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Translate of Intuitionistic M-Fuzzy Group

P.K. Sharma[1] and Manpreet Kaur[2]

[1]P.G. Department of Mathematics, D.A.V. College, Jalandhar City, Punjab, India.

[2]Department of Applied Science, Chandigarh Engineering College, Landran, Mohali, Punjab.

Email id: pksharma@davjalandhar.com ; manu.kr273@gmail.com

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***First Author / Corresponding Author; Paper ID: A16301**

Abstract

As an abstraction of the geometrical notion of translation, the first author has already introduced two operators $T_{\alpha+}$ *and* $T_{\alpha-}$ *called the intuitionistic fuzzy translation operators on the intuitionistic fuzzy sets and studied their properties and investigate the action of these operators on intuitionistic fuzzy subgroups of a group in [11]. The concept of intuitionistic M-fuzzy subgroup of a M-group and their properties are discussed by Zhan and Tan in [12]. Here in this paper we will study the action of these operators on intuitionistic M- fuzzy subgroup of a M- group.*

Keywords: Intuitionistic fuzzy set (IFS), Intuitionistic fuzzy subgroup (IFSG), Intuitionistic M-fuzzy subgroup (IMFSG), Translates, M- homomorphism.

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1. Introduction

The concept of intuitionistic fuzzy sets was introduced by K.T. Atanassov [1-2] as a generalization to the notion of fuzzy sets by L.A. Zadeh [13]. R. Biswas was the first to introduce the intuitionistic fuzzification of algebraic structure and developed the concept of intuitionistic fuzzy subgroup of a group in [4]. Later on many mathematicians worked in this area and developed the theory of intuitionistic fuzzy groups, for example see [3], [5-6], [7] and [10]. In [12] J. Zhan and Z. Tan has introduced and studied the notion of intuitionistic M-fuzzy subgroup of a M-group which was further studied by M.Oqla Massa'deh in [8]. Sharma in [11] has already introduced two translation operators in intuitionistic fuzzy sets and studied their effect on intuitionistic fuzzy subgroups of a group. Here in this paper, we will study the effect of these two operators on the intuitionistic M-fuzzy subgroups of a M-group.

2. Preliminaries

Atanassov introduced the concept of intuitionistic fuzzy set (IFS) defined on a non empty set X is an object having the form $A = \{< x, \mu_A(x), \nu_A(x) > : x \in X\}$, where $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and degree of non-membership of the element $x \in X$ respectively and for any $x \in X$, we have $0 \le \mu_A(x) + \nu_A(x) \le 1$.

Definition (2.1) [4](Intuitionistic fuzzy subgroup) An IFS A= (μ_A, ν_A) of a group G is said to be an intuitionistic fuzzy subgroup (IFSG) of G if

$$\text{(i)} \quad \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \quad \text{and} \quad \nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$$
$$\text{(ii)} \quad \mu_A(x^{-1}) = \mu_A(x) \quad \text{and} \quad \nu_A(x^{-1}) = \nu_A(x), \ \forall x, y \in G.$$

In other words, An IFS A of X is called an IFSG of G if and only if

$$\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\} \ \text{and} \ \nu_A(xy^{-1}) \leq \max\{\nu_A(x), \nu_A(y)\}, \forall x, y \in G.$$

Definition (2.2) [9] **(M-Group)** Let G be a group and M be a set of endomorphisms on G, then G is called M- Group if for any $g \in$G, $m \in$M, ∃ $mg \in$G such that $m(gh) = (mg)(mh)$ $\forall g, h \in$G, $m \in$M.

Example (2.3) Let G = (R - {0}, .), be the group of non-zero real numbers under multiplication. Let M = { f : G → G : $f(x) = x^{-1}$ }, the set of endomorphisms. Then G is a M-group.

Definition (2.4) [9] **(M-Subgroup)** A subgroup N of a M-group is said to be M-subgroup if $mx \in$N $\forall x \in N, m \in M$.

Example (2.5) Let G = (R - {0}, .), be a M-group as in Example (2.3). Then H = (Q –{0}, .) is M-subgroup of M-group G.

