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The Fluid Flow Motion through Porous Media for Slightly Compressible Materials

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Abstract

The multiphase flow in porous media is a topic of various big complexities for a long time in the field of fluid mechanics. This is a subject of important technical applications, most probably in oil recovery from petroleum reservoirs and also in others. The single phase fluid flow through a porous medium is generally defined by Darcy's law. In the petroleum industry and in other technical applications, the transport phenomenon is modeled by postulating a multiphase analysis of the Darcy's law. In this analysis, the distinct pressures are defined for each phase with the difference and well known as capillary pressure. That is determined by the interfacial tension, geometry of micro pore and the chemistry of the surface related to the solid medium. In regarding flow rates, the relative permeability is defined that gives the relationship between the volume flow rate of each fluid and the pressure gradient. In the present paper, there is an analysis about the mathematical laws and equations for the slightly compressible flow and rock and the analysis and important results have been founded. The analysis show that velocity of fluid related to any phase is inversely proportional to the viscosity of the fluid. The capillary pressure of the capillary tube is inversely proportional to the radius of tube and increases with increasing values of the surface tension of the fluid. It also varies inversely with the radii of curvature for the interface of the fluid. The pressure exerted by the fluid varies positively with its velocity and varies inversely with the absolute permeability of the porous medium.

Keywords: Multiphase flow motion, Porous media, Darcy's law, Compressible rocks.

2010 AMS Subject Classification: 76S05

1. Introduction

The consideration of porous media within a multi-scale framework is an emerging concept that takes advantage of the different mature state of understanding which applies at smaller length scales as the method to give the details of larger scale systems. Many physical systems can be concerned with a sequence of length scales; that is associated with a particular mathematical formulation which describes the system behavior at that scale. The multi-scale frameworks obtained the relations between these different details that give a series of mathematical formulations.

When these are applied to porous media the approach can be used to tie thermodynamic forms and conservation equations to which that apply at the pore-scale, otherwise known as the micro scale. This is of our use when macroscopic closure relationships are not suitable or incomplete then microscopic closure relations are usually better known. Microscopic simulations can be applied for giving insights into macroscopic phenomenon that gives simplifying assumptions, and generate suitable macroscopic closure relationships. These studies are heavily on computational analysis to give the real solutions for the microscopic analysis of porous medium flows. The computational assumptions give opportunities for incorporate big and more realistic solutions of micro-analysis behavior into macroscopic modeling analysis.

The Fluid mechanics analyses about the moving objects that include gases, liquids, plasmas and other solids. From a 'fluid-mechanical' point of view, the matter can be considered of consisting of fluid and solid in a single fluid system the difference between these two positions being that a solid can resist shear stress by a static deformation, which is impossible in fluid. But we can see that a distinction between the gas and liquid states of matter is impossible if temperature is above the critical point. The main differences between these two phases are due to the difference between their equilibrium density and compressibility, below the critical point. For the gaseous phase molecules move freely with respect to each other but in the controlled volume, the number of molecules changes continuously. Due to this analytical uncertainty, fluid density has no meaning unless the control volume is comparatively large for the intermolecular spacing. But, if the chosen is sufficiently large, then there could be variation in the bulk density of the molecules due to other effects. A reasonable value for such a volume is around 10^{-9} mm^3 for liquids and gases at the atmospheric pressure. The main analytical problems are due to their physical dimensions that are much larger than this size so that they display fluid properties with practically continuous spatial variation. These fluids can be said as continuum and well defined derivatives of the suitable variables can be used to explain its dynamical properties. The equilibrium of these fluids are produced and thus maintained by the collisions of the molecules that occur in a characteristic time scale τ. In the classical fluids, generally τ is of order of 10^{-10} to 10^{-14} seconds. As analyzed by *U. Aaltosalmi et. al.* [1], [8], the mean free path λ between collisions of the molecules is the related length scale. For the systems of various motions, the equilibrium states are non-homogeneous. The states of equilibrium due to the disturbances vary in time and length scales given by λ and *t* and the variables of the system have slow variations at the long wavelengths. For these variables, there are great amount of equilibrium collisions and some disturbances due to motion; these are small in space at all times for these variables. For determining that the above conditions the fluid dynamics are verified, the Knudsen number will be evaluated for the problem. The Knudsen number K_n is the ratio of the molecular mean free path λ to a physical length scale λ_0 of the obstacles and flow channels and is given by *U. Aaltosalmi et.al.* [1], [8] i.e.,

