Translate of Intuitionistic M-Fuzzy Group

P.K. Sharma¹ and Manpreet Kaur²

¹P.G. Department of Mathematics, D.A.V. College, Jalandhar City, Punjab, India.
²Department of Applied Science, Chandigarh Engineering College, Landran, Mohali, Punjab.

Email id: pksharma@davjalandhar.com ; manu.kr273@gmail.com

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Abstract

As an abstraction of the geometrical notion of translation, the first author has already introduced two operators $T_{\alpha^+}$ and $T_{\alpha^-}$ called the intuitionistic fuzzy translation operators on the intuitionistic fuzzy sets and studied their properties and investigate the action of these operators on intuitionistic fuzzy subgroups of a group in [11]. The concept of intuitionistic M-fuzzy subgroup of a M-group and their properties are discussed by Zhan and Tan in [12]. Here in this paper we will study the action of these operators on intuitionistic M-fuzzy subgroup of a M-group.

Keywords: Intuitionistic fuzzy set (IFS), Intuitionistic fuzzy subgroup (IFSG), Intuitionistic M-fuzzy subgroup (IMFS), Translates, M- homomorphism.

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1. Introduction

The concept of intuitionistic fuzzy sets was introduced by K.T. Atanassov [1-2] as a generalization to the notion of fuzzy sets by L.A. Zadeh [13]. R. Biswas was the first to introduce the intuitionistic fuzzification of algebraic structure and developed the concept of intuitionistic fuzzy subgroup of a group in [4]. Later on many mathematicians worked in this area and developed the theory of intuitionistic fuzzy groups, for example see [3], [5-6], [7] and [10]. In [12] J. Zhan and Z. Tan has introduced and studied the notion of intuitionistic M-fuzzy subgroup of a M-group which was further studied by M.Oqla Massâdeh in [8]. Sharma in [11] has already introduced two translation operators in intuitionistic fuzzy sets and studied their effect on intuitionistic fuzzy subgroups of a group. Here in this paper, we will study the effect of these two operators on the intuitionistic M-fuzzy subgroups of a M-group.

2. Preliminaries

Atanassov introduced the concept of intuitionistic fuzzy set (IFS) defined on a non empty set $X$ is an object having the form $A =\{(x, \mu_A(x), v_A(x)) : x \in X\}$, where $\mu_A : X \rightarrow [0,1]$ and $v_A : X \rightarrow [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \leq \mu_A(x) + v_A(x) \leq 1$. 

1 | Page
Definition (2.1) [4](Intuitionistic fuzzy subgroup) An IFS \( A = (\mu_A, \nu_A) \) of a group \( G \) is said to be an intuitionistic fuzzy subgroup (IFSG) of \( G \) if

(i) \( \mu_A(xy) \geq \min \{ \mu_A(x), \mu_A(y) \} \) and \( \nu_A(xy) \leq \max \{ \nu_A(x), \nu_A(y) \} \)

(ii) \( \mu_A(x^{-1}) = \mu_A(x) \) and \( \nu_A(x^{-1}) = \nu_A(x) \), \( \forall x, y \in G \).

In other words, An IFS \( A \) of \( X \) is called an IFSG of \( G \) if and only if

\[ \mu_A(xy^{-1}) \geq \min \{ \mu_A(x), \mu_A(y) \} \quad \text{and} \quad \nu_A(xy^{-1}) \leq \max \{ \nu_A(x), \nu_A(y) \}, \forall x, y \in G. \]

Definition (2.2) [9] (M-Group) Let \( G \) be a group and \( M \) be a set of endomorphisms on \( G \), then \( G \) is called M-Group if for any \( g \in G, m \in M \), \( \exists \ mg \in G \) such that \( m(gh) = (mg)h \) \( \forall g, h \in G, m \in M \).

Example (2.3) Let \( G = (R - \{ 0 \}, \cdot) \), be the group of non-zero real numbers under multiplication. Let \( M = \{ f : G \rightarrow G : f(x) = x^2 \} \), the set of endomorphisms. Then \( G \) is a M-group.

Definition (2.4) [9] (M-Subgroup) A subgroup \( N \) of a M-group is said to be M-subgroup if \( \exists x \in N \), \( m \in M \).

Example (2.5) Let \( G = (R - \{ 0 \}, \cdot) \), be a M-group as in Example (2.3). Then \( H = (Q - \{ 0 \}, \cdot) \) is M-subgroup of M-group \( G \).

