Differential Acoustic Resonance Spectroscopy Analysis of Fluids in Porous Media

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Abstract

The Acoustic Resonance Spectroscopy is a well established method and is used by the National Bureau of Standards for measuring the velocity and quality factor of the fluids. It was proposed to adapt the method to measure the velocity and quality factor of sound in rocks in the sonic frequency range by Harris (1996). The Differential Acoustic Resonance Spectroscopy (DARS) is an experimental method based on the change in the acoustic resonance frequency of a fluid-filled tube due to the involvement of an external object in a tube. The resonance frequency of the fluid-filled tube depends on the ratio of the sound velocity in the surrounding medium and the length of the cylinder. In the present paper there are analysis and comparative study with the help of tables and graphs about variations of frequencies by Aluminum and Lucite samples. The compressibility is also analyzed and compared by ultrasonic method and DARS methods for taking five different samples.

Keywords: DARS Theory, Fluids, Porous media.

1. Introduction

The Differential Acoustical Resonance Spectroscopy (DARS) uses the low frequency resonance i.e., (~1000 Hz) of a fluid column to measure the bulk acoustic properties of a small sample of rock and other porous material. The differential acoustic resonance spectroscopy (DARS) was developed to analyze the changes in the resonant frequencies of a cavity perturbed by the introduction of a centimeter-sized sample. The resonant frequency shifts, measured at different resonance modes and between empty and sample-loaded cavities, were used to infer the acoustic properties of the loaded samples in the low frequency range i.e., (0.5–2 kHz). Now, different principal experimental methods [1] exist to determine the acoustic properties as a function of frequency for the different samples ranging in compressibility from high i.e., elastic substances to low i.e., porous rock. The two common methods that measure the acoustic properties of samples are pulse transmission and resonant bar methods [2]. So, this experiment-based analytical technique fills an experimental gap between the low frequency stress strain method [3], [4]; i.e., quasi-static to several kHz and the ultrasonic transmission technique i.e., hundreds of kHz to MHz’s. The stress-strain measurements, that record the forced deformation exerted on rock samples, have been performed for nearly a century to analyzing the macroscopic mechanical properties. In these measurements, cyclic loading is used to subject a sample to deformations that are slow as compared to natural mechanical resonances of the sample. So, the sample is close to mechanical equilibrium at all times during the test. The elastic modulus of the sample can be inferred from measurements of both the applied stresses as well as the induced strains. Hence, the stress-strain method is limited by the lack of advancement, due to which it is difficult to monitor extremely sensitive deformation.

By considering of an effective perturbation model against the DARS system, the studies present a Green's function based theoretical derivation of a corrected DARS perturbation formula under the suitable boundary conditions. The numerical and experimental results show that the amended DARS perturbation is suitable to reflect the DARS operation mechanism suitably and more precisely than the previous method. Also the inversion was performed by fitting the resonance frequencies, measured at various locations inside the DARS resonance cavity, in a least square sense to determine the acoustic properties of a test sample. The inversion implementation at different resonance modes makes it to perform the direct dispersion analysis on reservoir rocks at various low frequency bands. Then the results had shown that the DARS laboratory device, in conjunction with the corrected perturbation formula and the proposed
inversion strategy are important for determining the acoustic properties of centimeter sized rock samples in a low frequency range.

Hence, the low frequency forced deformation system, in conjunction with a built-in pulse transmission assembly, can cover a much broadly frequency band i.e., from about 5 Hz to 800 kHz. Most of the laboratory analyses using this system have been taken for a variety of rock samples like: shales, siltstones, tight sandstones and carbonates for investigating their acoustic properties. Also, there are still big challenges in applying this system:

(1) There are very weak signals must be processed;
(2) The strain gauges are so sensitive to the surface preparation of samples;
(3) The preparation of samples is the time consuming process.

It is also important that the test samples, in the aforementioned laboratory measurement techniques, must have a relatively big size and a regular shape. For the pulse transmission measurements, the typical diameter and length of a cylindrical test sample are 2.5 cm x 5 cm, respectively. For the stress-strain method, the typical sample size is about 3.8 cm x 5 cm in diameter and length.