$$K_n = \lambda / \lambda_0 \tag{1}$$

The multiphase flow term is used for the fluid flow relating of more than one phase or component. The flows taken in the multiphase have some level of phase or component separation at a scale well above the molecular level. That drops the enormous spectrum of various multiphase flows. In other words, it can be classified according to the state of the different phases or components which refers to gas, solids, particle or bubbly flows etc. Some results deal for a specific category of fluid flow for low Reynolds number suspension flows, dusty gas dynamics etc. But others relate with a specific application like as slurry flows, cavitations flows, aerosols, debris flows and fluidized beds etc. For the multiphase flow phenomenon, there are wide ranges of flows with a lot of applications. Almost all the processing technologies deal with the multiphase flow, from the cavitations pumps, turbines to electro photographic processes to paper making the pellet form of almost raw plastics. The quantity of granular material, coal, ore, grain etc., is essential to flow. Clearly the ability for prediction of the fluid flow behavior of these methods is central to the efficiency and effectiveness of those processes. For example, the flow of toner is the major factor in the quality and speed of electro photographic printers. The multiphase flows found everywhere of our atmosphere whether one considers rain, fog, snow, avalanches, mud slides, sediment transport, debris flows, and countless other natural phenomenon. In the medical science, many biological and medical flows are multiphase too, from blood flow to semen, laser surgery cavitations etc. There are two types of flows known as disperse flows and separated flows. The disperse flows relating of finite particles, drops or bubbles distributed in a connected volume of continuous phase. But the separated flows relating of two or more continuous streams of different type of fluids separated by interfaces.

For studying the multiphase flows motion there is the necessity of a suitable model and the prediction for the characteristics of the flows and the phenomenon which they perform. The analysis was given by *C. E. Brenen et. al.* [6] that described the three different methods in which these models are as follows:

(i) By using laboratory models with the suitable instrumentation.

(ii) By using different equations and models related to flows.

(iii) By using the size and power of computers for illustrating the complex behavior of the flows.

So it is decided that in many applications in which full scale laboratory based models are needed. But, in most cases, the lab models must have a different type of scale than the prototype and so that a theoretical or computational model is much better for extrapolation for scaling the prototype. There are many cases, in which a laboratory model is unsuitable for different reasons. It is possible in many cases at distant time to use the Navier-Stokes equations for the various phases and to analyze the details of multiphase flow. If any of them phases become turbulent then the various type of challenge will be astronomical. So the simplification is much necessary in the real models for various types of multiphase flows.

The two types of models are considered for the disperse flow i.e., trajectory models and the two fluid models. In the motion of disperse phase there is analysis by the help of motion of actual particles or larger particles. The details of the flow nearer each of the particles are taking into assumed drag, lift and forces of moment acting on the trajectory motion of the particles. The thermal analysis of particles can also be taken if it is suitable for doing. The trajectory models are very useful to study the granular flows with respect of nature of interstitial fluid. In next approach, two fluid models, the disperse phase is taking into consideration due to next continuous phase mixed and interaction with continuous phase. Now the analysis of the equations for the conservation of mass, momentum and energy are derived for the two fluid flows which included the interaction terms modeling the exchange of momentum, mass and energy relating to flows. *C. E. Brenen et al.* [6] had solved the equations by the help of computational or theoretical methods. Then the two fluid models neglect the discrete nature of disperse phase and approximating the effect on the continuous phase of motion. The theory of the separated flows will be analyzed as single phase flow equations in the two streams and combining them through suitable kinematic conditions at interface.

The free streamline theory is the example of the implementation of strategy, and then the interface conditions used regarding this context are easy.