Definition (2.6) [12] (Intuitionistic M-Fuzzy Subgroup) Let \( G \) be a M-group. Then an intuitionistic fuzzy subgroup \( A \) of \( G \) is called an intuitionistic M-fuzzy subgroup (IMFSG) if

\[ \mu_A(mx) \geq \mu_A(x) \quad \text{and} \quad \nu_A(mx) \leq \nu_A(x), \quad \forall \ x \in G, \ m \in M. \]

Proposition (2.7) [12] Let \( A \) be an IMFSG of M-group \( G \), then for any \( x, y \in G, m \in M \), we have

(i) \( \mu_A(m(xy)) \geq \min \{ \mu_A(mx), \mu_A(my) \} \) and \( \nu_A(m(xy)) \leq \max \{ \nu_A(mx), \nu_A(my) \} \)

(ii) \( \mu_A(mx^{-1}) \geq \mu_A(x) \) and \( \nu_A(mx^{-1}) \leq \nu_A(x), \forall x, y \in G, m \in M. \)

Definition (2.8) [8] (Intuitionistic Normal M-Fuzzy subgroups) Let \( G \) be a M-group and \( A \) be an IMFSG of \( G \), then \( A \) is called an intuitionistic normal M-fuzzy subgroup (INMFSG) if

\[ \mu_A(mx^{-1}y^{-1}) \geq \mu_A(my) \quad \text{and} \quad \nu_A(mx^{-1}y^{-1}) \leq \nu_A(my), \quad \forall x, y \in G, \ m \in M. \]

Definition (2.9) [9, 12](M-homomorphism) Let \( G_1 \) and \( G_2 \) be two M-groups and \( f \) be a homomorphism from \( G_1 \) onto \( G_2 \). If \( f(mx) = m f(x) \), \( \forall x \in G_1 \), then \( f \) is called M-homomorphism.

3. Translation of intuitionistic M-fuzzy subgroups

In this section, we study the action of two operators \( T_{\mu} \) and \( T_{\nu} \) on intuitionistic M-fuzzy subgroup of a M-group \( G \). We prove that these operators take on IMFSG to IMFSG and INMFSG to INMFSG.

Definition (3.1) Let \( A = (\mu_A, \nu_A) \) be an IFS of a M-group \( G \) and \( \alpha \in [0, 1] \). We define
\[ T_{\alpha r}(x) = (\mu_{T_{\alpha r}}(x), \nu_{T_{\alpha r}}(x)) \]

\[ T_{\alpha r}(A)(x) = (\mu_{T_{\alpha r}}(A)(x), \nu_{T_{\alpha r}}(A)(x)) \], where

\[ \mu_{T_{\alpha r}}(x) = \min\{\mu_A(x) + \alpha, 1\} \quad \text{and} \quad \nu_{T_{\alpha r}}(x) = \max\{\nu_A(x) - \alpha, 0\} \]

\[ T_{\alpha r}(A) \] and \[ T_{\alpha l}(A) \] are respectively called the \( \alpha \)-up and \( \alpha \)-down intuitionistic fuzzy operator of \( A \).

**Results (3.2) The following results can be easily verified from definition**

(i) \( T_{0}(A) = T_{0}(A) = A \)

(ii) \( T_{1}(A) = 1 \)

(iii) \( T_{-1}(A) = 0 \)

**Remark (3.3) If \( A \) is an IFS of a M-group \( G \), then both \( T_{\alpha r}(A) \) and \( T_{\alpha l}(A) \) are IFS of \( G \). In other words \( 0 \leq \mu_{T_{\alpha r}}(x) + \nu_{T_{\alpha r}}(x) \leq 1 \) and \( 0 \leq \mu_{T_{\alpha l}}(x) + \nu_{T_{\alpha l}}(x) \leq 1 \), for all \( x \in G \).**

**Theorem (3.4) If \( A \) is an intuitionistic M-fuzzy subgroup of a M-group \( G \), then \( T_{\alpha r}(A) \) and \( T_{\alpha l}(A) \) are also an intuitionistic M-fuzzy subgroup of \( G \).**

**Proof.** Let \( A = (\mu_A, \nu_A) \) be an IFMSG and \( \alpha \in [0,1] \). Let \( x, y \in G, m \in M \)

\[ T_{\alpha r}(A)(x) = (\mu_{T_{\alpha r}}(A)(x), \nu_{T_{\alpha r}}(A)(x)) \quad \text{and} \quad T_{\alpha l}(A)(m) = (\mu_{T_{\alpha l}}(A)(x), \nu_{T_{\alpha l}}(A)(x)) \], where

\[ \mu_{T_{\alpha r}}(x) = \min\{\mu_A(x) + \alpha, 1\} \quad \text{and} \quad \nu_{T_{\alpha r}}(A)(x) = \max\{\nu_A(x) - \alpha, 0\} \]

\[ \mu_{T_{\alpha l}}(x) = \max\{\mu_A(x) - \alpha, 0\} \quad \text{and} \quad \nu_{T_{\alpha l}}(A)(x) = \min\{\nu_A(x) + \alpha, 1\} \]