2. DARS Perturbation theory

The theory is described by a fluid filled cylindrical cavity as shown in the figure-1 with both ends open will vibrate with a fundamental resonance such that the fluid column length is one half the wavelength of the sound wave. In the case of ideal cavity both end of the column must be a node for the pressure of the fluid as both the ends are open. In the case of fundamental mode there is single velocity node at the center. The frequency of the fundamental is given by the formula

\[ \omega_0 = \frac{\pi C_0}{L_0} \]  

Where \( C_0 \) is the acoustic velocity of the fluid that fills the cavity and \( L_0 \) is the length of cavity. The compressibility of the fluid and sample are defined as:

\[ k_0 = \frac{1}{\rho_0 C_0^2} \]  

\[ k_s = \frac{1}{\rho_s \left( u_p^2 - \frac{4u_s^2}{3} \right)} \]  

Where the meaning of the terms used are

\( u_p \) = p-wave velocity of sample

\( u_s \) = s-wave velocity of sample

\( \rho_0 \) = density of fluid

\( \rho_s \) = density of the sample

By introducing the sample perturbs of the resonance properties of the cavity, angular resonant frequency changes from \( \omega_0 \) to \( \omega_s \). The frequency perturbation is given as:

\[ \omega_s^2 - \omega_0^2 = \left[ \frac{p^2 (k_s - k_0) \omega_0^2}{k_0} + \omega_0^2 \frac{p (\rho_s - \rho_0)(u \rho_0 C_0)^2}{\rho_s} \right] \frac{V_s}{V_c} \]  

Where the terms used in the above equation are as follows:

\( \omega_0 \) = frequency of the cavity without sample

\( \omega_s \) = frequency of the cavity without sample

\( p^2 \) = average acoustic pressure

\( u^2 \) = particle vibration velocity of the fluid inside the cavity
$V_s = $ volume of the sample

$V_0 = $ volume of the cavity

$k_0 =$compressibility of the fluid

$k_s =$ compressibility of the sample

$\alpha =$ coefficient concerned with the structure of cavity

The compressibility contrast, $\delta_k$ and the density contrast, $\rho_k$ are as defined

$$\delta_k = \frac{(k_s - k_0)}{k_0} \quad (5)$$

$$\rho_k = \frac{(\rho_s - \rho_0)}{\rho_s} \quad (6)$$

Now we know that the most of the earth materials are harder and denser than the fluid inside the cavity so that the terms defined by eq. (5) and eq. (6) have opposite sign. So that, the compressibility and density contrasts between the tested sample and the fluid contributes oppositely to the frequency shift. This shows that at some particular locations inside the cavity, the frequency perturbation generated by the compressibility and the density contrast will cancel each other. The frequency shift is also linearly dependent with the sample volume $V_s$.

3. DARS Experimental Analysis

The experimental analysis was performed by Chuntang Xu in 2007 [3] under the supervision of Harris, J.M. The DARS apparatus used in his experiment was the cylindrical cavity resonator, immersed in a tank filled with fluid silicone oil. A schematic diagram of the DARS apparatus is shown in Figure-1. He was used a pair of piezoceramic discs to excite the resonator.

![Figure-1](The schematic representation of apparatus with computer that controlled the sample positioning and swept frequency data acquisition. The resonance frequency is measured with the help of sample at different positions in the cavity [2], [3]).

The apparatus shows that the disks are embedded in the wall at the longitudinal midpoint of the cavity, where the acoustic pressure is at its maximum for the fundamental mode, thus the disks can efficiently excite the first longitudinal mode. A high sensitivity hydrophone is used on the inner surface of the cavity wall and at the midpoint of the cavity separated by 90°from the two sources, detects acoustic pressure. The sample is moved vertically along the axis of the cavity to test the various stages of pressure and fluid flow. A computer controlled motor machine is used to give correct and suitable position of the sample. It also includes a computer that controlled
the position of the sample and swept frequency data acquisition and is used to scan the frequency to track and record a resonance curve. A special scan uses the frequency steps of 0.1 Hz from about 1035 – 1135 Hz to cover the first mode. The system is completely controlled by a computer. The dimension of the cavity is 15 inches in length and 3.1 inches of internal diameter. The fluid being used in the current system is Dow 200 silicone oil whose nominal acoustic velocity and density at 20°C are 986 m/sec and 918 kg/m³ respectively. The viscosity of the fluid is 5 cs.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>( L_S ) [cm]</th>
<th>( D_S ) [cm]</th>
<th>( V_S ) [cm³]</th>
<th>( \omega_{Alu}/2\pi ) [Hz]</th>
<th>( \omega_{Luc}/2\pi ) [Hz]</th>
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</thead>
<tbody>
<tr>
<td>1.</td>
<td>5.08</td>
<td>2.54</td>
<td>25.74</td>
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<td>1094</td>
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<td>31.15</td>
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<td>1096</td>
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<td>37.07</td>
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<td>3.30</td>
<td>43.50</td>
<td>1105</td>
<td>1101</td>
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<td>50.45</td>
<td>1108</td>
<td>1103</td>
</tr>
<tr>
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<td>5.08</td>
<td>3.81</td>
<td>57.92</td>
<td>1113</td>
<td>1107</td>
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<td>7.</td>
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<td>28.96</td>
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<td>1095</td>
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<tr>
<td>8.</td>
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<td>4.57</td>
<td>3.81</td>
<td>52.12</td>
<td>1109</td>
<td>1104</td>
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Table-1