Definition (2.6) [12] **(Intuitionistic M-Fuzzy Subgroup)** Let G be a M-group. Then an intuitionistic fuzzy subgroup A of G is called an intuitionistic M-fuzzy subgroup (IMFSG) if $\mu_A(mx) \geq \mu_A(x)$ and $\nu_A(mx) \leq \nu_A(x), \ \forall \ x \in G, \ m \in M.$

Proposition (2.7) [12] Let A be an IMFSG of M-group G, then for any $x,y \in G$, $m \in$M, we have

$$(i) \quad \mu_A(m(xy)) \geq \min\{\mu_A(mx), \mu_A(my)\} \ \text{and} \ \nu_A(m(xy)) \leq \max\{\nu_A(mx), \nu_A(my)\}$$
$$\text{(ii)} \ \mu_A(mx^{-1}) \geq \mu_A(x) \ \text{and} \ \nu_A(mx^{-1}) \leq \nu_A(x), \ \forall x, y \in G \ , m \in M.$$

Definition (2.8) [8] **(Intuitionistic Normal M-Fuzzy subgroups)** Let G be a M- group and A be an IMFSG of G, then A is called an intuitionistic normal M-fuzzy subgroup (INMFSG) if

$$\mu_A(m(xyx^{-1})) \geq \mu_A(my) \ \ and \ \ \nu_A(m(xyx^{-1})) \leq \nu_A(my), \ \forall x, y \in G, \ m \in M.$$

Definition (2.9) [9, 12](M-homomorphism) Let G_1 and G_2 be two M-groups and f be a homomorphism from G_1 onto G_2. If $f(mx) = m f(x)$ $\forall x \in G_1$ *and* $m \in M$, then f is called M-homomorphism.

3. Translation of intuitionistic M-fuzzy subgroups

In this section, we study the action of two operators $T_{\alpha+}$ and $T_{\alpha-}$ on intuitionistic M-fuzzy subgroup of a M-group G. We prove that these operators take on IMFSG to IMFSG and INMFSG to INMFSG.

Definition (3.1) Let A = (μ_A, ν_A) be an IFS of a M-group G and $\alpha \in [0, 1]$. We define

$T_{\alpha+}(A)(x) = (\mu_{T\alpha+(A)}(x), \nu_{T\alpha+(A)}(x))$ and $T_{\alpha-}(A)(x) = (\mu_{T\alpha-(A)}(x), \nu_{T\alpha-(A)}(x))$, where

$\mu_{T\alpha+(A)}(x) = \min\{\mu_A(x) + \alpha, 1\}$, $\nu_{T\alpha+(A)}(x) = \max\{\nu_A(x) - \alpha, 0\}$ and

$\mu_{T\alpha-(A)}(x) = \max\{\mu_A(x) - \alpha, 0\}$, $\nu_{T\alpha-(A)}(x) = \min\{\nu_A(x) + \alpha, 1\}, \forall\, x \in G$.

$T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are respectively called the α- up and α- down intuitionistic fuzzy operator of A. We shall call $T_{\alpha+}$ and $T_{\alpha-}$ as the intuitionistic fuzzy operator.

Results (3.2) The following results can be easily verified from definition

(i) $T_{0+}(A) = T_{0-}(A) = A$ (ii) $T_{1+}(A) = \tilde{1}$ (iii) $T_{1-}(A) = \tilde{0}$

Remark (3.3) If A is an IFS of a M-group G, then both $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are IFS of G. In other words $0 \le \mu_{T\alpha+(A)}(x) + \nu_{T\alpha+(A)}(x) \le 1$ and $0 \le \mu_{T\alpha-(A)}(x) + \nu_{T\alpha-(A)}(x) \le 1$, for all $x \in G$.

Theorem (3.4) If A is an intuitionistic M-fuzzy subgroup of a M-group G, then $T_{\alpha+}(A)$ and $T_{\alpha-}(A)$ are also an intuitionistic M-fuzzy subgroup of G.

Proof. Let $A = (\mu_A, \nu_A)$ be an IMFSG and $\alpha \in [0,1]$. Let $x, y \in G$, $m \in M$

$T_{\alpha+}(A)(x) = (\mu_{T\alpha+(A)}(x), \nu_{T\alpha+(A)}(x))$ and $T_{\alpha-}(A)(mx) = (\mu_{T\alpha-(A)}(x), \nu_{T\alpha-(A)}(x))$, where

$\mu_{T\alpha+(A)}(x) = \min\{\mu_A(x) + \alpha, 1\}$, $\nu_{T\alpha+(A)}(x) = \max\{\nu_A(x) - \alpha, 0\}$ and

$\mu_{T\alpha-(A)}(x) = \max\{\mu_A(x) - \alpha, 0\}$, $\nu_{T\alpha-(A)}(x) = \min\{\nu_A(x) + \alpha, 1\}$.