Now, \[ T_{\alpha r}(A)(xy^{-1}) = (\mu_{T_{\alpha r}}(A)(xy^{-1}), \nu_{T_{\alpha r}}(A)(xy^{-1})) \], here we have

\[ \mu_{T_{\alpha r}}(A)(xy^{-1}) = \min\{\mu_A(xy^{-1}) + \alpha, 1\} \]

\[ \geq \min\{\min\{\mu_A(x), \mu_A(y)\} + \alpha, 1\} \]

\[ = \min\{\min\{\mu_A(x) + \alpha, 1\}, \{\nu_A(y) + \alpha, 1\}\} \]

\[ = \min\{\mu_{T_{\alpha r}}(A)(x), \mu_{T_{\alpha r}}(A)(y)\} \]

\( \text{i.e.,} \quad \mu_{T_{\alpha r}}(A)(xy^{-1}) \geq \min\{\mu_{T_{\alpha r}}(A)(x), \mu_{T_{\alpha r}}(A)(y)\} \).

**Similarly, we have**

\[ \nu_{T_{\alpha r}}(A)(xy^{-1}) = \max\{\nu_A(xy^{-1}) - \alpha, 0\} \]

\[ \leq \max\{\max\{\nu_A(x), \nu_A(y)\} - \alpha, 0\} \]

\[ = \max\{\max\{\nu_A(x) - \alpha, 0\}, \{\nu_A(y) - \alpha, 0\}\} \]

\[ = \max\{\nu_{T_{\alpha r}}(A)(x), \nu_{T_{\alpha r}}(A)(y)\} \]

\( \text{i.e.,} \quad \nu_{T_{\alpha r}}(A)(xy^{-1}) \leq \max\{\nu_{T_{\alpha r}}(A)(x), \nu_{T_{\alpha r}}(A)(y)\} \).

Hence, \( T_{\alpha r}(A) \) is an IFSG of group of \( G \).

Now, \[ \mu_{T_{\alpha r}}(m) = \min\{\mu_A(m), \alpha, 1\} \geq \min\{\mu_A(x) + \alpha, 1\} = \mu_{T_{\alpha r}}(x) \]

Thus, \[ \mu_{T_{\alpha r}}(m) \geq \mu_{T_{\alpha r}}(x) \].
Also, \( \nu_{T^\alpha}(mx) = \max\{\nu_A(mx) - \alpha, 0\} \leq \max\{\nu_A(x) - \alpha, 0\} = \nu_{T^\alpha}(x) \).

So, \( \nu_{T^\alpha}(mx) \leq \nu_{T^\alpha}(x) \).

Thus, \( T^\alpha(A) \) is an IMFS of \( G \).

Similarly, we can prove that \( T^{-\alpha}(A) \) is an IMFS of \( G \).

Remark (3.5) If \( T^\alpha(A) \) or \( T^{-\alpha}(A) \) is an IMFS of a \( M \)-group \( G \) for a particular \( \alpha \in [0,1] \), then it cannot be deduced that \( A \) is an IMFS of \( G \).

Example (3.6) Let \( H \) be a \( M \)– subgroup of a \( M \)-group \( G \) and \( A \) be an IFS on \( G \) defined by

\[
\mu_A(x) = \begin{cases} 
0.2; & x \in H \\
0.6; & \text{otherwise} 
\end{cases}
\quad \text{and} \quad
\nu_A(x) = \begin{cases} 
0.8; & x \in H \\
0.3; & \text{otherwise} 
\end{cases}
\]

Take \( \alpha = 0.8 \), we have

\( \mu_{T^\alpha(A)}(x) = 1 \) and \( \nu_{T^\alpha(A)}(x) = 0 \), \( \forall x \in G \) \( \text{i.e.}, \) \( T^\alpha(A) = \overline{1} \).

Clearly, \( T^\alpha(A) \) is an IMFS of \( G \), however \( A \) is not an IMFS of \( G \).

Proposition (3.7) Let \( G \) be a \( M \)-group with identity element \( e \) and \( A \) be an IMFS of \( G \). Then the set

\( G_A = \{ x \in G : \mu_A(x) = \mu(e) \, \text{and} \, \nu_A(x) = \nu(e) \} \) is an \( M \)-subgroup of \( G \).

Proof: Clearly, \( G_A \neq \emptyset \), for \( e \in G_A \). So, let \( x, y \in G_A \) be any elements, then

\( \mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\} = \min\{\mu_A(e), \mu_A(e)\} = \mu_A(e) \).

But \( \mu_A(e) \geq \mu_A(xy^{-1}) \) always implies that \( \mu_A(xy^{-1}) = \mu_A(e) \).

Similarly, we can show that \( \nu_A(xy^{-1}) = \nu_A(e) \).

Now, \( \mu_A(mx) \geq \mu_A(x) = \mu_A(e) \), but \( \mu_A(e) \geq \mu_A(mx) \) implies \( \mu_A(mx) = \mu_A(e) \).

