Analytical Survey on the Two-Fluid Blood Flow through Stenosed Artery with Permeable Wall

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Abstract

The womersley flow of blood through constricted arteries is analyzed by considering the blood as a two-fluid model with the suspension of all the erythrocytes within the core region as a non-Newtonian fluid and also the plasma within the peripheral layer as a Newtonian fluid. The non-Newtonian fluid within the core region of the artery is assumed as a Herschel-Bulkley fluid and Casson fluid. The perturbation technique is employed to resolve the ensuing system of non-linear partial differential equations. Expressions for numerous flow quantities are obtained for the two-fluid Casson model. In the present paper, there's associate analysis on the varied researches. It's determined that the plug core radius, pressure drop, wall shear stress and also the resistance to flow are considerably terribly low for the two-fluid Casson model than those of the two-fluid Herschel-Bulkley model. Hence, the two-fluid Casson model would be a lot of helpful than the two-fluid Herschel-Bulkley model to research the blood flow through constricted arteries.

Keywords: Artery, Blood, Mathematical modeling, Permeable wall.

1. Introduction

The blood vessels are the part of the cardiovascular system that transports blood throughout the body of human. There are three major types of blood vessels: the arteries, that carry the blood removed from the heart; the capillaries, that modify the particular exchange of water and chemicals between the blood and therefore the tissues; and therefore the veins that carry blood from the capillaries back toward the guts. The word vascular that means regarding the blood vessels comes from the Latin word, which means “vessel”. Many structures like gristle and therefore the lens of the eye; don't contain blood vessels and are labeled.

The arteries are a part of the cardiovascular system, that is answerable for the delivery of O_2 and nutrients to any or all cells, similarly because the removal of CO_2 and waste merchandise, the upkeep of optimum pH scale, and therefore the circulation of proteins and cells of the system. In developed countries, the two leading causes of death, MI i.e., coronary failure, and stroke, might every directly result from associate degree blood vessel system that has been slowly and increasingly compromised by years of decay.

A stricture from Greek word “στένωσις” i.e., "narrowing" is the abnormal narrowing in a vessel or alternative hollow organ or structure. It is a conjointly generally known as a stricture. The stricture as a term is typically used once narrowing is caused by contraction of swish muscle e.g., achalasia, prinzmetal angina; stricture is typically used once narrowing is caused by lesion that reduces the house of lumen e.g., atherosclerosis. The term coarctation is another equivalent word, however is usually used solely within the context of aortal coarctation. Restenosis is that the repetition of stricture when a procedure. (Bertazzo et al., 2013), (Artery Wiki, 2017)

There are several evidences that vascular fluid dynamics plays a significant role within the development and progression of blood vessel stricture. Arteries are narrowed by the event of hardening of the arteries plaques that protrude into the lumen, ensuing blood vessel stricture. Oncethe obstruction developed in artery, one amongst the foremost serious consequences is that the accumulated resistance and therefore the associated reduction of the blood flow to the actual tube bed equipped by the artery. Thus, the presence of a stricture ends up in the intense circulatory disorder.

Several theoretical and experimental makes an attempt were created to review the blood flow characteristics within the presence of pathology. The belief of Newtonian behavior of blood is suitable for top shear rate flow through larger arteries. But, blood, being a suspension of cells in plasma, exhibits non-Newtonian behavior at low shear rate in little diameter arteries. In unhealthy state, the particular flow is clearly womersley. Several researchers studied the non-Newtonian behavior and womersley flow of blood through stenotic arteries.

Some researchers have shown by experimentation that for blood flowing through slender blood vessels, there is a peripheral layer of plasma and a core region of suspension of all the erythrocytes. Thus, for a sensible description of the blood flow, it's acceptable to treat blood as a two-fluid model with the suspension of all the erythrocytes within the core region as a non-Newtonian fluid and plasma within the peripheral region as a Newtonian fluid.

