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# Relationship Between Filters

**Priyanka Singh[1], Amitabh Banerjee[2], Purushotam Jha[3]**

[1]Department of Engineering ITM University, Raipur India
[2]Department of Mathematics Govt. D.B.Girls College, Raipur India
[3]Department of Mathematics Govt. J.Y.C.G.College, Raipur India

[1]Email: singhpriyanka2107@gmail.com
[2]Email: amitabh61@yahoo.com
[3]Email: purushjah@gmail.com

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**[1]First Author / Corresponding Author; Paper ID: A16203**

**Abstract**

*In this paper we investigate the relationship between filters such as Positive Filter (PF), Associative Filter(AF), Fantastic Filter (FF), and Fuzzy Implicative Filter(FIF) in a lattice implication algebra. Moreover we investigate a filter F of L is* $(i)\ FF \Leftrightarrow F,\ (ii) PF \Leftrightarrow F\ ,(iii) AF\ \Leftrightarrow F,\ (iv) PF \Leftrightarrow FF,\ (v) AF \Leftrightarrow PF,\ (vi) AF \Leftrightarrow FF.$

**Keywords:** Lattice implication algebra, Filter, Implicative filter, Positive fillter, Fantastic filter and associative fillter.

## 1. Intorduction

According to Bosman Fuzzy filter is an innovative and cost effective fillter suitable for removing suspended solids from waste water. Because the fuzzy filter has a modular construction and contains a compressible porous filter medium, it is a highly versatile, compact system that can be operated at extremely high filltration rates. These unique characteristic allow the filter to be employed for many diverse applications one of them show in figure 4.1.

### 1.1 Application of Fuzzy Filter

The fuzzy filter can be used in a large number of situations such as.

1. Post filtration of effluent from waste purification.
2. Filtration of pre settled waste water.
3. Pre- Filtration for other systems including membrane filtration.
4. Industrial water treatment.
5. Cooling water treatment.
6. Filtration for disinfection.
7. Water recycling systems.
8. Pulp and paper process water treatment

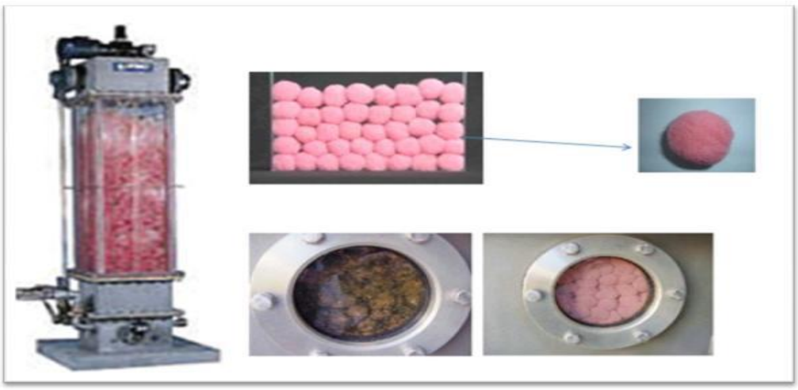


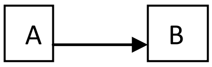
Figure 1: Fuzzy Filter

Xu and Qin [2] introduced the notion of filters and implicative filters in a lattice implication algebra and investigated their properties, Y.B Jun [3] gave an equivalent condition of a filter and provided some equivalent conditions for a filter to be an implication filter. In [4] Liu and Xu. defined the notion of prime filters and studied decomposition theorem of lattice implication algebras. Y. B. Jun [6] introduced the concepts of a positive implicative filter in a lattice H-implication algebra and proved that

(i) Every positive implicative filter is an implicative filter

(ii) Every associative filter is a filter.

They also provided equivalent condition for both a positive implicative filter and an associative filter. In [7] Jun et. al. defined an LI- ideal of a lattice implication algebra and showed that every LI- ideal is a lattice ideal. They gave an example that a lattice ideal may not be an LI-ideal, and showed that every lattice ideal is an LI- ideal in a lattice implication algebra. They discussed the relationship between filter and LI-ideals. In[12] Song and Jun introduce the n-fold positive implication filter in a lattice implication algebra, and gave a relation among a filter, an n-fold implicative filter and an n-fold positive implicative filter. They represent the relationship between filter through graph, see figure 1.

The symbol 

mean A should be B. Moreover they gave a characterization of n-fold positive implicative filters, and establish the extension property for n-fold positive implicative filters.

