Where

$$\Psi = \int_0^4 [\emptyset(z)]_{\frac{R}{R_0}=1}dz + \int_d^{d+L_0/2} [\emptyset(z)]_{\frac{R}{R_0}from(1)}dz + \int_{d+L_0/2}^{L_0} [\emptyset(z)]_{\frac{R}{R_0}from(2)}dz + \int_{L_0}^{L} [\emptyset(z)]_{\frac{R}{R_0}=1}dz$$

Now the analytical evaluation of the first and fourth integrals of the above expression for $\Psi$ is easy but the evaluation of the second and the third integrals a much difficult job so that it would be better to evaluate these quantities numerically. As discussed by (Young, 1979), (Srivastava, 2007), the expression for the impedance, i.e., flow resistance $\lambda$, the wall shear stress in the stenotic region $\tau_w$ and the shearing stress at the stenosis throat $\tau_s$ is formulated as (Ku, D. N., 1997), (Medhavi, A, 2011).

$$\lambda = (1-C)\mu \left[ \frac{1-\frac{L_0}{L}}{1+\beta} - \frac{R_0 L_0}{2\beta\delta L}\left\{1 - \frac{1}{1-\frac{\delta}{R_0}} + \frac{1}{\beta}tan^{-1}\frac{\frac{\delta\sqrt{\beta}}{R_0}}{1+\beta-\frac{\delta}{R_0}}\right\} + \frac{L_0}{2\pi L}\int_0^{\pi}\frac{d\theta}{(a+bcos\theta)^2[(a+bcos\theta)^2+\beta]} \right] \tag{17}$$

$$\tau_w = \frac{(1-C)\mu}{(R/R_0)^3 + \beta(R/R_0)} \tag{18}$$

$$\tau_s = \frac{(1-C)\mu}{(1-\delta/R_0)^3 + \beta(1-\delta/R_0)} \tag{19}$$

$$where\ \lambda = \frac{\bar{\lambda}}{\lambda_0}, \quad (\tau_w, \tau_s) = \frac{(\overline{\tau_w}, \overline{\tau_s})}{\tau_0}, \quad \bar{\lambda} = \frac{\Delta p}{Q}, \quad \overline{\tau_w} = -\frac{R}{2}\left(\frac{dp}{dz}\right),$$

$$\overline{\tau_s} = \left[-\frac{R}{2}\frac{dp}{dz}\right]_{R/R_0=(1-\delta/R_0)}, \quad \mu = \frac{\mu_s}{\mu_o}, \quad \lambda_0 = \frac{8\mu_0 L}{\pi R_0^3}, a = 1-\frac{\delta}{2R_0}, \quad b = \frac{\delta}{2R_0},$$

$$\theta = \pi - \frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)$$

In above expressions $\lambda_0$ and $\tau_0$ are the flow resistance and wall shear stress for the normal artery i.e., artery having no stenosis in the absence of the particle phase i.e., C=0, Newtonian fluid (Nadeem, S. et al., 2011), (Ponalagusamy, R. et al., 2011).

By the discussed equations; we can conclude that, in the absence of the particles, i.e., C=0, the results for a Newtonian fluid are as follows:

$$\lambda_N = 1 - \frac{L_0}{6\delta}\left[1 - \frac{1}{\left(1-\frac{\delta}{R_0}\right)^3}\right] + \frac{L_0}{2\pi L}\int_0^{\pi}\frac{d\theta}{(a+bcos\theta)^4} \tag{20}$$

$$\tau_{wN} = \frac{1}{(R/R_0)^3}\ ; \ \tau_{sN} = \frac{1}{(1-\delta/R_0)^3} \tag{21}$$

## 4. Numerical Results and Discussion

Hence for observing the quantitatively effects of hematocrit and other parameters for the blood flow Computational analysis of the results obtained by various equations for the tube of radius 0.01 cm at temperature of $37^0$C is done. The values of different parameters are as follows:

d(cm) = 0; L0 (cm ) = 1; L(cm) = 1, 2, 5 ; C = 0, 0.2, 0.4, 0.6; $\delta/R_0$ =0, 0.05, 0.10, 0.15, 0.20.

It will be noted that the present study corresponds to a case of the Newtonian fluid and no stenosis for the parametric values C=0 and δ/R = 0 respectively.

## 5. Conclusions

A large two-phase model of blood has been wont to observe the consequences of hematocrit on blood flow characteristics owing to the presence of a light pathology. The varied properties of the flow of blood like, the flow resistance, the wall shear stress within the stenosed region and also the shear stress at the pathology throat increase with the hematocrit also like the pathology size. The shear stress at the pathology throat has similar properties to it of the resistivity with reference to the other parameter.

