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THE OPTIMAL ASSET ALLOCATION PROBLEMFOR AN INVESTOR THROUGH UTILITY MAXIMIZATION

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Abstract

This paper studies the optimal asset allocation problem for an investor through utility maximization. A power utility function is adopted for this sake, and the model takes into account, taxes, and dividends and transaction costs. The assets available in the market are assumed to be risky asset, whose price follows a geometric Brownian motion, and riskless asset, given by the money market account. Interest rates are deterministic, and increase linearly over time with a slope equal to half the volatility of the risky asset. Transaction costs and Taxes are assumed to be proportional to the whole investment in the risky asset.

Keywords: Dividend, Optimal investment, Tax, Transaction cost, Power Utility function, Hamilton-Jacobi-Bellman (HJB) equation.

1. Introduction

Optimal portfolio allocations are derived when an investor maximizes the utility of his wealth. The portfolio problem is a problem involving choice. The choice set typically contains elements such as how much to invest, what to invest in, what point in one's life-cycle one should invest in what assets, how should one invest in order to have maximum tax benefits over the life-cycle, etc. An investor saves from current earnings for future consumption and invests the savings in investment vehicles, broadly classified into asset classes like stocks, bonds, gold, real estate, etc. The investor evaluates the performance of the investments using some criterion; this is called the investor's objective. Let us denote the investor's objective by a function, U(.); this can be a function of returns or wealth or any other aspect of the investment the investor values. Moreover, since the future returns of the asset classes is uncertain, the wealth one will have at future times is uncertain; in this case the objective is taken to be the expected value of the function, U(.).Earlier work in this area on optimal portfolio selection problem can be traced to Markowitz's mean variance model (Markowitz, 1959); Samuelson (Sameulson, 1969) extended the work of Markowitz to a dynamic set up. He used a dynamic stochastic programming approach; he succeeded in obtaining the optimal decision for consumption investment model. Merton (Merton, 1971) used the stochastic optimal control methods in continuous finance to obtain a closed form solution to the problem of optimal portfolio strategy under specific assumptions about the asset returns and the investor's preferences. These days, investors invest both in the money market and stocks. Due to the high risks involved in the stock market, investment strategies and risk management are becoming more important. Using the Hamilton-Jacobi-Bellman equation, (Hipp et al., 2000) determined the strategy of investment which minimizes the probability of ruin modeling the price of the stocks by geometric Brownian motion. Gaier et al. (Gaier, et al., 2002) under the same hypothesis obtained an exponential bound with a rate that improves the classical

Lundberg parameter. The optimal trading strategy they found involved investing in the stock a constant amount of money independent of the reserve.

Castillo and Parrocha (Castillo et al., 2008) considered an insurance business with a fixed amount available for investment in a portfolio consisting of one non-risky asset and one risky asset. They presented the Hamilton-Jacobi-Bellman (HJB) equation and demonstrated its use in finding the optimal investment strategy based on some given criteria. The objective of the resulting control problem was to determine the investment strategy that minimized infinite ruin probability. The existence of a solution to the resulting HJB equation was then shown by verification theorem. A numerical algorithm is also given for analysis. Promislow and Young (Promislow et al., 2005) minimized the probability of ruin of an insurer facing a claim process modeled by a Brownian motion with drift. They consider two controls to minimize the probability of ruin;

1. Investing in a risky asset (constrained and the non-constrained cases)

2. Purchasing quota-share reinsurance.

They obtained an analytic expression for the minimum probability of ruin and their corresponding optimal controls. They also demonstrated their results with numerical results.

In (Bayraktar et al., 2008), a minimization problem using stochastic optimal control techniques with the assumption that an agent's rate of consumption is ratcheted; that is it forms a non-decreasing process, was solved. Liu and Yang (Liu et al., 2004) studied optimal investment strategies of an insurance company where they assumed that an insurance company receives premiums at a constant rate. The total claims are modeled by a compound Poison process, and the insurance company can invest in the money market (bonds) and in a risky asset such as stocks. In (Oksendal et al., 2002), a market with one risk-free and one risky asset in which the dynamics of the risky asset are governed by a geometric Brownian motion was investigated with the objective to maximize the cumulative expected utility of consumption over planning horizon. Kostadinova (Kostadinova, 2007) considered a stochastic model for the wealth of an insurance company which has the possibility to invest into a risky asset and a risk-less asset under constant mix strategy. Here the resulting integrated risk process and the corresponding discounted net loss process were investigated. This opened up a way to measure the risk of a negative outcome of the integrated risk process in a stationary way. It also provided an approximation of the optimal investment strategy that maximizes the expected wealth of the insurance company under the risk constraint on the Value-at-Risk.