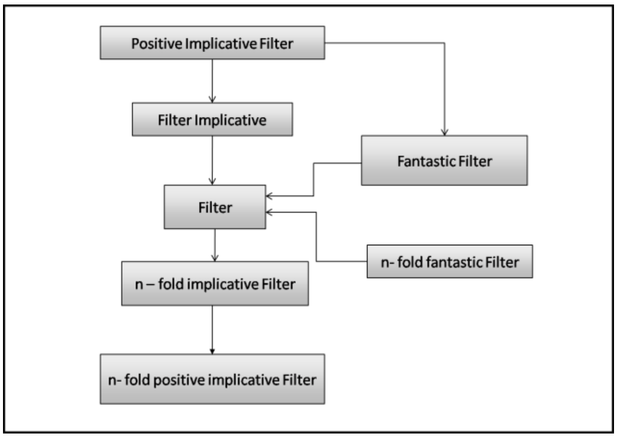
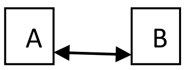



Figure 2: FIF [11]

In this paper we introduce the relationship between various types of filter through graph, see figure 2 and figure 3 which shows the relationship between various types of filters, the symbol



means A should be B and B should be A. This Work inspired by the Xu[1], Quin[2], Jan[3] et.al.

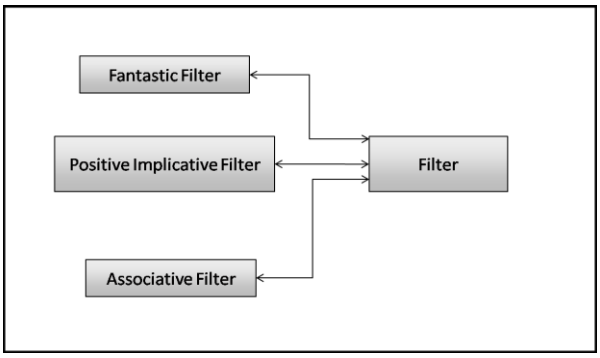



Figure 3: Relationship between PF , AF and FF to Filter

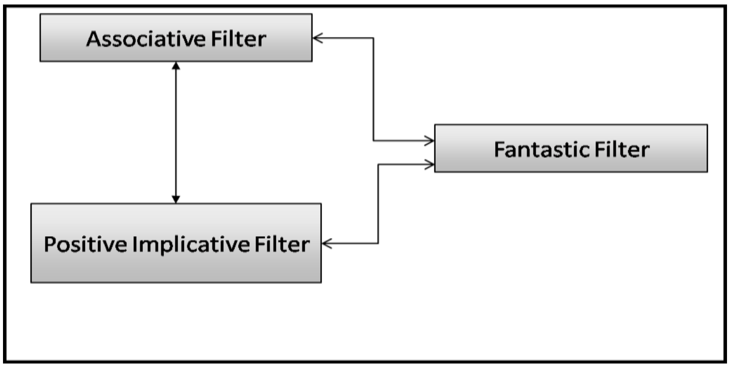



Figure 4: Relationship between PF , AF and FF

## 2. Preliminaries

We display basic definitions and properties of Fuzzy implicative filter that will be used in this paper. For more details of Fuzzy implicative filter, we refer the reader to [1], [2], [3], [6], and [9].

**Definition 1** [1]: By a lattice implication algebra we mean a bounded lattice $(L, \vee, \vee, 0, 1)$ with order reversing involution $'$ and a binary operation $\rightarrow$ satisfying the following axioms.

1. $\boldsymbol{x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)}$
2. $\boldsymbol{x \rightarrow x = 1}$
3. $\boldsymbol{x \rightarrow y = y \rightarrow x}$
4. $\boldsymbol{x \rightarrow y = y \rightarrow x = 1 \Rightarrow x = y}$
5. $\boldsymbol{(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x}$
6. $\boldsymbol{(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)}$
7. $\boldsymbol{(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)}$

Note that condition (6) and (7) are equivalent to the conditions.

8. $\boldsymbol{x \rightarrow (y \wedge z) = (x \rightarrow y) \wedge (x \rightarrow z)}$
9. $\boldsymbol{x \rightarrow (y \vee z) = (x \rightarrow y) \vee (x \rightarrow z)}$ respectively.

**Theorem 1 [1]:** In a lattice implication algebra $L$ hold the following condition for all $x, y, z \in L$

1. $0 \rightarrow x = 1$ ; $1 \rightarrow x = x$ $and$ $x \rightarrow 1 = 1$
2. $x \leq y$ $implies$ $(y \rightarrow z) \rightarrow (x \rightarrow z)$ $and$ $(z \rightarrow x) \leq (z \rightarrow y)$
3. $(x \rightarrow y) \rightarrow ((y \rightarrow z) \rightarrow (x \rightarrow z)) = 1$
4. $x \rightarrow ((x \rightarrow y) \rightarrow y) = 1$

   In [2] Y. Xu and K .Y Qin defined the notions of a filter and an implication filter in a lattice implication algebra.