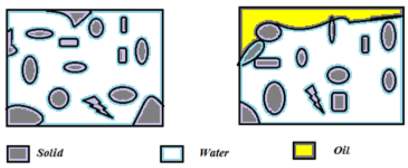

Figure-1 (Porous medium filled with water and oil)

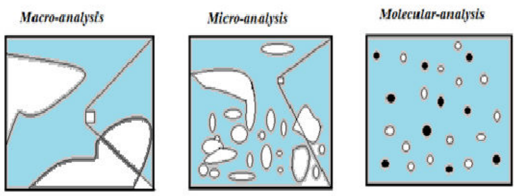

Figure-2 (Analytical scales in porous medium)

The multiphase flow in the porous media is the important topic in the field of fluid mechanics due to its important applications in oil recovery from petroleum reservoirs. The single phase flow in the porous medium is governed by Darcy's law. On the other hand, the modeling of multiphase flow remains an enormous technical difficulties and challenges. There is a big difference between the model equations in industrial applications and the general understanding of the micro scale physics analysis. For the petroleum industry and in some other technical applications the transport is modeled with the help of multiphase generalization of *Darcy's law* given by *P. M. Adler et.al.* [2]. The distinct pressures are analyzed for every constituent phase with the difference said as capillary pressure, and determined with the help of micro pore geometry, interfacial tension and chemistry of the surface of the solid medium. In flow rates the relative permeability is defined which relates the volume flow rate of every fluid to the pressure gradient.

The classic theories of the multiphase flow in porous media have been given by *Wooding & Morel-Seytoux* [12] in 1976 and *Adler & Brenner* in 1988 [2], [6]. The Darcy's law is linear, but the true governing equation is nonlinear as derived by the experiment and micro scale analysis. In addition to surface tension, the fluid flow is shear thinning even if the fluids are Newtonian. The flow will lead to jamming if the pressure gradient is not enough to overcome surface tension and remove the droplets from a pore surface. The jamming phenomenon gives a difference between flows at constant pressure gradient and constant flow rate.

2. Governing Equations and Conditions

The fluid motion is given by the basic hydrodynamic equations, i.e., the equation of continuity is given by

$$D\,\rho + \nabla.(\rho\, v) = 0, \qquad D = \frac{\partial}{\partial t} \tag{2}$$

That gives the conservation of mass and momentum given by the equations:

$$D(\rho\, v) + \nabla.(\rho v\, v) = -\nabla p + \nabla.\tau + \rho g \tag{3}$$

Where ρ is the density, v is the velocity, p is the hydrostatic pressure, τ is the fluid shear tensor and g is acceleration due to external forces including gravity on the fluid. The equation of conservation of energy can be written as:

$$\rho\frac{d\hat{u}}{dt} + p(\nabla.v) = \nabla.(k\,\nabla T) + \phi \tag{4}$$

Where k the coefficient of thermal conductivity of the fluid, T is temperature, ϕ is the viscous dissipation function and $\hat{u}$ is the density of thermal energy. Here $\hat{u}$ is given by

$\hat{u}=\hat{u}\,(p, T)$,

This is approximated as:

$d\hat{u} \approx c_v\ dT$,

Where c_v is the specific heat.

In the Newtonian fluids the viscous stresses varies with velocity derivatives *i.e.*,

$$\tau_{\alpha\beta} = \partial_\alpha u_\beta \tag{5}$$

Now the equation of momentum will be reduced to Navier-Stokes equation given by

$$\frac{\partial(\rho v)}{\partial t} + \nabla.(\rho v\, v) = -\nabla p + \mu \nabla^2 v + \rho g \tag{6}$$

Here μ denotes the viscosity of the fluid. For the incompressible fluids, ρ is constant, and then the equation takes the form

$$\frac{\partial v}{\partial t} + (v.\nabla)v = -\frac{1}{\rho}\nabla p + \frac{\mu}{\rho}\nabla^2 v + g \tag{7}$$

Where $\mu/\rho = \nu$, denotes the kinematics viscosity of the fluid.