Now, $T_{\alpha^+}(A)(xy^{-1}) = (\mu_{T_{\alpha^+}(A)}(xy^{-1}), \nu_{T_{\alpha^+}(A)}(xy^{-1}))$, here we have

$$\begin{aligned}
\mu_{T_{\alpha^+}}(A)(xy^{-1}) &= \min\{\mu_A(xy^{-1}) + \alpha, 1\} \\
&\ge \min\{\min\{\mu_A(x), \mu_A(y)\} + \alpha, 1\} \\
&= \min\{\min\{\mu_A(x) + \alpha, 1\}, \{\mu_A(y) + \alpha, 1\}\} \\
&= \min\{\mu_{T_{\alpha^+}}(x), \mu_{T_{\alpha^+}}(y)\}
\end{aligned}$$

$$i.e.,\ \mu_{T_{\alpha^+}}(A)(xy^{-1}) \ge \min\{\mu_{T_{\alpha^+}}(A)(x), \mu_{T_{\alpha^+}}(A)(y)\}.$$

Similarly, we have

$$\begin{aligned}
\nu_{T_{\alpha^+}}(A)(xy^{-1}) &= \max\{\nu_A(xy^{-1}) - \alpha, 0\} \\
&\le \max\{\max\{\nu_A(x), \nu_A(y)\} - \alpha, 0\} \\
&= \max\{\max\{\nu_A(x) - \alpha, 0\}, \{\nu_A(y) - \alpha, 0\}\} \\
&= \max\{\nu_{T_{\alpha^+}}(x), \nu_{T_{\alpha^+}}(y)\}
\end{aligned}$$

$$i.e.,\ \nu_{T_{\alpha^+}}(A)(xy^{-1}) \le \max\{\nu_{T_{\alpha^+}}(A)(x), \nu_{T_{\alpha^+}}(A)(y)\}.$$

Hence, $T_{\alpha^+}(A)$ is an IFSG of group of G.

Now, $\mu_{T_{\alpha^+}}(mx) = \min\{\mu_A(mx) + \alpha, 1\} \ge \min\{\mu_A(x) + \alpha, 1\} = \mu_{T_{\alpha^+}}(x)$

Thus, $\mu_{T_{\alpha^+}}(mx) \ge \mu_{T_{\alpha^+}}(x)$.

Also, $\nu_{T_{\alpha^+}}(mx) = \max\{\nu_A(mx) - \alpha, 0\} \le \max\{\nu_A(x) - \alpha, 0\} = \nu_{T_{\alpha^+}}(x).$
So, $\nu_{T_{\alpha^+}}(mx) \le \nu_{T_{\alpha^+}}(x).$

Thus, $T_{\alpha^+}(A)$ is an IMFSG of G.

Similarly, we can prove that $T_{\alpha^-}(A)$ is an IMFSG of G.

Remark (3.5) If $T_{\alpha^+}(A)$ or $T_{\alpha^-}(A)$ is an IMFSG of a M-group G for a particular α ∈[0,1], then it cannot be deduced that A is an IMFSG of G.

Example (3.6) Let H be a M – subgroup of a M-group G and A be an IFS on G defined by

$$\mu_A(x) = \begin{cases} 0.2; & x \in H \\ 0.6; & otherwise \end{cases} \quad \text{and} \quad \nu_A(x) = \begin{cases} 0.8; & x \in H \\ 0.3; & otherwise \end{cases}.$$

Take $\boldsymbol{\alpha = 0.8}$, we have

$\mu_{T_{\alpha^+}(A)}(x) = 1$ and $\nu_{T_{\alpha^+}(A)}(x) = 0$, $\forall\, x \in G$ i.e., $T_{\alpha^+}(A) = \tilde{1}.$

Clearly, $T_{\alpha_+}(A)$ is an IMFSG of G, however A is not an IMFSG of G.

Proposition (3.7) Let G be a M-group with identity element *e* and A be an IMFSG of G. Then the set $G_A = \{x \in G:\ \mu_A(x) = \mu(e)\ and\ \nu_A(x) = \nu(e)\}$ is an M-subgroup of G.

Proof: Clearly, $G_A \neq \varnothing$, for $e \in G_A$. So, let $x, y \in G_A$ be any elements, then

$\mu_A(xy^{-1}) \ge \min\{\mu_A(x), \mu_A(y)\} = \min\{\mu_A(e), \mu_A(e)\} = \mu_A(e).$
But $\mu_A(e) \ge \mu_A(xy^{-1})$ always implies that $\mu_A(xy^{-1}) = \mu_A(e).$
Similarly, we can show that $\nu_A(xy^{-1}) = \nu_A(e).$
Now, $\mu_A(mx) \ge \mu_A(x) = \mu_A(e),$ but $\mu_A(e) \ge \mu_A(mx)$ implies $\mu_A(mx) = \mu_A(e)$
Simliarly, we can show that $\nu_A(mx) = \nu_A(x).$
Thus, we get $xy^{-1}, mx \in G_A$, $\forall\, x, y \in G$ and $m \in$ M.
Hence G_A is a M-subgroup of G.