## References


1. Ahmed, P. S. and Giddens, D. P., (1983), Velocity measurements in steady flow through axisymmetric stenosis at moderate Reynolds number. J. Biomech. 16, 505-516.
2. Bandyopadhyay, S. and Layek, G. C., (2012), Study of magneto-dydrodynamic pulsatile flow in a constricted channel. Commum. Nonlinear Sci. Numer. Simulat. 17, 2434-2446.
3. Caro, C. G., Pedley, T. J., Schroter, R.C. and Seed, W.A., (1978), The Mechanics of the circulation, Oxford Medical, N.Y.
4. Charm, S. E. and Kurland, G. S., (1965), Blood Rheology in Cardiovascular Fluid Dynamics, Academic Press, London.
5. Cokelet, G. R., (1972), The Rheology of Human Blood: In Biomechanics, Prentice-Hall, Englewood Cliffs, N. J.
6. Deshpande, M. D. Giddens, D. P. and Mabon, R. F., (1976), Steady laminar flow through modeled vascular stenoses. J. Biomech. 9, 165-174.
7. Huckaba, C. E. and Hahu A. W., (1968), A generalized approach to the modeling of arterial blood flow. J. Appl. Physiol. 27, 27-34.
8. Ku, D. N., (1997), Blood flow in arteries. Ann. Rev. Fluid Mech. 29, 399-434.
9. Medhavi, A., (2011), On macroscopic two-phase arterial blood flow through an overlapping stenosis. E-Journal of Science and Technology 6, 19-31.
10. Medhavi, A., Srivastav, R. K., Ahmad, Q. S., and Srivastava, V. P., (2012), TWO-PHASE ARTERIAL BLOOD FLOW THROUGH A COMPOSITE STENOSIS. e-JST, (4), 7, pp. 83-95.
11. Mekheimer, Kh. S. and El-Cot, M. A., (2008), Magnetic field and hall current on blood flow through stenotic artery. Appl. Math. And Mech. 29, 1-12.
12. Mekheimer, Kh. S., Harun, M. H. and Elkot, M. A., (2011), Induced magnetic field influences on blood flow through an anisotropically tapered elastic arteries with overlapping stenosis in an annulus. Can. J. Phys. 89, 210-212.

13. Mishra, B. K. and Verma, N., (2007), Effect of porous parameter and stenosis on wall shear stress for the flow of blood in human body. Res. J. Medicine and Medical Sciences 2, 98-101.

14. Mishra, S. and Siddiqui, S. U. (2011), A Mathematical model for blood flow and diffusion through stenotic capillary-tissue exchange system. E-J. Sci. & Tech. 6, 1-17.

15. Nadeem, S., Akbar, N. S., Hendi, A. A. and Hayat, T. Power law fluid model for blood flow through a tapered artery with a stenosis. Appl. Math. Comput. 217, 7108-7116.

16. Pant, P. K., Gupta, A. K. and Miyan, M., (2017), A Review on the Multi-phase Blood Flow through Composite Stenosis, International Journal of Dynamics of Fluids, ISSN 0973-1784 Volume 13, Number 1, pp. 53-59.

17. Ponalagusamy, R. and Selvi, R. T., (2011), Blood flow through stenosed arteries: new formula for computing peripheral layer thickness. Int. J. Bio-Sci. & Bio-Tech. 3, 27-37.

18. Ponalagusamy, R., (2007), Blood flow through an artery with mild stenosis: A two layered model, different shapes of stenosis and slip velocity at the wall. J Appl. Sci. 7(7), 1071-1077.

19. Sankar, D. S. and Lee, U., (2009), Mathematical modeling of pulsatile flow of Non-Newtonian fluid in stenosed arteries. Commum. Nonlinear Sci. Numer. Simulat. 14, 2971-2981.

20. Singh, B., Joshi, P. and Joshi, B. K., (2010), Blood flow through an artery having radially non-summetric mild stenosis. Appl. Math. Sci. 4(22), 1065-1072, 2010.

21. Srivastava, V. P., (2007), A Theoretical model for blood flow in small vessels. Applc. Appl. Maths. 2, 51-65.

22. Srivastava, V.P. and Rastogi, R., (2009), Effects of hematocrit on impedance and shear stress during stenosed artery catheterization. Applc. Appl. Math. 4, 98- 113.

23. Srivastava, V. P., Vishnoi, R. and Sinha, P., (2012), Response of a composite stenosis to non-Newtonian blood flow in arteries. E-J. Sci. & Tech. 7(2), 61-70.

24. Young, D. F., (1979), Fluid mechanics of arterial stenosis. J. Biomech. Eng. 101, 157- 175.