(The measured values of the dimensions and resonant frequency of two reference solids are shown. The samples lengths \( L_S \) and diameters \( D_S \) are measured. The volume-dependent resonant frequencies at \( x = 0 \) for Aluminum and Lucite are \( \omega_{Alu}/2\pi \) and \( \omega_{Luc}/2\pi \) respectively. The resonant frequency of the oil-filled cylinder \( \omega_0/2\pi \) is 1083 Hz. The measurements are acquired by Xu in 2007 [3]).
Table-2
(Experimental results of the reference solids. The resonant frequencies $\omega_0$ and $\omega_S$ are the oil-filled and sample-loaded values at the pressure antinode respectively. The volume of the resonating cylinder $V_0$ is 1855 cm$^3$. The ultrasonic compressibilities $k_{US}$ are calculated from table. The DARS compressibility $k_{DARS}$ are calculated. Errors are attributed to the uncertainty in $\lambda_3$ [3],[4]).

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Solid</th>
<th>$\omega_0/2\pi$[Hz]</th>
<th>$\omega_S/2\pi$[Hz]</th>
<th>$V_0$[cm$^3$]</th>
<th>$k_{DARS}$[GPa$^{-1}$]</th>
<th>$k_{US}$[GPa$^{-1}$]</th>
<th>$E%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Alum</td>
<td>1083.0</td>
<td>1092.4</td>
<td>19.27 ± 0.01</td>
<td>0.006</td>
<td>0.013</td>
<td>-16.667</td>
</tr>
<tr>
<td>2.</td>
<td>Teflon</td>
<td>1083.8</td>
<td>1090.2</td>
<td>19.20 ± 0.01</td>
<td>0.367</td>
<td>0.345</td>
<td>6.377</td>
</tr>
<tr>
<td>3.</td>
<td>PVC</td>
<td>1082.7</td>
<td>1090.5</td>
<td>19.21 ± 0.01</td>
<td>0.190</td>
<td>0.190</td>
<td>0</td>
</tr>
<tr>
<td>4.</td>
<td>Lucite</td>
<td>1082.9</td>
<td>1090.8</td>
<td>19.17 ± 0.02</td>
<td>0.179</td>
<td>0.178</td>
<td>-0.562</td>
</tr>
<tr>
<td>5.</td>
<td>Delrin</td>
<td>1083.3</td>
<td>1091.2</td>
<td>19.16 ± 0.01</td>
<td>0.172</td>
<td>0.160</td>
<td>-7.5</td>
</tr>
</tbody>
</table>

Figure-2
(Variation of resonant frequency $\omega_{Alu}/2\pi$ and $\omega_{Luc}/2\pi$ with respect to diameter $D_s$ of the sample by taking length $L_s$ =5.08, $\omega_0/2\pi =1083$ Hz for Aluminum and Lucite)
Figure-3
(Variation of resonant frequency $\omega_{Alu}/2\pi$ and $\omega_{Luc}/2\pi$ with respect to length $V_S$ of the sample by taking diameter $L_S = 5.08$, $\omega_0/2\pi = 1083$ Hz for Aluminum and Lucite)