Theorem (3.8) Let A be an IFS of a M-group G such that $T_{\alpha_+}(A)$ be an IMFSG of G, for some α∈$[\mathbf{0}, \mathbf{1}]$ with $\boldsymbol{\alpha < \min\{1 - p, q\}}$, then A is an IMFSG of G, where p = max{$\mu_A(x)$: x∈ G-G_A} and q = min{$\nu_A(x)$: x ∈G-G_A}.

Proof: Let $T_{\alpha_+}(A)$ be an IMFSG of G for some $\alpha \in [0,1]$ with $\boldsymbol{\alpha < \min\{1 - p, q\}}$ for any x ,y ∈G , m∈M. We have, $T_{\alpha_+}(A)(mx) = (\mu_{T_{\alpha^+}(A)}(mx), \nu_{T_{\alpha^+}(A)}(mx)), where$

$\mu_{T_{\alpha^+}(A)}(mx) = \min\{\mu_A(mx) + \alpha, 1\}$ and $\nu_{T_{\alpha^+}}(mx) = \max\{\nu_A(mx) - \alpha, 0\}.$

Since $T_{\alpha^+}(A)$ is an IMFSG of G , therefore we have

$\mu_{T_{\alpha^+}(A)}(mx) \geq \mu_{T_{\alpha^+}(A)}(x)$ and $\nu_{T_{\alpha^+}(A)}(mx) \leq \nu_{T_{\alpha^+}(A)}(x)$ ....(*)

Case I: *when* $\mu_{T_{\alpha^+}(A)}(x) = 1$ and $\nu_{T_{\alpha^+}(A)}(y) = 1.$

As, $0 \leq \mu_{T_{\alpha^+}(A)}(x) + \nu_{T_{\alpha^+}(A)}(x) \leq 1 \Rightarrow \nu_{T_{\alpha^+}(A)}(x) = 0.$ Similarly, we have $\nu_{T_{\alpha^+}(A)}(y) = 0.$

Also, $\mu_{T_{\alpha^+}(A)}(mx) \geq \mu_{T_{\alpha^+}(A)}(x) = 1$ but $\mu_{T_{\alpha^+}(A)}(mx) \leq 1$ (always) $\Rightarrow$ $\mu_{T_{\alpha^+}(A)}(mx) = 1$ and so, $\nu_{T_{\alpha^+}(A)}(mx) = 0.$

Similarly, we get $\mu_{T_{\alpha^+}(A)}(my) = 1$ and $\nu_{T_{\alpha^+}(A)}(my) = 0.$

Now, $\mu_{T_{\alpha^+}(A)}(x) = 1$ *and* $\nu_{T_{\alpha^+}(A)}(x) = 0$

$\Rightarrow \min\{\mu_A(x) + \alpha, 1\} = 1$ *and* $\max\{\nu_A(x) - \alpha, 0\} = 0$

$\Rightarrow \mu_A(x) + \alpha \geq 1$ *and* $\nu_A(x) - \alpha \leq 0$

$\Rightarrow \mu_A(x) \geq 1 - \alpha$ *and* $\nu_A(x) \leq \alpha.$

Similarly, we get $\mu_A(y) \geq 1 - \alpha$ *and* $\nu_A(y) \leq \alpha$ ....(1)

Since, $\alpha < \min\{1 - p, q\}$ $\Rightarrow$ $\alpha < 1 - p$ and $\alpha < q$ $\Rightarrow$ $p < 1 - \alpha$ and $q > \alpha$

$\Rightarrow \max\{\mu_A(x): x \in G - G_A\} < 1 - \alpha$ and $\min\{\nu_A(x): x \in G - G_A\} > \alpha.$

Therefore, from (1), we get $x \in G_A$ and $y \in G_A$, but G_A is a M-subgroup of G.

Therefore, $xy^{-1} \in G_A$ and $mx \in G_A$, where $m \in M$ be any element

$\Rightarrow \mu_A(xy^{-1}) = \mu_A(e) = min\{\mu_A(e), \mu_A(e)\} = min\{\mu_A(x), \mu_A(y)\}$

$\Rightarrow \mu_A(xy^{-1}) \geq min\{\mu_A(x), \mu_A(y)\}.$

Similarly, we have $\nu_A(xy^{-1}) \leq max\{\nu_A(x), \nu_A(y)\}.$

Also, $\mu_A(mx) = \mu_A(e) = \mu_A(x) \Rightarrow \mu_A(mx) \geq \mu_A(x).$

Similarly, we can show that $\nu_A(mx) \leq \nu_A(x).$

Hence A is an IMFSG of G.