**Definition 2 [2]**: Let $(L, \vee, \wedge, 0, 1)$ be a lattice implication algebra. A subset F of L is called a filter of L if it satisfies for all $x, y \in L$ .

(F1) $1 \in F$

(F2) $x \in F$ $and$ $xy \in F$ $implies$ $y \in F$

**Lemma 2** [3]: Every filter F of L has the following property $x \leq y$ and $x \in F$ $imply$ $y \in F$ for all $x, y \in F$.

**Definition 3 [6]:** Let x be a fixed element of L. A subset F of L is called an associative filter of L with respect to x if it satisfies (F1) and

(F3) $x(yz) \in F$ $and$ $xy \in F$ $imply$ $z \in F$

For all $x, y, z \in F$. An associative filter of L with respect to all $x \neq 0$ is called an associative filter of L.

**Theorem 3[6]:** Let F be a filter of L. Then F is an associative filter if and only if it satisfies the following property.

$$x(yz) \in F \text{ implies } (xy)z \in F \text{ for all } x, y, z \in L$$

**Example 1 [9]:** Let $L = \{ 0, a, b, c, 1\}$. Define the partially ordered relation on L $0 < a < b < c < 1$ and define $x \wedge y = \min\{x, y\}$, $x \vee y = \max\{x, y\}$ for all $x, y \in L$ the parameter $'$ and " $\rightarrow$ " is defined in table 1.1. Then $(L, \vee, \wedge, ', \rightarrow)$ is a lattice implication algebra.

| x | $x'$ | | $\rightarrow$ | 0 | a | b | c | 1 |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | 0 | 1 | 1 | 1 | 1 | 1 |
| a | c | | a | c | 1 | 1 | 1 | 1 |
| b | b | | b | b | c | 1 | 1 | 1 |
| c | a | | c | a | b | c | 1 | 1 |
| 1 | 0 | | 1 | 0 | a | b | c | 1 |

Table 1.1

In the sequel the binary operation will denoted by $\rightarrow$ by juxtaposition. We can define a partial ordering $\leq$ on a lattice implication algebra L by $x \leq y$ if and only if $xy = 1$.

In a lattice implication algebra L, the following hold;

1. $0x = 1,\ 1x = x\ and\ x1 = 1,$
2. $x' = x0$
3. $xy \leq (yz)(xz)$
4. $x \vee y = (xy)y$
5. $\left((yx)y'\right)' = x \wedge y = \left((xy)x'\right)'$
6. $x \leq y\ implies\ yz \leq\ xz\ and\ zx \leq zy$
7. $x \leq (xy)y$

**Definition 4 [2]:** A subset F of L is called and implicative filter of L if it satisfy (F1) and

$(F4)\ x(yz) \in F\ and\ xy \in F\ imply\ xz \in F\ for\ all\ x, y \in L.$

**Proposition 4 [2]:** Every filter F of L has the following properties

$x \leq y\ and\ x \in F\ imply\ y \in F.$

**Definition 5 [6]:** A subset F of L is called a positive implicative filter of L if it satisfy (F1) and

$(F5)\ x\left((yz)y\right) \in F\ and\ x \in F\ imply\ y \in F\ for\ all\ x, y, z \in L.$

**Proposition 5[6]** Let F be a filter of L. Then F is a positive implicative filter of L if and only if

$(F6)\ \left((xy)x\right) \in F\ and\ x \in F\ x, y, z \in L.$

**Proposition 6[6]:** Let F be a non-empty subset of L. Then F is a filter of L if and only if it satisfies for all

$xy \in F$ and $z \in L$ ;

$(F7)\ x \leq yz\ implies\ z \in F,$

**Definition 6 [9]:** A non-empty subset of F of L is called a fantastic filter of L if it satisfies

$(F1)\ and$

$(F8)\ z(yx) \in F\ and\ z \in F\ imply\ \left((xy)y\right) \rightarrow x \in F\ for\ all\ x, y, z \in L$

**Theorem 7 [9]**: A filter F of L is fantastic if and only if it

$(F9)\ yx \in F\ implies\ \left((xy)y\right)x \in F, for\ all\ x, y \in L$

**3. Relationship between Filters**

In this section we describe relationship between various filters.

**Corollary 8 [9]:** Every fantastic filter of L is a filter.

**Theorem 9:** A filter F of L is Fantastic filter if and only if it is fillter.

**Proof:** Let F be a filter of a lattice implication algebra L. Let $x, y \in F$.

Then $x \in F \text{ and } xy \in F$ this implies that $y \in F$. It follows from (F8) that

$$xy = x(1y) \in F \text{ and } x \in F$$

This implies that $y = ((y,1),1)y \in F \text{ for all } z \in F$, so that F is a fantastic filter.