Many researchers analyzed that Casson fluid model and Herschel-Bulkley fluid model are the fluid models with nonzero yield stress and that they are a lot of appropriate for the studies of the blood flow through slim arteries. It's been rumored by Iida that Casson fluid model is easy to use for blood flow issues, due to the actual style of its organic equation, whereas, Herschel-Bulkley fluid model's organic equation isn't simple to use due to the shape of its empirical relation, since, it contains an additional parameter than the Casson fluid model. It’s been incontestable by Scott-Blair and Copley that the parameters acceptable to Casson fluid-viscosity, yield stress and power law-are adequate for the illustration of the straightforward shear behavior of blood. It's been established by Merrill et al. that Casson fluid model holds satisfactorily for blood flowing in tubes of diameter 130-1300, whereas Herschel-Bulkley fluid model may be employed in tubes of diameter 20-100. (Sankar D. S. & Ismail; 2009, 2010)

2. Literature Review

Sharan, M. et al., (Sharan, 2001) have delineated a two-phase model for the flow of blood in slender tubes. The model consists of a central core of suspended erythrocytes and a non-cellular layer encompassing the core. It’s assumed that the body within the non-cellular layer differs from that of plasma as results of extra dissipation of energy close to the wall caused by the red blood corpuscle motion close to the non-cellular layer. A regular system of nonlinear equations is resolved numerically to estimate the effective dimensionless body within the non-cellular layer, the thickness of the non-cellular layer and also the core hematocrit.

Misra J. C. et al., have developed a mathematical model for finding out the non-Newtonian flow of blood through a stenotic blood vessel section. Herschel-Bulkley equation has been taken to represent the non-Newtonian character of blood. The matter is investigated by a combined use of analytical and numerical techniques. It’s noticed that the resistance of flow and also the skin-friction increase because the stricture height will increase. The results are compared with information offered within the existing literature conferred by previous researchers (Mishra et al., 2006).

In 2007, Ponalagusamy, R. et al., have thought of a mathematical model for blood flow through constricted arteries with axially variable peripheral layer thickness and variable slip at the wall. The model consists of a core encircled by a peripheral layer. It’s assumed that the fluids of each the regions i.e., core and peripheral are Newtonian having totally different viscosities. For such models, in literature, the peripheral layer thickness and slip are assumed a priori supported experimental observations (Ponalagusamy et al., 2007). Mandal, P. K. et al., have resolved the problems of non-Newtonian and nonlinear pulsatile flow through an irregular basis constricted blood vessel phase numerically, whereas the non-Newtonian physical science of the flowing blood is characterized by the generalized Power-law model wherever each of the shear-thinning and shear-thickening models of the streaming blood are taken into consideration (Mandal et al., 2007). Politis, A. K. et al., have thought about the composite blood vessel coronary grafts, transport phenomena and pure mathematics that alter blood flow dynamics. In their analysis, the procedure fluid dynamics (CFD) techniques are applied for the simulation of multi-branched CACGs underneath physiologically realistic influx waveforms. The numerical answer is obtained by a finite-volume methodology developed in non-orthogonal, curved coordinates and a multi-grid approach (Politis et al., 2008).

Singh, A. K. et al., have shown the consequences of Hematocrit, height of stricture, porous parameter and rate of blood on wall shear stress of the flow of blood through tapered artery. The study reveals that wall shear stress reduces for increasing Hematocrit share. It's conjointly discovered that wall shear stress will increase as stricture height and porous parameter increase whereas it decreases with the increasing values of rate of blood and slope of tapered artery (Singh et al., 2013).

In 2013, Ellahia, R. et al., have analyzed with the theoretical study of unsteady blood flow of a Jeffery fluid. The two kinds of arteries, i.e., composite constricted artery and anistropically tapered constricted artery with porous walls are thought of. The extremely nonlinear momentum equations of the Jeffery fluid model are simplified by considering the case of delicate stricture, and at last the precise solutions are supported. Medhavi, A. et al., (Medhavi, et al., 2013) have conferred the paper considerations with the fluid mechanical study on the results of the permeableness of the wall through overlapping stricture in associate degree artery assumptive that the flowing blood is drawn by a megascopic two-phase model. The expressions for the blood flow characteristics, the electric resistance, the wall shear stress distribution within the stenosed region, cut stress at the stricture throats and at the stricture essential height are derived (Elahia et al., 2013).