Case II: When $\mu_{T\alpha+(A)}(x) < 1$ and $\mu_{T\alpha+(A)}(y) < 1.$

$\min\{\mu_A(x) + \alpha, 1\} < 1$ and $\min\{\mu_A(y) + \alpha, 1\} < 1 \Rightarrow \mu_A(x) + \alpha < 1$ and $\mu_A(y) + \alpha < 1.$

$\min\{\mu_A(mx) + \alpha, 1\} < 1$ and $\min\{\mu_A(my) + \alpha, 1\} < 1 \Rightarrow \mu_A(mx) + \alpha < 1$ and $\mu_A(my) + \alpha < 1.$

As $T_{\alpha^+}(A)$ is a IMFSG of G. Therefore, for any $x, y \in G$, we have

$\mu_{T_{\alpha^+}(A)}(xy^{-1}) \geq \min\{\mu_{T_{\alpha^+}(A)}(x), \mu_{T_{\alpha^+}(A)}(y)\}$ and $\nu_{T_{\alpha^+}(A)}(xy^{-1}) \leq \max\{\nu_{T_{\alpha^+}(A)}(x), \nu_{T_{\alpha^+}(A)}(y)\}$........(2)

$Now,\ \mu_{T_{\alpha^+}(A)}(xy^{-1}) \geq \min\{\mu_{T_{\alpha^+}(A)}(x), \mu_{T_{\alpha^+}(A)}(y)\}$

$\Rightarrow \min\{\mu_A(xy^{-1})+\alpha,1\} \geq \min\left\{\min\{\mu_A(x)+\alpha,1\}, \min\{\mu_A(y)+\alpha,1\}\right\} = \min\{\mu_A(x)+\alpha, \mu_A(y)+\alpha\}$

$\Rightarrow \mu_A(xy^{-1})+\alpha \geq \min\{\mu_A(x), \mu_A(y)\}+\alpha$ *i.e.,* $\mu_A(xy^{-1}) \geq \min\{\mu_A(x), \mu_A(y)\}$.

$Also,\ \nu_{T_{\alpha^+}(A)}(xy^{-1}) \leq \max\{\nu_{T_{\alpha^+}(A)}(x), \nu_{T_{\alpha^+}(A)}(y)\}$

$\Rightarrow \max\{\nu_A(xy^{-1})-\alpha,0\} \leq \max\left\{\max\{\nu_A(x)+\alpha,1\}, \max\{\nu_A(y)+\alpha,1\}\right\} = \max\{\nu_A(x)-\alpha, \nu_A(y)-\alpha\}$

$\Rightarrow\ \nu_A(xy^{-1})-\alpha \leq \max\{\nu_A(x), \nu_A(y)\}-\alpha$, *i.e.,* $\nu_A(xy^{-1}) \leq \max\{\nu_A(x), \nu_A(y)\}$.

Thus A is an IFSG of G.

Also, as $T_\alpha+(A)$ is an IMFSG so, $\mu_{T_{\alpha^+}(A)}(mx) \geq \mu_{T_{\alpha^+}(A)}(x)$ and $\nu_{T_{\alpha^+}(A)}(mx) \leq \nu_{T_{\alpha^+}(A)}(x)$, $\forall\ m \in M$.

$\Rightarrow \min\{\mu_A(mx)+\alpha,1\} \geq \min\{\mu_A(x)+\alpha,1\}$ and $\max\{\nu_A(mx)-\alpha,0\} \leq \max\{\nu_A(x)-\alpha,0\}$

$\Rightarrow \mu_A(mx)+\alpha \geq \mu_A(x)+\alpha$ and $\nu_A(mx)-\alpha \leq \nu_A(x)-\alpha$

$\Rightarrow \mu_A(mx) \geq \mu_A(x)$ and $\nu_A(mx) \leq \nu_A(x)$.

Hence A is an IMFSG of G.

Case III: When $\mu_{T\alpha+(A)}(x) = 1$ and $\mu_{T\alpha+(A)}(y) < 1$.

As in case (i), we get $x \in G_A$, so $\mu_A(x) = \mu_A(e)$ and $\nu_A(x) = \nu_A(e)$.

As $T_{\alpha^+}(A)$ is an IMFSG of G. So, we have

$\mu_{T_{\alpha^+}(A)}(xy^{-1}) \geq \min\{\mu_{T_{\alpha^+}(A)}(x), \mu_{T_{\alpha^+}(A)}(y)\} = \min\{1, \mu_{T_{\alpha^+}(A)}(y)\} = \mu_{T_{\alpha^+}(A)}(y)$.

$\Rightarrow \min\{\mu_A(xy^{-1})+\alpha,1\} \geq \min\{\mu_A(y)+\alpha,1\}$

$\Rightarrow \mu_A(xy^{-1})+\alpha \geq \mu_A(y)+\alpha$

$\Rightarrow \mu_A(xy^{-1}) \geq \mu_A(y) = \min\{\mu_A(e),\ \mu_A(y)\} = \min\{\mu_A(x),\ \mu_A(y)\}$.