The equations for momentum and continuity are independent of T and will be solved with the help of the energy equation. The Navier-Stokes equation gives the nonlinear second order differential equations of four variables, *i.e.* pressure and three components of velocity which can be solved in time and space. The system is solved with the help of the equation of continuity and by using some boundary conditions such as known pressure or velocity at inlet and outlet or at the free surfaces. At solid fluid interfaces, the no-slip boundary condition is the main characteristic of all the viscous flows. That means, at a solid wall the fluid velocity is zero with respect to wall. So, we can solve the flow equations. If we consider the flow as frictionless then Navier-Stokes equation changes to the Euler equation that is as given:

$$g - \frac{1}{\rho}\nabla p = \frac{\partial v}{\partial t} + (v.\nabla)v \tag{8}$$

On integrating along the streamlines in the gravitational field given by

$$g = \hat{g}\hat{e}_\alpha \tag{9}$$

We get a relationship between velocity, pressure and the elevation of flow of fluid, the Bernoulli equation for the incompressible and steady flow is as given:

$$p + \frac{1}{2}\rho v^2 + \rho g \alpha = C_1 \tag{10}$$

Where C_1 is the constant and α is the angle of elevation.

The Bernoulli equation is related to the steady flow energy equation and is used with some conditions. in the case of stationary flow with low inertial forces, the term on the left hand side of eq.(6) can be neglecting and it changes to Stokes equation *i.e.,*

$$\nabla p - \rho\, g = \mu\, \nabla^2 v \tag{11}$$

The above equation is very important in experimental and theoretical work relating to flows in porous media, for that the fluid velocities are very low. In flow system given by the Stokes equation, the pressure drop varies with the fluid velocity and an experiment is used to cover this range. If the flow is symmetric and force is reversed then the streamlines remain unchanged. For the Stokes flow the fluid motion is regular and smooth *i.e.,* the laminar flow. If the inertial forces are comparable to viscous forces *i.e.,* by increasing the flow velocity, the flow begins to have instable and moves to the transitional state. If we still increase the velocity of flow then it becomes turbulent. [1], [8].

The ratio of viscous and inertial forces gives some parameter for the nature of all Newtonian fluids, and the ratio is called the Reynolds number *i.e.,* dimensionless and is defined as:

$$Re = \frac{\rho VL}{\mu} = \frac{VL}{\nu} \qquad (12)$$

Where V is characteristic velocity and L is the length scales of flow.

The Reynolds number *Re* gives criterion for dynamic similarity. For the two similar types of flow systems of different sizes, the flow rates have the same Reynolds number then they will form a similar flow pattern. There also exist some other dimensionless numbers and are important as the results or experiments performed to the real systems. In most of the cases, we have advantage to write the relevant quantities and equations in dimensionless form. For example, the dimensionless form of equation of continuity will be as given

$$\frac{\partial \rho^*}{\partial t^*} + \nabla^*.(\rho^* v^*) = 0 \qquad (13)$$

3.1 Equations for the slightly compressible materils

The equation of state in terms of fluid compressibility c_f is given as

$$c_f = -\frac{1}{R}\left[\frac{\partial V}{\partial p}\right]_T = \frac{1}{\rho}\left[\frac{\partial \rho}{\partial p}\right]_T \qquad (14)$$

C. Zhangxin et.al. [13] had expressed the fluid compressibility c_f as a constant on a certain range of pressures [13] and integrating the equation (14), we have

$$\rho = \rho_0\, e^{c_f(p-p_0)} \qquad (15)$$

Here ρ_0 is the density of the fluid at the pressure p_0. By the Taylor' theorem, we have

$$\rho = \rho_0 \left\{1 + c_f(p-p_0) + \frac{{c_f}^2(p-p_0)^2}{2!} + \cdots \right\} (16)$$

By neglecting the terms of higher powers and approximating, we get

$$\rho \approx \{1 + c_f(p-p_0)\}$$

The compressibility is given by

$$c_R = \frac{1}{\phi}\frac{d\phi}{dp} \qquad (17)$$

Again integrating both the side, we get

$$\phi = \phi_0\, e^{c_R(p-p_0)} \qquad (18)$$

Here ϕ_0 is the porosity at pressure p_0. Also by the Taylor' theorem, we have

$$\phi = \phi_0 \left\{1 + c_R(p-p_0) + \frac{{c_R}^2(p-p_0)^2}{2!} + \cdots \right\} (19)$$