Similarly, we can show that \( \nu_A(mx) = \nu_A(x) \).

Thus, we get \( xy^{-1}, \, mx \in G_A \), \( \forall x, y \in G \) and \( m \in M \).

Hence \( G_A \) is a \( M \)-subgroup of \( G \).

Theorem (3.8) Let \( A \) be an IFS of a \( M \)-group \( G \) such that \( T^\alpha(A) \) be an IMFS of \( G \), for some \( \alpha \in [0,1] \) with \( \alpha < \min\{1 - p, q\} \), then \( A \) is an IMFS of \( G \), where \( p = \max\{\mu_A(x) : x \in G - G_A\} \) and \( q = \min\{\nu_A(x) : x \in G - G_A\} \).

Proof: Let \( T^\alpha(A) \) be an IMFS of \( G \) for some \( \alpha \in [0,1] \) with \( \alpha < \min\{1 - p, q\} \) for any \( x, y \in G \), \( m \in M \). We have, \( T^\alpha(A)(mx) = (\mu_{T^\alpha(A)(mx)}, \nu_{T^\alpha(A)(mx)}) \), where

\[
\mu_{T^\alpha(A)(mx)} = \min\{\mu_A(mx) + \alpha, 1\} \quad \text{and} \quad \nu_{T^\alpha(A)(mx)} = \max\{\nu_A(mx) - \alpha, 0\}.
\]
Since \( T_{A}^{*}(A) \) is an IMFSG of G, therefore we have
\[
\mu_{T_{A}^{*}(A)}(mx) \geq \mu_{T_{A}^{*}(A)}(x) \quad \text{and} \quad \nu_{T_{A}^{*}(A)}(mx) \leq \nu_{T_{A}^{*}(A)}(x) \quad \quad \quad \text{(b)}
\]

Case I: when \( \mu_{T_{A}^{*}(A)}(x)=1 \) and \( \nu_{T_{A}^{*}(A)}(y)=1 \).

As, \( 0 \leq \mu_{T_{A}^{*}(A)}(x)+\nu_{T_{A}^{*}(A)}(x) \leq 1 \) \( \Rightarrow \nu_{T_{A}^{*}(A)}(x)=0 \). Similarly, we have \( \nu_{T_{A}^{*}(A)}(y)=0 \).

Also, \( \mu_{T_{A}^{*}(A)}(mx) \geq \mu_{T_{A}^{*}(A)}(x)=1 \) but \( \mu_{T_{A}^{*}(A)}(mx) \leq 1 \) (always) \( \Rightarrow \mu_{T_{A}^{*}(A)}(mx)=1 \) and so, \( \nu_{T_{A}^{*}(A)}(mx)=0 \). Similarly, we get \( \mu_{T_{A}^{*}(A)}(my)=1 \) and \( \nu_{T_{A}^{*}(A)}(my)=0 \).

Now, \( \mu_{T_{A}^{*}(A)}(x)=1 \) and \( \nu_{T_{A}^{*}(A)}(x)=0 \)
\[
\Rightarrow \min\{\mu_{A}(x)+\alpha,1\}=1 \quad \text{and} \quad \max\{\nu_{A}(x)-\alpha,0\}=0
\]
\[
\Rightarrow \mu_{A}(x)+\alpha \geq 1 \quad \text{and} \quad \nu_{A}(x)-\alpha \leq 0
\]
\[
\Rightarrow \mu_{A}(x) \geq 1-\alpha \quad \text{and} \quad \nu_{A}(x) \leq \alpha
\]

Similarly, we get \( \mu_{A}(y) \geq 1-\alpha \) and \( \nu_{A}(y) \leq \alpha \) \( \quad \text{.........(l)} \)

Since, \( \alpha < \min\{1-\ p, \ q\} \Rightarrow \alpha < 1-\ p \ \text{and} \ \alpha < q \Rightarrow p < 1-\ \alpha \ \text{and} \ q > \alpha \)
\[
\Rightarrow \max\{\mu_{A}(x): x \in G-A\} < 1-\ \alpha \ \text{and} \ \min\{\nu_{A}(x): x \in G-A\} > \alpha.
\]
Therefore, from (1), we get \( x \in G_{A} \) and \( y \in G_{A} \), but \( G_{A} \) is a M-subgroup of G.

Therefore, \( xy^{-1} \in G_{A} \) and \( mx \in G_{A} \), where \( m \in M \) be any element
\[
\Rightarrow \mu_{A}(xy^{-1}) = \mu_{A}(e) = \min\{\mu_{A}(e), \mu_{A}(e)\} = \min\{\mu_{A}(x), \mu_{A}(y)\}
\]
\[
\Rightarrow \mu_{A}(xy^{-1}) \geq \min\{\mu_{A}(x), \mu_{A}(y)\}.
\]

Similarly, we have \( \nu_{A}(xy^{-1}) \leq \max\{\nu_{A}(x), \nu_{A}(y)\} \).
Also, \( \mu_{A}(mx) = \mu_{A}(e) = \mu_{A}(x) \Rightarrow \mu_{A}(mx) \geq \mu_{A}(x) \).