Conversely, let F be a fantastic filter of L and let $zx \in F \text{ and } z \in F$.

Then $z(1x) \in F \text{ and } z \in F$.

It follows form (F8) $x = ((x1)1)x \in F$ so that F is a filter.

**Corollary 10 [6]:** Every positive implicative filter F of L is a filter.

**Theorem 11:** A filter F of L is positive implicative filter if and only if it is a filter.

**Proof:** Let F be a positive implicative filter of L and let $x(yz)y \in F \text{ and } x \in F$.

then from (F5) $y \in F$ this implies that

$x(y1)y \in F \text{ i.e. } (xy) \in F \text{ and } x \in F \text{ for all } x, y, z \in L$

It follows that $y = 1(y1)y \in F$. so that F is a filter.

Conversely, suppose F is a filter of L then $xz \in F \text{ and } x \in F$ it follows from (F2) that

$$z \in F \text{ i.e. } xz = x(1z)1 \in F \text{ and } 1 \in F.$$

This implies that

$$xy = (xy)1 \in F \text{ if } z = 1 \text{ then } (xy)z \in F.$$

Hence F is a positive implicative filter of L.

**Corollary 12 [6]:** Every associative filter is a filter.

**Theorem 13:** A filter F of L is associative filter if and only if it is a filter.

**Proof:** Let F be a associative filter of L and $x(yz) \in F \text{ then } (xy)z \in F$

It follows that if $z = 1 \text{ then } x(y1) = xy \in F \text{ and } x \in F$.

This implies that $y \in F \text{ for all } x, y, z \in F$. Hence F is a filter.

Conversely, suppose that F is a filter then we have to show that F is associative filter of L

This implies that $xy = (xy)1 \in F \text{ if } z = 1 \text{ then } (xy)z \in F$. It follows that $x(yz) \in F$.

Hence F is a associative filter of L.

**Corollary 14** [9]: Every positive implicative fillter of L is fantastic.

**Theorem 15:** A filter F of L is positive implicative filter if and only if it is fantastic.

**Proof:** Let F be a positive implicative fiter of L. Then F is a filter of L by (corollary 14).

Let $x, y \in L$ be such that $yx \in F$ .It is sufficient to show that $((xy)y)x \in F$.

Since $x \leq ((xy)y)x$ we get

$\left(((xy)y)x\right)y \leq xy$ put $p = ((xy)y)x$ we obtain.

$(py)p = \left(((xy)y)x\right)y\left(((xy)y)x\right)$

$\geq (xy)\left(((xy)y)x\right)$ $\qquad [\therefore x \leq ((xy)y)x]$

$\geq ((xy)y)((xy)x)$

$\geq yx$

It follows (Proposition-5) that $(py)p \in F$so from (corollary 14) $a \in F$ $i.e.$ i.e. $\left(((xy)y)x\right) \in F$.

Hence F is a fantastic filter.

Conversely, suppose F is a fantastic filter then we know that by (corollary-8) F is filter on L.

Now we have to show that F is positive implicative filter, for this it is sufficient to prove that $y \in F$

This implies from (F8) that

$x \leq z(yx)$

$\Rightarrow xy \leq z(yx)y$

$\Rightarrow xy \leq x(yz)y$

$\Rightarrow xy \leq z(yx)y$

$\Rightarrow x \in F$ $therefore$ $x(yz)y \in F$

This implies that
$y \in F$ $for\ all$ $x, y, z \in L$

Hence F is a positive implicative filter.

**Theorem 16:** A filter F of L is associative filter if and only if it is a positive filter.

**Proof:** Let F be a associative filter then we know that by (Thm. 11) it is filter. Now we have to

Prove that F is a positive implicative filter for this we have to show that

$x(yz)y \in F$ $and$ $x \in F$ this implies that $y \in F$ $for\ all$ $x, y, z \in F$.

Let F be a associative filter then from (F3) we know that $x(yz) \in F$ this implies that $x(yz)y \in F$.

where $x \in F$ this implies that $y \in F$. Hence F is a positive implicative filter.

Conversely it is trivially prove that every positive implicative filter is associative filter.

**Theorem 17:** A filter F of L is associative filter if and only if it is fantastic.

**Proof**: Let F be a associative filter then from (F3) $(xy)z \in F$ this implies that $x(yz) \in F$. Let $(xy)z \in F$ then by definition of associative filter $z(yx) \in F$ $and$ $z \in F$ $imply\ that$ $z = 1$ $then$ $(xy) = ((xy)y)x \in F$ $for\ all$ $x, y, z \in L$. Hence F is a fantastic filter.

Conversely, suppose that F is a fantastic filter of L then from (F8) we know that $z(yx) \in F$ $and$ $z \in F$ .

Hence it is trivial that F is a associative filter.

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