Liu, Bai and Yiu (Bai et al., 2007) considered a constrained investment problem with the objective of minimizing the ruin probability and formulated the cash reserve and investment model for the insurance company and analyzed the Value-at-Risk in a short time horizon.

In this paper however, optimal investment problem of utility maximization with taxes, dividends and transaction costs under power Utility function is studied. With the application of the Ito lemma, we obtained the Hamilton-Jacobi-Bellman equation associated with the optimization problem for the power utility maximization of the investor. Furthermore, the properties of the optimal strategies are analyzed and the effects of market parameters on the strategies discussed.

This work aims at optimization of an investor’s investment returns, build a foundation for further research and also contribute to improved business ventures.

2. The problem formulation

We shall assume that the investor can trade two assets continuously in an economy. The first asset is the money market account (bond) growing at a rate r_t that is linear function of time ($r_t = \alpha + \sigma t$), instead of a constant as in (Nie, 2010). $r_t = \alpha + \sigma t$, *(*$\alpha > 0, 0 \leq \sigma < \infty$*) is a decreasing (or an increasing) linear* function of t as $t \to \infty$. If $\sigma > 0$, then r_t is increasing and constant if $\sigma = 0$. This property is now valid for any t not only when $t \to \infty$. Hence r_t is simply a deterministic linear process and σ is the slope and not just the acceleration coefficient which is the volatility (variance) of the process and is proportional to the level of the interest rate. If the rate is assumed to follow the Ornstein-Uhlenbeck process

$$dr_t = \alpha(\beta - r_t)dt + \sigma dZ(t), \quad r_{t_0} = r_0.$$

Then

$$r_t = (r_0 - \beta)e^{-\alpha t} + \sigma \int_{t_0}^{t} e^{-\alpha(t-u)} dZ(u),$$

where α is the speed of mean reversion, β the mean level attracting the interest rate (taxes) and σ is the constant volatility of interest rate. The investor takes these prices as given and chooses quantities without and with transaction costs. Further assumptions are that the securities pay dividend and taxes on the amount invested in the risky asset. Throughout this research work, we assume a probability space $(\Omega, \mathcal{F}, \mathrm{P})$ and a filtration$\{\mathcal{F}_t\}$.Uncertainty in the models is generated by standard Brownian motion$Z(t)$. The two equations governing the dynamics of the risk free and risky assets are given as;

$$dP_O(t) = (\alpha + \sigma t)P_O(t)dt \text{ or } P_O(t) = P_O(0)exp\left\{\alpha t + \frac{\sigma t^2}{2}\right\}, \tag{1}$$

and

$$dP_1(t) = P_1(t)\{\mu dt + \sigma dZ(t)\} or P_1(t) = P_1(0)exp\left\{\sigma Z(t) + \left(\mu - \frac{\sigma^2}{2}\right)t,\right\}, \forall t \in [0, \infty). \tag{2}$$

The investor is allowed to invest in the risky asset and the risk-free asset. Let $S(t)$ be the money amount invested in the risky asset at time t, then$[W(t) - S(t)]$is the money amount invested in the risk-free asset, where $W\ (t)$ is the total investment.

2.1 Assumptions

It is assumed that transaction cost, taxes and dividends are paid on the amount invested in the risky asset.

Therefore for any policy, S, the total wealth process of the insurance company evolves according to the stochastic differential equation;

$$dW^S(t) = S(t)\frac{dP_1(t)}{P_1(t)} + \big(W(t) - S(t)\big)\frac{dP_O(t)}{P_O(t)} - (\vartheta + \beta - d)S(t)dt. \tag{3}$$

Substituting the expressions for, $\frac{dP_1(t)}{P_1(t)}$, and$\frac{dP_O(t)}{P_O(t)}$, the stochastic differential equation for the wealth process of the investor becomes;

$$dW^S(t) = \{[W(t)(\alpha + \sigma t) + [(\mu + d - (\alpha + \sigma t) - \vartheta - \beta)S(t)]\}dt + \sigma S(t)dZ(t). \tag{4}$$

Here, the rate of the taxes in the financial market is $\beta$and $d$is the dividend income. The rate of transaction costs is ϑ. Correspondingto a trading strategy $S(t)$and an initial capital,$W(0)$, the wealth process $W(t)$of the investor follows (4)

Suppose the investor has a utility function$U(.)$ which is strictly concave and continuously differentiableon $(-\infty; +\infty)$andwishes to maximize the expected utility of his terminal wealth. Then, the investor's problem can therefore be written as;

$$\left.\begin{aligned} \mathrm{Max}_S\{\mathcal{B}^S V(\mathrm{t},w)\} = 0;\ V(T,w) = U(w) \\ V(T,w) = \underset{S}{\mathrm{Max}} E^{t,w}|U(w^s(t))| \end{aligned}\right\} \quad (5)$$

where $\mathcal{B}^{\mathrm{S}}$ is an operator that will be obtained using Ito's lemma, subject to (4).