Figure-4
(Variation of resonant frequency $\omega_{Alu}/2\pi$ and $\omega_{Luc}/2\pi$ with respect to diameter length $L_S = 5.08$ of the sample by taking $D_S = 3.81$, $\omega_0/2\pi = 1083$ Hz for Aluminum and Lucite)
Figure-5
(Variation of resonant frequency $\omega_{Alu}/2\pi$ and $\omega_{Luc}/2\pi$ with respect to diameter $V_S$ of the sample by taking length $D_S=3.81$, $\omega_0/2\pi=1083$ Hz for Aluminum and Lucite)

Figure-6
(Surface representation of resonant frequencies $\omega_0/2\pi$ and $\omega_s/2\pi$, $V_0=1855$ cm$^3$ for the five metals samples from table-2)
Figure-7
(Variations of compressibilities $k_{US}$ and $k_{DARS}$, $V_0 = 1855 \text{ cm}^3$ for the five metals samples calculated from table-2)

Figure-8
(Variation of percent errors of compressibilities $k_{US}$ and $k_{DARS}$, $V_0 = 1855 \text{ cm}^3$ for the five metals samples calculated from table-2)
4. Discussion and Results

The table-1 shows the measured values of the dimensions and resonant frequency of two reference solids are shown. The samples lengths \( L_5 \) and diameters \( D_3 \) are measured. The volume-dependent resonant frequencies at \( x = 0 \) for Aluminum and Lucite are \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) respectively. The resonant frequency of the oil-filled cylinder \( \omega_0/2\pi \) is 1083 Hz. The table-2 shows the resonant frequencies \( \omega_0 \) and \( \omega_S \) are the oil-filled and sample-loaded values at the pressure antinode respectively. The volume of the resonating cylinder \( V_0 \) is 1855 cm\(^3\). The ultrasonic compressibilities \( k_{US} \) are calculated from table. The DARS compressibility \( k_{DARS} \) are calculated. Errors are attributed to the uncertainty in \( \lambda_k \). With the help of these tables the figures are constructed. The figure-2 shows the variation of resonant frequency \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) with respect to diameter \( D_3 \) of the sample by taking length \( L_5 = 5.08 \), \( \omega_0/2\pi = 1083 \) Hz for the two samples Aluminum and Lucite. The figure-3 shows the variation of resonant frequency \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) with respect to length \( V_5 \) of the sample by taking diameter \( L_5 = 5.08 \), \( \omega_0/2\pi = 1083 \) Hz for samples. The figure-4 shows the variation of resonant frequency \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) with respect to diameter length \( L_5 = 5.08 \) of the sample by taking \( D_3 = 3.81 \), \( \omega_0/2\pi = 1083 \) Hz for samples. The figure-5 shows the variation of resonant frequency \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) with respect to diameter \( V_5 \) of the sample by taking length \( D_3 = 3.81 \), \( \omega_0/2\pi = 1083 \) Hz for the samples. The figure-6 shows the surface representation of resonant frequencies \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \), \( V_0 = 1855 \) cm\(^3\) for the five metals samples from table-2. The figure-7 shows the variations of compressibilities \( k_{US} \) and \( k_{DARS} \), \( V_0 = 1855 \) cm\(^3\) for the five metals samples calculated from table-2. The figure-8 shows the variation of percent errors of compressibilities \( k_{US} \) and \( k_{DARS} \), \( V_0 = 1855 \) cm\(^3\) for the five metals samples calculated from table-2.

5. Conclusion

The resonant frequencies increase with respect to diameter \( D_3 \) and length \( V_5 \) of the samples Aluminum and Lucite. The variation of resonant frequency \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) with respect to diameter length \( L_5 = 5.08 \) of the sample by taking \( D_3 = 3.81 \), \( \omega_0/2\pi = 1083 \) Hz and the variation of resonant frequency \( \omega_{\text{Alu}}/2\pi \) and \( \omega_{\text{Luc}}/2\pi \) with respect to diameter \( V_5 \) of the sample by taking length \( D_3 = 3.81 \), \( \omega_0/2\pi = 1083 \) Hz, first decreases then increases. The variations of compressibilities \( k_{US} \) and \( k_{DARS} \), \( V_0 = 1855 \) cm\(^3\) for the five metals, is low in Aluminum, high in Teflon and normal in others. The variation of percent errors of compressibilities \( k_{US} \) and \( k_{DARS} \), \( V_0 = 1855 \) cm\(^3\) for the five metals samples, negative in Aluminum, positive in Teflon and negligible in others, i.e., PVC, Lucite and Derlin.

References:


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