This can also be approximated as:

$$\phi \approx \phi_0 \{1 + c_R(p-p_0)\} \qquad (20)$$

So we have

$$\frac{d\phi}{dp} = c_R\phi_0 \tag{21}$$

After taking the differentiation on the left hand side of (22), we have

$$\left\{\phi\frac{\partial\rho}{\partial p} + \rho\frac{d\phi}{dp}\right\}\frac{\partial p}{\partial t} = \nabla.\left(\rho\frac{1}{\mu}K(\nabla p - \rho\hat{g}\nabla z)\right) + q \tag{22}$$

Put the approximated calculated values [13] in (32), we have

$$\rho\{\phi c_f + c_R\phi_0\}\frac{\partial p}{\partial t} = \nabla.\left(\rho\frac{1}{\mu}K(\nabla p - \rho\hat{g}\nabla z)\right) + q \tag{23}$$

Let us taking, the total compressibility c_t as given

$$c_t = c_f + \frac{\phi_0}{\phi}c_R \tag{24}$$

We have

$$\rho_0\, e^{c_f(p-p_0)}\phi c_t\frac{\partial p}{\partial t} = \nabla.\left(\rho_0\, e^{c_f(p-p_0)}\frac{1}{\mu}K\left(\nabla p - \rho_0\, e^{c_f(p-p_0)}\hat{g}\nabla z\right)\right) + q \tag{25}$$

4. Numerical Analysis

Table-4.1 (Numerical values of the pressure exerted by the kerosene oil, water and turpentine oil in a porous media having the values of K between 1x10^{-11} to 25x10^{-11} m^2)

The table gives the values of calculated pressure of the three fluids, kerosene oil, water and turpentine oil.

S.No.	Fluid	Viscosity μ(Pa s)	Velocity v m/sec.	p(Pa) x10^7 for K=1 x10$^{-11}m^2$	p(Pa) x10^7 for K=2 x10$^{-11}m^2$	p(Pa) x10^7 for K=5 x10$^{-11}m^2$	p(Pa) x10^7 for K=10 x10$^{-11}m^2$	p(Pa)x10^7 for K=25x10^{-11} m^2
1.	Kerosene oil	640x10^{-6}	1	6.4	3.2	1.28	0.64	0.26
			2	12.8	6.4	2.56	1.28	0.52
			5	32	16	6.4	3.2	1.30
			10	64	32	12.8	6.4	2.60
			25	160	80	32	16	6.4
2.	Water	890x10^{-6}	1	8.9	4.45	2.97	0.89	0.36
			2	17.8	8.9	3.35	1.78	0.72
			5	44.5	22.25	8.9	4.45	1.80
			10	89.0	44.5	17.8	8.9	3.60
			25	222.5	111.25	44.5	22.25	9.00
3.	Turpantine oil	1375x10^{-6}	1	13.75	6.875	2.75	1.38	0.55
			2	27.5	13.75	5.5	2.75	1.1
			5	68.75	34.375	13.75	6.88	2.75
			10	137.5	68.75	27.5	13.75	5.5
			25	343.75	171.875	68.75	34.50	13.75

5. Conclusion

In the slightly compressible flow and rock, the motion is always governed by the law of conservation of mass and the Darcy's law given by equation (30), which shows that the velocity of the fluid at phase α is inversely proportional to the viscosity of the fluid. The capillary pressure p_c is inversely proportional to the radius of the tube and increases with increasing values of the surface tension σ of the fluid. It is also inversely proportional to the radii of curvature of the interface of fluid. The results also show that compressibilityc_R depends on the porosity ϕ of the fluid and varies directly with the values of ϕ. The partial differential equation of the equation of motion for the slightly compressible flow and rock is given by equation (49). The variations of pressure through fluid with respect to velocity of fluid and permeability factor of porous medium are shown by the table. That shows that the pressure is directly proportional to the velocity of fluid and is inversely proportional to the permeability factor *K* of the medium.

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