Similarly, we can show that $\nu_A(xy^{-1}) \leq \max\{\nu_A(x), \nu_A(y)\}$.

Therefore, A is an IFSG of G.

Moreover, as $\mu_{T\alpha+(A)}(x) = 1 \Rightarrow \mu_{T\alpha+(A)}(mx) = 1$ as in case (i) and hence $\mu_A(mx) \geq \mu_A(x)$.

Similarly, we can show that $\nu_A(mx) \leq \nu_A(x)$. Hence A is an IMFSG of G.

Proposition(3.9) Let A be an IFS of a M-group G such that $T_{\alpha^-}(A)$ be IMFSG of G, for some $\boldsymbol{\alpha} \in [\mathbf{0}, \mathbf{1}]$ with $\boldsymbol{\alpha < \min\{1-p, q\}}$, then A is an IMFSG of G, where p = max{$\mu_A(x)$: $x \in$ G-G_A }, q = min{ $\nu_A(x)$: $x \in$ G-G_A }.

Proof: Similar as in proposition (3.8)

Theorem (3.10) If A be an INMFSG of M-group G if and only if $T_{\alpha^+}(A)$ and $T_{\alpha^-}(A)$ are INFMSG of G for $\alpha \in [0,1]$.

Proof: Firstly, let A be an INMFSG of a M-group G and $\alpha \in [0,1]$ be any real number. Then

$\mu_A(m(xyx^{-1})) \geq \mu_A(my)$ *and* $\nu_A(m(xyx^{-1})) \leq \nu_A(my)\ \forall\ x, y \in G,\ m \in M$.

We have already proved that $T_{\alpha^+}(A)$ and $T_{\alpha^-}(A)$ are IMFSGs of G (See Theorem (3.4))

$\mu_A(m(xyx^{-1})) \geq \mu_A(my) \quad \Rightarrow \quad \min\{\mu_A(m(xyx^{-1}))+\alpha,1\} \geq \min\{\mu_A(my)+\alpha,1\}$

i.e., $\mu_{T_{\alpha^+}(A)}(m(xyx^{-1})) \geq \mu_{T_{\alpha^+}(A)}(my)$.

Similarly, we can show that $\nu_{T_{\alpha^+}(A)}(m(xyx^{-1})) \geq \nu_{T_{\alpha^+}(A)}(my)$.

Hence, $T_{\alpha^+}(A)$ is also an INMFSG of G.

Conversely, let $T_{\alpha^+}(A)$ and $T_{\alpha^-}(A)$ are INMFSG of G for $\alpha \in [0,1]$

Take $\alpha = 0$, we get $T_{0^+}(A) = A = T_{0^-}(A)$.

Hence, A is an INMFSG of G.

4. M- homomorphism of an intuitionistic M-fuzzy subgroups

Lemma (4.1) Let f : X → Y be a mapping and $\alpha\in[0,1]$ be any real number. If A and B be any IFSs on X and Y respectively, then

$$(i) \quad f^{-1}(T_{\alpha+}(\mathrm{B})) = T_{\alpha+}\left(f^{-1}(\mathrm{B})\right);$$

$$(ii) \quad f(T_{\alpha+}(\mathrm{A})) = T_{\alpha+}\left(f(\mathrm{A})\right).$$

Proof. (i) Now, $f^{-1}(T_{\alpha+}(\mathrm{B}))(x) = \left(\mu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(x),\ \nu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(x)\right)$, where

$$\mu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(x) = \mu_{T_{\alpha+}(\mathrm{B})}(f(x)) = \min\{\mu_{\mathrm{B}}(f(x))+\alpha,\ 1\}$$
$$= \min\{\mu_{f^{-1}(\mathrm{B})}(x)+\alpha,\ 1\} = \mu_{T_{\alpha+}\left(f^{-1}(\mathrm{B})\right)}(x) \text{ and}$$
$$\nu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(x) = \nu_{T_{\alpha+}(\mathrm{B})}(f(x)) = \mathrm{Max}\{\nu_{\mathrm{B}}(f(x))-\alpha,\ 0\}$$
$$= \max\{\nu_{f^{-1}(\mathrm{B})}(x)-\alpha,\ 0\} = \nu_{T_{\alpha+}\left(f^{-1}(\mathrm{B})\right)}(x).$$

Thus, $f^{-1}(T_{\alpha+}(\mathrm{B})) = T_{\alpha+}\left(f^{-1}(\mathrm{B})\right)$.