In 2013, Tripathi, D. et al., have shown an analytical and procedure studies on transient peristaltic heat flow through a finite length porous channel. The results for the temperature field, axial speed, cross speed, pressure gradient, wall shear stress, volume rate of flow, averaged volume flow, mechanical potency, and stream operate are obtained beneath the belief of low painter variety and long wavelength approximation (Tripathi et al., 2013). Eldesoky, I. M. et al., have studied the pulsatile flow of blood through porous medium in an artery beneath the influence of periodic body acceleration and slip condition by considering blood as incompressible Newtonian electrically conducting fluid within the presence of field of force. In their analysis, a replacement technique of differential construction technique is introduced to search out numerical answer of non-linear partial differential equations like the equation of motion of this drawback "Navier-Stokes equation". The presence of the nonlinearity within the drawback ends up in severe difficulties within the answer approximation. In construction of the numerical theme "a new algorithm" a generalized differential construction technique is to use for spinoffs with relevance house variables of differential equations and for the time derivative applying fourth order Runge-Kutta method (Eldesoky et al., 2013).

In 2014, Eldesoky, I. M. et al., have studied the flow through porous boundaries each in technological and biophysical, involved with the study of unsteady pulsatile flow of blood through porous medium in a very time dependent constricted porous channel subjected to time dependent suction at the walls of the channel (Eldesoky et al., 2014). Srivatava, R. K. et al., have investigated the fluid mechanical study on the consequences of the porousness of the wall through axisymmetric stricture in artery forward that the flowing blood is described by a two-fluid model. The expressions for the blood flow characteristics, the resistance, the wall shear stress distribution inside the constricted region and additionally the cut stress at the stricture throat are derived (Srivatava et al., 2014). Shit, G. C. et al., have given a theoretical study of blood flow through a tapered and overlapping constricted artery below the action of an externally applied field of force (Shit et al., 2014).

The fluid i.e., blood is assumed to be porous in nature. The governing equation for bedded, incompressible and Newtonian fluid subject to the boundary conditions is resolved by employing a documented Frobenius methodology.

By Vardakis, J. C. et al., Multiple Network Poroelastic Theory (MPET) is employed to develop a completely unique spatio-temporal model of fluid regulation and tissue displacement at intervals the varied scales of the cerebral setting. The model is applied through 2 formats, a one-dimensional finite distinction – process Fluid Dynamics (CFD) coupling framework, moreover as a two-dimensional Finite component methodology (FEM) formulation (Vardakis et al., 2015).

Raza, T. et al., have delineated the laminar, turbulent, transitional of viscous flow that will occur downstream of a stricture relying upon the painter range and constriction form parameter. The flow was thought of to be steady, axisymmetric, and incompressible (Raza et al., 2016).

3. Formulation of Problem

Let we consider the two-layered axisymmetric flow of blood through axisymmetric stricture. The geometry of the stenosis that is assumed to be manifested in the arterial wall segment is described as:

$$\frac{R(z),R_1(z)}{R_0} = (1,\beta) - \frac{(\delta\delta_1)}{2R_0}\left\{1 + cos\frac{2\pi}{L_0}\left(z - d - \frac{L_0}{2}\right)\right\} \quad ; \quad d \le z \le d + L_0 \qquad (3.1)$$

=(1, β) ; otherwise, (3.2)

where z is the axial coordinate, R(z), R_1(z)) are the radius of the tube and, interface with constriction respectively; R_0 is the radius of the artery without stenosis; L_0 is the stenosis length, L is the tube length and d indicates the location of stenosis, β is the ratio of the central core radius to the tube radius in the unobstructed region and δ, δ_1 are the maximum height of the stenosis and the bulging of the interface respectively. The flowing blood is assumed to be represented by a two-layered Newtonian fluid. The equations describing the laminar, steady, one-dimensional flow in the case of a mild stenosis $\delta \ll R_0$ are expressed as:

$$\frac{dp}{dz} = \frac{\mu_p}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_p, \qquad R_1(z) \le r \le R(z) \qquad (3.3)$$

$$\frac{dp}{dz} = \frac{\mu_c}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_c, \qquad 0 \le r \le R_1(z) \qquad (3.4)$$

Where R_1(z)=radius of the central layer;

μ= viscosity;

u= shows velocity of fluid;

Whereas the suffix p and c are due to peripheral and central layers, p is the pressure and (r, z) are coordinates in the two-dimensional cylindrical polar coordinate system.