The quadratic variation of the wealth process is;

$$< dW^S(t)) >= S^2(t)\sigma^2 dt. \quad (6)$$

The optimization problem being considered is for the power utility function given in the form;

$$U(w) = \frac{w^{1-\phi} - k}{1-\phi}, \quad (7)$$

where ϕ and k are constants and, $\phi \neq 1$.

3. Optimal investment strategy for the power utility function

In this section, the explicit solutions for the optimization problem are obtained using stochastic control and Ito lemma.

3.1 General framework

Define the value function as;

$$J(t,s,w) = \mathrm{Max}_S\{\mathcal{B}^S V(\mathrm{t},w)\} = 0;\ V(T,w) = U(w), 0 < t < T. \quad (8)$$

The corresponding HJB equation is given by;

$$J_t + J_w\{[W(t)(\alpha + \sigma t) + [(\mu + d) - (\alpha + \sigma t + \vartheta + \beta)]S(t)]\} + J_{ww}\left[\frac{S^2(t)\sigma^2}{2}\right] = 0, \quad (9)$$

where J_t, J_w, and J_{ww} denote the first partial derivatives of J respect to t, the wealth w respectively and second order partial derivatives with respect to the wealth w, with the boundary condition that at the terminal time T,

$$J(T,s,w) = U(w). \quad (10)$$

The differentiation of (9) with respect to $S(t)$ gives the optimal policy;

$$J_w(\mu + d - (\alpha + \sigma t) - \vartheta - \beta) + \sigma^2 S(t) J_{ww} = 0. \quad (11)$$

This simplifies to,

$$S^*(t) = \frac{[(\vartheta + \beta + \alpha + \sigma t) - (\mu + d)]J_w}{\sigma^2 J_{ww}}. \quad (12)$$

3.2 The optimal strategy for the power utility function

Considering the power utility function described by (7), to eliminate dependency on $w$let the solution to the HJB equation (9) is of the form,

$$J(t,s,w) = h(t,T)\frac{w^{1-\phi} - k}{1-\phi}, \quad (13)$$

with the boundary condition,

$$h(T,T) = 1. \quad (14)$$

Then,

$$J_t = h_t \frac{w^{1-\phi} - k}{1-\phi}, J_w = hw^{-\emptyset}, \text{ and } J_{ww} = -\emptyset w^{-\emptyset - 1}. \quad (15)$$

The HJB equation (9) becomes;

$$h_t \frac{w^{1-\phi}-k}{1-\phi} + hw^{-\emptyset}\{[W(t)(\alpha+\sigma t) + [(\mu+d)-(\alpha+\sigma t+\vartheta+\beta)]S^*(t)]\} + \sigma S^*(t)dZ(t) - \emptyset w^{-\emptyset-1} h\left[\frac{S^{*2}(t)\sigma^2}{2}\right] = 0, \quad (16)$$

which simplifies to,

$$\frac{h_t}{h} = -\begin{Bmatrix} \frac{(1-\phi)w^{-\emptyset}}{w^{1-\phi}-k}\{[W(t)(\alpha+\sigma t) + [(\mu+d)-(\alpha+\sigma t+\vartheta+\beta)]S^*(t)]\} - \\ \frac{(1-\phi)\emptyset w^{-\emptyset-1}}{w^{1-\phi}-k}\left[\frac{S^{*2}(t)\sigma^2}{2}\right] \end{Bmatrix}. \quad (17)$$

Differentiating (16) with respect to $S^*(t)$ obtained the optimal investment in the risky asset as;

$$S_{d,\vartheta,\beta}{}^*(t) = \frac{[(\mu+d)-(\alpha+\sigma t+\vartheta+\beta)]}{\emptyset\sigma^2} w \quad (18)$$

From equation (17) obtained;

$$\int_t^T \frac{dh}{h} ds = \int_t^T (A+Bs)ds, \quad (19a)$$

Where

$$(A+Bt) = -\left\{\frac{(1-\phi)w^{-\emptyset}}{w^{1-\phi}-k}\{[W(t)(\alpha+\sigma t) + [(\mu+d)-(\alpha+\sigma t+\vartheta+\beta)]S^*(t)]\} - \frac{(1-\phi)\emptyset w^{-\emptyset-1}}{w^{1-\phi}-k}\left[\frac{S^{*2}(t)\sigma^2}{2}\right]\right\}. \quad (19b)$$