Similarly, we can show that \( \nu_{A}(mx) \leq \nu_{A}(x) \).

Hence A is an IMFSG of G.

Case II: When \( \mu_{T_{A}^{*}(A)}(x)<1 \) and \( \mu_{T_{A}^{*}(A)}(y)<1 \).
\[
\min\{\mu_{A}(x)+\alpha,1\} < 1 \ \text{and} \ \min\{\mu_{A}(y)+\alpha,1\} < 1 \ \Rightarrow \mu_{A}(x)+\alpha < 1 \ \text{and} \ \mu_{A}(y)+\alpha < 1.
\]
\[
\min\{\mu_{A}(mx)+\alpha,1\} < 1 \ \text{and} \ \min\{\mu_{A}(my)+\alpha,1\} < 1 \ \Rightarrow \mu_{A}(mx)+\alpha < 1 \ \text{and} \ \mu_{A}(my)+\alpha < 1.
\]
As \( T_{A}^{*}(A) \) is an IMFSG of G. Therefore, for any \( x, y \in G \), we have
\[
\mu_{T_{A}^{*}(A)}(xy^{-1}) \geq \min\{\mu_{T_{A}^{*}(A)}(x), \mu_{T_{A}^{*}(A)}(y)\} \quad \text{and} \quad \nu_{T_{A}^{*}(A)}(xy^{-1}) \leq \max\{\nu_{T_{A}^{*}(A)}(x), \nu_{T_{A}^{*}(A)}(y)\} \quad \text{.........(2)}
\]
Now, \( \mu_{T_x(A)}(x^{-1}) \geq \min \{ \mu_{T_x(A)}(x), \mu_{T_y(A)}(y) \} \)

\[
\Rightarrow \min \{ \mu_A(xy^{-1}) + \alpha, 1 \} \geq \min \{ \min \{ \mu_A(x) + \alpha, 1 \}, \min \{ \mu_A(y) + \alpha, 1 \} \} = \min \{ \mu_A(x) + \alpha, \mu_A(y) + \alpha \}
\]

\[
\Rightarrow \mu_A(xy^{-1}) + \alpha \geq \min \{ \mu_A(x), \mu_A(y) \} + \alpha \quad \text{i.e.,} \quad \mu_A(xy^{-1}) \geq \min \{ \mu_A(x), \mu_A(y) \}.
\]

Also, \( v_{T_x(A)}(x^{-1}) \leq \max \{ v_{T_x(A)}(x), v_{T_y(A)}(y) \} \)

\[
\Rightarrow \max \{ v_A(xy^{-1}) - \alpha, 0 \} \leq \max \{ \max \{ v_A(x) + \alpha, 1 \}, \max \{ v_A(y) + \alpha, 1 \} \} = \max \{ v_A(x) - \alpha, v_A(y) - \alpha \}
\]

\[
\Rightarrow v_A(xy^{-1}) - \alpha \leq \max \{ v_A(x), v_A(y) \} - \alpha, \quad \text{i.e.,} \quad v_A(xy^{-1}) \leq \max \{ v_A(x), v_A(y) \}.
\]

Thus \( A \) is an IFSG of \( G \).

Also, as \( T_{\alpha^+}(A) \) is an IMFSG so, \( \mu_{T_{\alpha^+}(A)}(mx) \geq \mu_{T_{\alpha^+}(A)}(x) \) and \( v_{T_{\alpha^+}(A)}(mx) \leq v_{T_{\alpha^+}(A)}(x) \), \( \forall \ m \in M \).

\[
\Rightarrow \min \{ \mu_A(mx) + \alpha, 1 \} \geq \min \{ \mu_A(x) + \alpha, 1 \} \quad \text{and} \quad \max \{ v_A(mx) - \alpha, 0 \} \leq \max \{ v_A(x) - \alpha, 0 \}
\]

\[
\Rightarrow \mu_A(mx) + \alpha \geq \mu_A(x) + \alpha \quad \text{and} \quad v_A(mx) - \alpha \leq v_A(x) - \alpha
\]

\[
\Rightarrow \mu_A(mx) \geq \mu_A(x) \quad \text{and} \quad v_A(mx) \leq v_A(x).
\]

Hence \( A \) is an IMFSG of \( G \).

**Case III:** When \( \mu_{T_{\alpha^+}(A)}(x) = 1 \) and \( \mu_{T_{\alpha^+}(A)}(y) < 1 \).

As in case (i), we get \( x \in G_{\alpha} \), so \( \mu_A(x) = \mu_A(e) \) and \( v_A(x) = v_A(e) \).