The assumed boundary conditions are

$$\frac{\partial u_c}{\partial r} = 0 \ \ atr = 0 \qquad (3.5)$$

$$u_p = u_c and \mu_p\frac{\partial u_p}{\partial r} = \mu_c\frac{\partial u_c}{\partial r} atr = R_1(z) \qquad (3.6)$$

$$u_p = u_B and\frac{\partial u_p}{\partial r} = \frac{\alpha}{\sqrt{k}}\left(u_B - u_{porous}\right)atr = R(z), \qquad (3.7)$$

Where

$u_{porous} = -\frac{k}{\mu_p}\frac{dp}{dz}$ = velocity in permeable boundry;

u_B = slip velocity;

μ_p= fluid viscosity in the peripheral layer;

k= Darcy's number;

α= slip parameter.

4. Mathematical Analysis

The solution of differential equations under the boundary conditions yields the expressions for velocity as given

$$u_p = -\frac{R_0^2}{4\mu_p}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2 - 2\frac{R}{R_0}\frac{\sqrt{k}}{\alpha R_0} + 4\frac{K}{R_0^2}\right\} \tag{4.1}$$

$$u_c = -\frac{R_0^2}{4\mu_p}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^2 - \mu\left(\frac{r}{R_0}\right)^2 - (1-\mu)\left(\frac{R_1}{R_0}\right)^2 - 2\frac{R}{R_0}\left(\frac{\sqrt{k}}{\alpha R_0}\right) + 4\frac{K}{R_0^2}\right\} \tag{4.2}$$

$$Q = -\frac{\pi R_0^4}{8\mu_p}\frac{dp}{dz}\left\{\left(\frac{R}{R_0}\right)^4 - (1-\mu)\left(\frac{R_1}{R_0}\right)^2 + \frac{8k}{R_0^2}\left(\frac{R}{R_0}\right)^2 - \frac{4\sqrt{k}}{\alpha R_0}\left(\frac{R}{R_0}\right)^3\right\} \tag{4.3}$$

$$\frac{dp}{dz} = -\frac{8\mu_p Q}{\pi R_0^4}\varphi(z) \tag{4.4}$$

Where, Q =volumetric flow rate;

$$\varphi(z) = 1/\left\{[1-(1-\mu)\beta^4](R/R_0)^4 + 8k(R/R_0)^2/R_0^2 - 4\sqrt{k}(R/R_0)^3/\alpha R_0\right\}$$

5. Conclusions

The results of the porousness of the artery wall and therefore the peripheral layer on blood flow characteristics owing to the presence of a pathology, a two-fluid blood flow of Newtonian fluid through axisymmetric pathology in associate degree artery with leaky wall has been studied. The study permits one to watch the synchronic effects of the wall porousness and therefore the peripheral layer on blood flow characteristics owing to the presence of pathology. For any given set of parameters, the blood flow characteristics i.e., impedance, wall shear stress, etc. assume lower magnitude in two-fluid model than its corresponding price in one-fluid analysis. The electrical resistance decreases with increasing Darcy range from its top magnitude within the case of imperviable wall i.e., at zero Darcy range. It’s thus all over that the existence of porousness within the artery wall and therefore the presence of the peripheral layer within the artery facilitate the functioning of the pathological artery. It's determined that the pressure drop, plug core radius, wall shear stress and therefore the resistance to flow square measure considerably terribly low for the two-fluid Casson model than those of the two-fluid Herschel-Bulkley model. Therefore, the two-fluid Casson model would be very helpful than the two-fluid Herschel-Bulkley model to research the blood flow through stenotic arteries.

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