The solution of equation (19a) gives;

$$\ln\left[\frac{h(T,T)}{h(t,T)}\right] = A(T-t) + \frac{B(T^2-t^2)}{2}, \quad (20)$$

which simplifies to

$$h(t,T) = h(T,T)\exp\left\{-[A(T-t) + \frac{B(T^2-t^2)}{2}]\right\}. \quad (21a)$$

Applying the boundary condition (14) on (21a), obtained;

$$h(t,T) = \exp\left\{-[A(T-t) + \frac{B(T^2-t^2)}{2}]\right\}, \quad (21b)$$

from which the optimal value function of the investor is given as;

$$J^*(t,s,w) = \exp\left\{-[A(T-t) + \frac{B(T^2-t^2)}{2}]\right\}\frac{w^{1-\phi}-k}{1-\phi}, \quad (22)$$

which at terminal date, T, equals

$$J^*(T,s,w) = \frac{w^{1-\phi}-k}{1-\phi}. \quad (23)$$

The effects of the parameters d, ϑ, and β are as follows;

When there are no, dividend, (d), transaction costs, (ϑ), taxes financial market is (β), the optimal investment in the risky asset is,

$$S^*(t) = \frac{[\mu-(\alpha+\sigma t)]}{\emptyset\sigma^2}. \quad (24)$$

The introduction of dividends only gives;

$$S_d{}^*(t) = \frac{[(\mu+d)-(\alpha+\sigma t)]}{\emptyset\sigma^2} \quad (25)$$

from which,

$$S_d{}^*(t) = \frac{[(\mu + d) - (\alpha + \sigma t)]}{\emptyset\sigma^2}$$

$$= S^*(t) + \frac{d}{\emptyset\sigma^2}. \quad (26)$$

The introduction of transaction costs leads to;

$$S^*_{d,\vartheta,}(t) = \frac{[(\mu + d) - (\alpha + \sigma t + \vartheta)]}{\emptyset\sigma^2}$$

$$= S_d{}^*(t) - \frac{\vartheta}{\emptyset\sigma^2}$$

$$= S^*(t) + \frac{d-\vartheta}{\emptyset\sigma^2}.$$

(27)

Introducing tax leads to;

$$S^*_{d,\vartheta,\beta}(t) = \frac{[(\mu + d) - (\alpha + \sigma t + \vartheta + \beta)]}{\emptyset\sigma^2}$$

$$= S_{d,\vartheta}{}^*(t) - \frac{\beta}{\emptyset\sigma^2}$$

$$= S_d{}^*(t) - \frac{(\vartheta + \beta)}{\emptyset\sigma^2}$$

$$= S^*(t) + \frac{d-(\vartheta+\beta)}{\emptyset\sigma^2}.$$

(28)

Equation (25) shows that dividends increased the invested amount. Equations (26) through (28) show how the pension investor's investment is depleted by the introduction of each market parameter. Further they show that the investor can only hold investment in the risky asset whenever,

$[(\mu + d) > (\alpha + \sigma t + \vartheta + \beta)]$;

that is, if the sum of the mean return on the risky asset and the dividend ratio is greater than the sum of financial market tax rate, transaction costs, and the rate of return of risk free asset.

The portfolio problem may be described in broad terms as determining the set of decisions to maximize this objective function E [U (.)]. Mathematically this may be written as

$$\min_{X_t, C_t, t=0,T-1} \sum_{t=0}^{T} E[U_t(W_t, C_t, R_t, \dots)], \quad (29)$$

where T is the planning horizon, W_t is the wealth at time t, C_t is the consumption at time t, Rt is a vector of returns on the asset classes. Ut (.) is the utility function at time t for Wealth, Return on assets, Consumption or any other properties at each decision point t. The vector X_t is the wealth invested in each asset class at time t; i.e., if there are N asset classes, X_t will be a vector of size N. If instead of a discrete-time problem this were a continuous-time problem, the $\sum U(.)$ would be replaced by an $\int_0^T U(.)$.

If the optimization problem is one-dimensional, i.e., x $\in\mathbb{R}$, and has no constraints or bounds, a solution x^* is characterized by the following conditions, $U'(x^*) = 0$ and $U''(x^*) < 0$, assuming sufficient differentiability of function U. If derivatives of the objective function are available, an approach for finding optima is to find a zero of the function $U^{'}(.)$, which is a necessary condition for optima. Conditions for sufficiency can then be checked at these points. Some standard and well-known methods for finding the zeros of a function are the bisection method, the secant method and Newton's method.