As \( T_{\alpha}(A) \) is an IMFSG of \( G \). So, we have

\[
\mu_{T_{\alpha}(A)}(x^{-1}) \geq \min \{ \mu_{T_{\alpha}(A)}(x), \mu_{T_{\alpha}(A)}(y) \} = \min \{ 1, \mu_{T_{\alpha}(A)}(y) \} = \mu_{T_{\alpha}(A)}(y).
\]

\[
\Rightarrow \min \{ \mu_A(xy^{-1}) + \alpha, 1 \} \geq \min \{ \mu_A(y) + \alpha, 1 \}
\]

\[
\Rightarrow \mu_A(xy^{-1}) + \alpha \geq \mu_A(y) + \alpha
\]

\[
\Rightarrow \mu_A(xy^{-1}) \geq \mu_A(y) = \min \{ \mu_A(e), \mu_A(y) \} = \min \{ \mu_A(x), \mu_A(y) \}.
\]

Similarly, we can show that \( v_A(xy^{-1}) \leq \max \{ v_A(x), v_A(y) \} \).

Therefore, \( A \) is an IFSG of \( G \).

Moreover, as \( \mu_{T_{\alpha^+}(A)}(x) = 1 \Rightarrow \mu_{T_{\alpha^+}(A)}(mx) = 1 \) as in case (i) and hence \( \mu_A(mx) \geq \mu_A(x) \).

**Similarly,** we can show that \( v_A(mx) \leq v_A(x) \). Hence \( A \) is an IMFSG of \( G \).

**Proposition (3.9)** Let \( A \) be an IFS of a \( M \)-group \( G \) such that \( T_{\alpha}(A) \) be IMFSG of \( G \), for some \( \alpha \in [0, 1] \) with \( \alpha < \min \{ 1 - p, q \} \), then \( A \) is an IMFSG of \( G \), where \( p = \max \{ \mu_A(x) : x \in G - G_{\alpha} \} \), \( q = \min \{ v_A(x) : x \in G - G_{\alpha} \} \).

**Proof:** Similar as in proposition (3.8)

**Theorem (3.10)** If \( A \) be an INMFSG of \( M \)-group \( G \) if and only if \( T_{\alpha}(A) \) and \( T_{\alpha^{-}}(A) \) are INFMSG of \( G \) for \( \alpha \in [0,1] \).

**Proof:** Firstly, let \( A \) be an INMFSG of a \( M \)-group \( G \) and \( \alpha \in [0,1] \) be any real number. Then

\[
\mu_A(m(xy^{-1})) \geq \mu_A(my) \quad \text{and} \quad v_A(m(xy^{-1})) \leq v_A(my) \quad \forall \ x, y \in G, \ m \in M.
\]

We have already proved that \( T_{\alpha}(A) \) and \( T_{\alpha^{-}}(A) \) are IMFSGs of \( G \) (See Theorem (3.4))
\[ \mu_x(m(x^{-1}y)) \geq \mu_y(mx) \Rightarrow \min \{ \mu_x(m(x^{-1}y)) + \alpha, 1 \} \geq \min \{ \mu_y(mx) + \alpha, 1 \} \]

i.e., \[ \mu_{v_x^{-1}}(m(x^{-1}y)) \geq \mu_{v_y^{-1}}(mx). \]

Similarly, we can show that \[ v_{v_x^{-1}}(m(x^{-1}y)) \geq v_{v_y^{-1}}(mx). \]

Hence, \( T_{a^+} \) is also an INMFSF of \( G \).

Conversely, let \( T_{a^+}(A) \) and \( T_{a^+}(A) \) are INMFSF of \( G \) for \( \alpha \in [0,1] \).

Take \( \alpha = 0 \), we get \( T_{0^+}(A) = A = T_{0^+}(A). \)

Hence, \( A \) is an INMFSF of \( G \).

4. **M- homomorphism of an intuitionistic M-fuzzy subgroups**

**Lemma (4.1)** Let \( f : X \rightarrow Y \) be a mapping and \( \alpha \in [0,1] \) be any real number. If \( A \) and \( B \) be any IFSs on \( X \) and \( Y \) respectively, then

\[
(i) \quad f^{-1}(T_{a^+}(B)) = T_{a^+}(f^{-1}(B)); \\
(ii) \quad f(T_{a^+}(A)) = T_{a^+}(f(A)).
\]

**Proof.** (i) Now, \( f^{-1}(T_{a^+}(B))(x) = \left( \mu_{f^{-1}(v_x^{-1}(B))}(x), v_{f^{-1}(v_x^{-1}(B))}(x) \right) \), where

\[
\mu_{f^{-1}(v_x^{-1}(B))}(x) = \mu_{v_x^{-1}(B)}(f(x)) = \min \{ \mu_y(f(x)) + \alpha, 1 \} = \min \{ \mu_{v_x^{-1}(B)}(f^{-1}(B)) \}(x) \text{ and}
\]

\[
v_{f^{-1}(v_x^{-1}(B))}(x) = v_{v_x^{-1}(B)}(f(x)) = \max \{ \nu_y(f(x)) - \alpha, 0 \} = v_{v_x^{-1}(B)}(f^{-1}(B))(x).
\]