(ii) Now, $f(T_{\alpha+}(\mathrm{A}))(y) = \left(\mu_{f(T_{\alpha+}(\mathrm{A}))}(y),\ \nu_{f(T_{\alpha+}(\mathrm{A}))}(y)\right)$, where

$$\mu_{f(T_{\alpha+}(\mathrm{A}))}(y) = sup\{\mu_{T_{\alpha+}(\mathrm{A})}(x) : f(x) = y\}$$
$$= sup\left\{\min\{\mu_A(x)+\alpha,\ 1\} : f(x) = y\right\}$$
$$= \min\ \left\{\sup\left\{\mu_A(x)+\alpha : f(x) = y\right\},\ 1\right\}$$
$$= \min\ \left\{\sup\left\{\mu_A(x) : f(x) = y\right\}+\alpha,\ 1\right\}$$
$$= \min\left\{\mu_{f(\mathrm{A})}(y)+\alpha,\ 1\right\}$$
$$= \mu_{T_{\alpha+}(f(\mathrm{A}))}(y).$$

Theorem (4.2) Let G_1 and G_2 be two M-groups and let $f: G_1 \rightarrow G_2$ be a M – homomorphism. Let B be an IFS on G_2 such that $T_{\alpha+}(\mathrm{B})$ is an IMFSG of G_2, then $f^{-1}(T_{\alpha+}(\mathrm{B}))$ is an IMFSG of G_1.

Proof: As we have already proved that $f^{-1}(T_{\alpha+}(\mathrm{B})) = T_{\alpha+}\left(f^{-1}(\mathrm{B})\right)$. So, we will prove that $T_{\alpha+}\left(f^{-1}(\mathrm{B})\right)$ is an IMFSG of G_1. Since $T_{\alpha+}\left(f^{-1}(\mathrm{B})\right)(x) = (\mu_{T_{\alpha^+}(f^{-1}(B)}(x), \nu_{T_{\alpha^+}(f^{-1}(B)}(x))$

$\mu_{T_{\alpha^+}(f^{-1}(B)}(x) = \min\{\mu_{f^{-1}(B)}(x)+\alpha,1\}$ and $\nu_{T_{\alpha^+}(f^{-1}(B)}(x) = \max\{\nu_{f^{-1}(B)}(x)-\alpha,0\}$.

Let $x, y \in$ G1 and $m \in$ M be any elements, then

$$\mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(xy^{-1}) = \min\{\mu_{f^{-1}(\mathrm{B})}(xy^{-1}) + \alpha,\ 1\}$$
$$= \min\{\mu_{\mathrm{B}}(f(xy^{-1})) + \alpha,\ 1\} = \min\{\mu_{\mathrm{B}}(f(x)(f(y))^{-1}) + \alpha,\ 1\}$$
$$= \mu_{T_{\alpha+}(\mathrm{B})}(f(x)(f(y))^{-1})$$
$$\geq \min\{\mu_{T_{\alpha+}(\mathrm{B})}(f(x)),\ \mu_{T_{\alpha+}(\mathrm{B})}(f(y))\}$$
$$= \min\{\mu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(x),\ \mu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(y)\}$$
$$= \min\{\mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(x),\ \mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(y)\}.$$

Thus, $\mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(xy^{-1}) \geq \min\{\mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(x),\ \mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(y)\}$.

Similarly, we can show that $\nu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(xy^{-1}) \leq \max\{\nu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(x),\ \nu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(y)\}$.

Further, $\mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(mx) = \min\{\mu_{f^{-1}(\mathrm{B})}(mx) + \alpha,\ 1\}$

$$= \min\{\mu_{\mathrm{B}}(f(mx) + \alpha,\ 1\}$$
$$= \min\{\mu_{\mathrm{B}}(mf(x) + \alpha,\ 1\}$$
$$= \mu_{T_{\alpha+}(\mathrm{B})}(mf(x))$$
$$\geq \mu_{T_{\alpha+}(\mathrm{B})}(f(x))$$
$$= \mu_{f^{-1}(T_{\alpha+}(\mathrm{B}))}(x)$$
$$= \mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(x)\ [\text{Using lemma (4.1)}(i)]$$

Thus, $\mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(mx) \geq \mu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(x)$.

Similarly, we can show that $\nu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(mx) \leq \nu_{T_{\alpha+}(f^{-1}(\mathrm{B}))}(x)$.

Thus, $T_{\alpha+}(f^{-1}(\mathrm{B}))$ and hence $f^{-1}(T_{\alpha+}(\mathrm{B}))$ is an IFMSG of G_1.

Theorem (4.3) Let G_1 and G_2 be two M-groups and let $f : G_1 \rightarrow G_2$ be a M-homomorphism. Let A be an IFS on G_1 such that $T_{\alpha+}(\mathrm{A})$ is an IMFSG of G_1, then $f(T_{\alpha+}(\mathrm{A}))$ is an IMFSG of G_2.