4. Conclusion

In this work we have studied the optional investment problem for an investor when transaction costs, dividend (d), and financial market tax rate (β), are involved. We have shown that the introduction of the various market parameters depleted the investment in the risky asset except the dividend which yielded an increment that is a fraction of the wealth.

It is observed that the investor's investment and the value function are horizon dependent, and should be taken into consideration when policy decisions are being made.

If we decide against the linearity of r_t such that the price process of the riskless asset is governed by

$$dP_o = r_t P_o dt \tag{30}$$

where r_t is driven by Ornstein-Uhlenbeck process;

$$dr_t = \alpha(\beta - r_t)dt + \sigma dz(t).\ r_{t_0} = r_0 \tag{31a}$$

or

$$r_t = (r_0 - \beta)e^{-\alpha t} + \sigma \int_{t_0}^{t} e^{-\alpha(t-u)}\, dZ(u), \tag{31b}$$

then, with (2), and using the maximum principle, the Hamilton-Jacobi-Bellman (HJB) equation becomes;

$$J_t + \alpha(\beta - r_t)J_r + [wr_t + [(\mu + d) - (r_t + \sigma + \beta)]s]J_w + r^2 sJ_{rw} + \frac{\sigma^2}{2}J_{rr} + \frac{\sigma^2 s^2}{2}J_{ww} = 0 \tag{32}$$

Differentiating (32) with respect to s gives

$$[(\mu + d) - (r_t + \vartheta + \beta)]J_w + \sigma^2 J_{rw} + \sigma^2 sJ_{ww} \tag{33}$$

from which

$$S^* = \frac{[(\mu+d)-(r_t+\vartheta+\beta)]J_w}{\sigma^2 J_{ww}} - \frac{\sigma^2 J_{rw}}{\sigma^2 J_{ww}}. \tag{34}$$

To eliminate dependency on w are conjecture that

$$J(t, r, w) = G(t, r)\frac{[w^{1-\emptyset}]}{1-\phi}; \phi \neq 1 \tag{35a}$$

with the boundary condition

$$G(T, r) = 1. \tag{35b}$$

From (35a) obtain

$$J_w = w^{-\phi}G,\ J_{ww} = -\phi w^{-\phi-1}GandJ_{rw} = w^{-\phi}G_r \tag{35c}$$

Applying (35c) to (34) yields

$$S^*{}_{d,\vartheta,\beta}(t) = \frac{w}{\phi}\left[\frac{[(\mu+d)-(r_t+\vartheta+\beta)]}{\sigma^2} + \frac{G_r}{G}\right]. \tag{36}$$

Further to eliminate dependency on$r$ we conjecture that

$$G(t,r) = h(t)\left[\frac{r^{1-\phi}}{1-\phi}\right], \tag{37a}$$

such that

$$h(T) = \frac{1-\phi}{r^{1-\phi}}. \tag{37b}$$

We obtain from (37a)

$$G_r = r^{-\phi}h(t). \tag{37c}$$

The application of (37a) and (37c) in (36) yields

$$S^*{}_{d,\vartheta,\beta}(t) = \frac{w}{\phi}\left[\frac{[(\mu+d)-(r_t+\vartheta+\beta)]}{\sigma^2} + (1-\phi)r_t\right] \tag{38}$$

Notice that the optimal investment in the risky asset, using (31b), will become

$$S^*{}_{d,\vartheta,\beta}(t) = \frac{w}{\phi}\left[\frac{[(\mu+d)-(\vartheta+\beta)]}{\sigma^2} + \left(1-\frac{1}{\sigma^2}-\phi\right)[(r_0-\beta)e^{-\propto t} + \sigma\int_{t_0}^{t} e^{-\propto(t-u)}dZ(u)\right]. \tag{39}$$

Notice that

$$\lim_{t-\infty} S^*{}_{d,\vartheta,\beta}(t) = \frac{w}{\phi}\left[\frac{[(\mu+d)-(\vartheta+\beta)]}{\sigma^2}\right], \tag{40}$$

while

$$\lim_{t-0} S^*{}_{d,\vartheta,\beta}(t) = \frac{w}{\phi}\left[\frac{[(\mu+d)-(\vartheta+\beta)]}{\sigma^2} + \left(1-\frac{1}{\sigma^2}-\phi\right)\left[(r_0-\beta) + \sigma\int_{t_0}^{T} e^{\propto u}dZ(u)\right]\right]. \tag{41}$$

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