Thus, \( f^{-1}(T_{a^+}(B)) = T_{a^+}(f^{-1}(B)). \)

(ii) Now, \( f(T_{a^+}(A))(y) = \left( \mu_{f(v_x^{-1}(A))}(y), v_{f(v_x^{-1}(A))}(y) \right) \), where

\[
\mu_{f(v_x^{-1}(A))}(y) = \sup \{ \mu_{v_x^{-1}(A)}(x) : f(x) = y \}
\]

\[
= \sup \{ \min \{ \mu_A(x) + \alpha, 1 \} : f(x) = y \}
\]

\[
= \min \{ \sup \{ \mu_A(x) + \alpha : f(x) = y \}, 1 \}
\]

\[
= \min \{ \sup \{ \mu_A(x) : f(x) = y \} + \alpha, 1 \}
\]

\[
= \mu_{f(v_A)}(y) + \alpha, 1 \}
\]

\[
= \mu_{v_x^{-1}(f(A))}(y).
\]

**Theorem (4.2)** Let \( G_1 \) and \( G_2 \) be two M-groups and let \( f : G_1 \rightarrow G_2 \) be a M – homomorphism. Let \( B \) be an IFS on \( G_2 \) such that \( T_{a^+}(B) \) is an IMFSF of \( G_2 \), then \( f^{-1}(T_{a^+}(B)) \) is an IMFSF of \( G_1 \).

**Proof:** As we have already proved that \( f^{-1}(T_{a^+}(B)) = T_{a^+}(f^{-1}(B)) \). So, we will prove that \( T_{a^+}(f^{-1}(B)) \) is an IMFSF of \( G_1 \). Since \( T_{a^+}(f^{-1}(B))(x) = (\mu_{v_x^{-1}(B)}(f^{-1}(B))(x), v_{v_x^{-1}(B)}(f^{-1}(B))(x)) \)

\[
\mu_{v_x^{-1}(B)}(f^{-1}(B))(x) = \min \{ \mu_{v_x^{-1}}(B)(x) + \alpha, 1 \} \text{ and } v_{v_x^{-1}(B)}(f^{-1}(B))(x) = \max \{ v_{v_x^{-1}}(B)(x) - \alpha, 0 \}.
\]
Let \( x, y \in G_1 \) and \( m \in M \) be any elements, then
\[
\mu_{T_{\alpha+}(f^{-1}(B))}(xy^{-1}) = \min\{\mu_{f^{-1}(B)}(xy^{-1}) + \alpha, 1\}
\]
\[
= \min\{\mu_B(f(xy^{-1})) + \alpha, 1\} = \min\{\mu_B(f(x)f(y)^{-1}) + \alpha, 1\}
\]
\[
= \mu_{T_{\alpha+}(B)}(f(x)f(y)^{-1})
\]
\[
\geq \min\{\mu_{T_{\alpha+}(B)}(f(x)), \mu_{T_{\alpha+}(B)}(f(y))\}
\]
\[
= \min\{\mu_{f^{-1}(T_{\alpha+}(B))}(x), \mu_{f^{-1}(T_{\alpha+}(B))}(y)\}
\]
\[
= \min\{\mu_{T_{\alpha+}(f^{-1}(B))}(x), \mu_{T_{\alpha+}(f^{-1}(B))}(y)\}.
\]
Thus, \( \mu_{T_{\alpha+}(f^{-1}(B))}(xy^{-1}) \geq \min\{\mu_{T_{\alpha+}(f^{-1}(B))}(x), \mu_{T_{\alpha+}(f^{-1}(B))}(y)\} \).

Similarly, we can show that \( \nu_{T_{\alpha+}(f^{-1}(B))}(xy^{-1}) \leq \max\{\nu_{T_{\alpha+}(f^{-1}(B))}(x), \nu_{T_{\alpha+}(f^{-1}(B))}(y)\} \).

Further, \( \mu_{T_{\alpha+}(f^{-1}(B))}(mx) = \min\{\mu_{f^{-1}(B)}(mx) + \alpha, 1\} \)
\[
= \min\{\mu_B(f(mx) + \alpha, 1\}
\]
\[
= \min\{\mu_B mf(x) + \alpha, 1\}
\]
\[
= \mu_{T_{\alpha+}(B)}(mf(x))
\]
\[
= \mu_{f^{-1}(T_{\alpha+}(B))}(x)
\]
\[
= \mu_{T_{\alpha+}(f^{-1}(B))}(x) \text{ [Using lemma (4.1)(i)]}
\]
Thus, \( \mu_{T_{\alpha+}(f^{-1}(B))}(mx) \geq \mu_{T_{\alpha+}(f^{-1}(B))}(x) \).