Proof: As we have already proved that $f(T_{\alpha+}(\mathrm{A})) = T_{\alpha+}(f(\mathrm{A}))$.

Now, we show that $T_{\alpha+}(f(\mathrm{A}))$ is an IMFS(G_2). Let $x^*, y^* \in G_2$, and $m \in$ M be any elements, then ∃'s $x, y \in G_1$ such that $f(x) = x^*$, $f(y) = y^*$. Now $T_{\alpha+}(f(\mathrm{A})) = (\mu_{T_{\alpha+}(f(\mathrm{A}))}(xy^{-1}), \nu_{T_{\alpha+}(f(\mathrm{A}))}(xy^{-1}))$

$$\mu_{T_{\alpha+}(f(\mathrm{A}))}(xy^{-1}) = \min\{\mu_{f(\mathrm{A})}(xy^{-1}) + \alpha,\ 1\}$$
$$= \min\left\{\sup\{\mu_{\mathrm{A}}(xy^{-1}):\ f(x) = x^*, f(y) = y^*\} + \alpha,\ 1\right\}$$
$$= \min\left\{\sup\{\mu_{\mathrm{A}}(xy^{-1}) + \alpha :\ f(x) = x^*, f(y) = y^*\},\ 1\right\}$$
$$= \sup\left\{\mu_{T_{\alpha+}(\mathrm{A})}(xy^{-1}):\ f(x) = x^*, f(y) = y^*\right\}$$
$$\geq \sup\left\{\min\{\mu_{T_{\alpha+}(\mathrm{A})}(x), \mu_{T_{\alpha+}(\mathrm{A})}(y)\}:\ f(x) = x^*, f(y) = y^*\right\}$$
$$= \min\left\{\sup\{\mu_{T_{\alpha+}(\mathrm{A})}(x), \mu_{T_{\alpha+}(\mathrm{A})}(y)\}:\ f(x) = x^*, f(y) = y^*\right\}$$
$$= \min\left\{\sup\{\mu_{T_{\alpha+}(\mathrm{A})}(x):\ f(x) = x^*\}, \sup\{\mu_{T_{\alpha+}(\mathrm{A})}(y):\ f(y) = y^*\}\right\}.$$

Thus, $\mu_{T_{\alpha+}(f(\mathrm{A}))}(xy^{-1}) \geq \min\left\{\mu_{T_{\alpha+}(f(\mathrm{A}))}(x), \mu_{T_{\alpha+}(f(\mathrm{A}))}(y)\right\}.$

Similarly, we can show that $\nu_{T_{\alpha+}(f(\mathrm{A}))}(xy^{-1}) \leq \max\left\{\nu_{T_{\alpha+}(f(\mathrm{A}))}(x), \nu_{T_{\alpha+}(f(\mathrm{A}))}(y)\right\}.$

$$\mu_{T_{\alpha+}(f(\mathrm{A}))}(mx) = \min\{\mu_{f(\mathrm{A})}(mx^*) + \alpha,\ 1\}$$
$$= \min\left\{\sup\{\mu_{\mathrm{A}}(mx):\ f(mx) = mx^*\} + \alpha,\ 1\right\}$$
$$= \min\left\{\sup\{\mu_{\mathrm{A}}(mx) + \alpha :\ f(mx) = mx^*\},\ 1\right\}$$
$$= \sup\left\{\min\{\mu_{\mathrm{A}}(mx) + \alpha, 1\}:\ f(mx) = mx^*\right\}$$
$$= \sup\left\{\mu_{T_{\alpha+}(\mathrm{A})}(mx):\ f(mx) = mx^*\right\}$$
$$\geq \sup\left\{\mu_{T_{\alpha+}(\mathrm{A})}(x):\ f(x) = x^*\right\}$$
$$= \mu_{f(T_{\alpha+}(\mathrm{A}))}(x^*) = \mu_{T_{\alpha+}(f(\mathrm{A}))}(x^*).$$

Thus, $\mu_{T_{\alpha+}(f(\mathrm{A}))}(mx^*) \geq \mu_{T_{\alpha+}(f(\mathrm{A}))}(x^*).$

Similarly, we can show that $\nu_{T_{\alpha+}(f(\mathrm{A}))}(mx^*) \leq \nu_{T_{\alpha+}(f(\mathrm{A}))}(x^*).$

Thus, $T_{\alpha+}\left(f(\mathrm{A})\right)$ and hence $f(T_{\alpha+}(\mathrm{A}))$ is an IMFSG on G_2.

5. Conclusions

In this paper, we have investigated the effect on the IMFSG of a M-group G under the two translation operators and concluded that it remains invariant under the two translation operators. We have also observed the effect of translation of IMFSG of M-group G under M- homomorphism.

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