Similarly, we can show that \( \nu_{T_{\alpha+}(f^{-1}(B))}(mx) \leq \nu_{T_{\alpha+}(f^{-1}(B))}(x) \).

Thus, \( T_{\alpha+}(f^{-1}(B)) \) and hence \( f^{-1}(T_{\alpha+}(B)) \) is an IFMSG of \( G_1 \).

**Theorem (4.3)** Let \( G_1 \) and \( G_2 \) be two \( M \)-groups and let \( f : G_1 \to G_2 \) be a \( M \)-homomorphism. Let \( A \) be an IFS on \( G_1 \) such that \( T_{\alpha+}(A) \) is an IMFSG of \( G_1 \), then \( f(T_{\alpha+}(A)) \) is an IMFSG of \( G_2 \).

**Proof:** As we have already proved that \( f(T_{\alpha+}(A)) = T_{\alpha+}(f(A)) \).

Now, we show that \( T_{\alpha+}(f(A)) \) is an IMFSG of \( G_2 \). Let \( x^*, y^* \in G_2 \), and \( m \in M \) be any elements, then \( \exists \)’s \( x, y \in G_1 \) such that \( f(x) = x^*, f(y) = y^* \). Now \( T_{\alpha+}(f(A)) = (\mu_{T_{\alpha+}(f(A))}(xy^{-1}), \nu_{T_{\alpha+}(f(A))}(xy^{-1})) \)
\[ \mu_{\alpha+}(f(A))(xy^{-1}) = \min \{ \mu_f(A)(xy^{-1}) + \alpha, 1 \} \]
\[ = \min \left\{ \sup \{ \mu_A(xy^{-1}) : f(x) = x^*, f(y) = y^* \} + \alpha, 1 \right\} \]
\[ = \min \left\{ \sup \{ \mu_A(xy^{-1}) + \alpha : f(x) = x^*, f(y) = y^* \}, 1 \right\} \]
\[ = \sup \left\{ \mu_{\alpha+}(A)(xy^{-1}) : f(x) = x^*, f(y) = y^* \right\} \]
\[ \geq \sup \left\{ \min \{ \mu_{\alpha+}(A)(x), \mu_{\alpha+}(A)(y) \} : f(x) = x^*, f(y) = y^* \right\} \]
\[ = \min \left\{ \sup \{ \mu_{\alpha+}(A)(x) : f(x) = x^* \}, \sup \{ \mu_{\alpha+}(A)(y) : f(y) = y^* \} \right\}. \]

Thus, \( \mu_{\alpha+}(f(A))(xy^{-1}) \geq \min \left\{ \mu_{\alpha+}(f(A))(x), \mu_{\alpha+}(f(A))(y) \right\}. \)

Similarly, we can show that \( \nu_{\alpha+}(f(A))(xy^{-1}) \leq \max \left\{ \nu_{\alpha+}(f(A))(x), \nu_{\alpha+}(f(A))(y) \right\}. \)

\[ \mu_{\alpha+}(f(A))(mx) = \min \{ \mu_f(A)(mx^*) + \alpha, 1 \} \]
\[ = \min \left\{ \sup \{ \mu_A(mx) : f(mx) = mx^* \} + \alpha, 1 \right\} \]
\[ = \min \left\{ \sup \{ \mu_A(mx) + \alpha : f(mx) = mx^* \}, 1 \right\} \]
\[ = \sup \left\{ \min \{ \mu_A(mx) + \alpha, 1 \} : f(mx) = mx^* \right\} \]
\[ = \sup \left\{ \mu_{\alpha+}(A)(mx) : f(mx) = mx^* \right\} \]
\[ \geq \sup \left\{ \mu_{\alpha+}(A)(x) : f(x) = x^* \right\} \]
\[ = \mu_{f(T_{\alpha+}(A))}(x^*) = \mu_{\alpha+}(f(A))(x^*). \]

Thus, \( \mu_{\alpha+}(f(A))(mx^*) \geq \mu_{\alpha+}(f(A))(x^*). \)

Similarly, we can show that \( \nu_{\alpha+}(f(A))(mx^*) \leq \nu_{\alpha+}(f(A))(x^*). \)

Thus, \( T_{\alpha+}(f(A)) \) and hence \( f(T_{\alpha+}(A)) \) is an IMFSG on \( G_2 \).

5. Conclusions

In this paper, we have investigated the effect on the IMFSG of a M-group \( G \) under the two translation operators and concluded that it remains invariant under the two translation operators. We have also observed the effect of translation of IMFSG of M-group \( G \) under M-homomorphism.

References


