Propagation of Characteristic Wave Front through a Two Phase Mixture of Gas and Dust Particles

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Abstract

The aim of the present paper is to discuss weak-non-linear waves through a two-phase mixture of gas and dust particles, when particle-volume-fraction appears as an additional variable. The solutions based on an asymptotic procedure are obtained under the approximation that the characteristic length of the signal is much shorter that the characteristic length of the medium.

Keywords: Propagation, Two phase, Gas particles.

1. Introduction

In the recent technological advancements in different branches of engineering and science, the compressible flows of a dusty gas have been encountered. When a gas carries a lot of solid particles, the two-phase relaxation phenomenon significantly affects the flow field. It has been shown by Lick (1967) and Parker (1969) that a disturbance having a time-scale comparable to the attenuation time may produce partially and fully dispersed shock in relaxing gases. However in some physical situations a signal with characteristic time much shorter than the attenuation time may suffer continual profile distortion leading to a shock formation. In some disturbances this catastrophic effect is delayed by dispersion but in non-dispersive systems it arises due to non-linearity provided that the propagation distances are sufficiently large (1975, 1971,1972). General discussion on small amplitude waves with considerations of non-linear effects are typified in works of Light-hill (1949), Witham (1952) and Lin (1955).

It is known that a gas flow with an appreciable amount of small solid particles may exhibit significantly relaxation effects, as a result of the inability of the particles to follow the rapid changes in velocity and temperature of the gas. Such effects are predominant in case of wave propagation in dusty gases. The problem of acoustical damming in dusty gases has been dealt by Epstein with linearized analysis (1955, 1941). Bhutani and Chandran (1977) have discussed the weak waves in dusty gases using the characteristic coordinate system; they have analyzed the decay of the plane, cylindrical and spherical weak-waves in gas particle system.

The aim of the paper is to discuss weak-non-linear waves through a two-phase mixture of gas and dust particles, when particle-volume fraction appears as an additional variable. The solutions based on an asymptotic simple wave procedure, are obtained under the approximation that the characteristic length of the signal is much shorter that the characteristic length of the medium.

2. Basic equation and boundary conditions

The mathematical analysis of two-phase flows is considerably more difficult than that of pure gas flows, and one of usual simplifying assumptions is that the volume occupied by the particle can be neglected. At high gas densities (high pressure) or at high particle mass fraction, the particle-volume fraction may become sufficiently large so that it formulated under the following assumptions-

1. The gas obeys the perfect gas law and specific heats are constant.
2. The particles are spherical, of uniform size and are uniformly distributed initially. The specific heat is constant and temperature is uniform within each particle.
3. Particles do not interact on each other and their motion is negligible.
4. The viscosity and heat conductivity of gas are neglected except for the interaction with solid particles.
5. The particles do not contribute to the pressure.
6. No external forces (such as gravity) or heat exchange affects the mixture and no mass transfer takes place between the gas and particle.

Equations governing the motion of gas particle mixture, under above assumptions are given by (1969).

$$u,_x + uu,_x + \frac{RT}{\rho(1-\in)}\rho,_x + \frac{R}{(1-\in)}T,_x + \frac{\in \rho_p}{\rho(1-\in)^2}\frac{(u-v)}{\tau_v} = 0, \tag{2.1}$$

$$\rho,_t + \rho u,_x + u\rho,_x + \frac{\in \rho}{(1-\in)}v,_x + \frac{\rho}{(1-\in)}(v-u)\in,_x = 0, \tag{2.2}$$

$$T,_t + (\gamma-1)Tu,_x + \frac{T}{\rho}\frac{\in(\gamma-1)(v-u)}{(1-\in)}\rho,_x + \frac{\left[u(1-\gamma\in)+\in v(\gamma-1)\right]}{(1-\in)}T,_x$$
$$+\frac{\in T(\gamma-1)}{(1-\in)}v,_x + \frac{T(\gamma-1)(v-u)}{(1-\in)}\in,_x + \frac{\in \rho_p}{(1-\in)\rho C_v}\left[\frac{C_m(T-T_p)}{\tau_T} - \frac{(u-v)^2}{(1-\in)\tau_v}\right] = 0, \tag{2.3}$$

$$v,_t + vv,_x + \frac{(v-u)}{(1-\in)\tau_v} = 0, \tag{2.4}$$

$$\in,_t + \in v,_x + v\in,_x = 0, \tag{2.5}$$

$$T_{p,t} + vT_{p,x} \frac{\left(T_p - T\right)}{\tau_T} = 0, \qquad (2.6)$$

where $p, \rho, T, u, C_p, C_{v,}\gamma$ be the pressure, density, temperature, velocity, specific heats and specific heat ratio of the gas and $\in, T_p, v, C_m$ be the volume-fraction, temperature, velocity and specific heat of the dust particles respectively. τ_v is relaxation time for particle velocity and τ_T is relaxation time for heat transfer. A Comma followed by an index denotes the partial differentiation with respect to index.

Let $a_0 = \left[\gamma\left(p_0 / \rho_0\right)\right]^{1/2}$, be the speed of sound in undisturbed gas, where subscript '0' is used to indicate a constant reference value. If the time 't' is non dimensionalized by $'\tau_v'$ the distance 'x' by $'a_0\tau_v'$, the speed of sound $'a'$ by $'a_0'$ the gas velocity 'u' and particle velocity 'v' by $'a_0'$, gas temperature 'T' and particle temperature $'T_p'$ by $'T_0'$ equation (2.1) to (2.6) can be written in the following matrix form :

$$U_{,t} + AU_{,x} + B = 0, \qquad (2.7)$$

where

$$U = \begin{bmatrix} u \\ \rho \\ T \\ v \\ \in \\ T_p \end{bmatrix},$$

$$A = \begin{pmatrix} u & \dfrac{T}{\rho\gamma(1-\in)} & \dfrac{1}{\gamma(1-\in)} & 0 & 0 & 0 \\ \rho & u & 0 & \dfrac{\in\rho}{(1-\in)} & \dfrac{\rho(v-u)}{(1-\in)} & 0 \\ T(\gamma-1) & \dfrac{T\in(\gamma-1)(v-u)}{\rho(1-\in)} & \dfrac{\left[u\left(1-\in_\gamma\right)+\in v\left((\gamma-1)\right)\right]}{(1-\in)} & \dfrac{\in T(\gamma-1)}{(1-\in)} & \dfrac{T(v-u)(\gamma-1)}{(1-\in)} & 0 \\ 0 & 0 & 0 & \mathrm{v} & 0 & 0 \\ 0 & 0 & 0 & \in & \mathrm{v} & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathrm{v} \end{pmatrix}$$

$$B = \begin{bmatrix} \dfrac{\in \rho_p (u-v)}{(1-\in)^2 \tau_v} \\ 0 \\ \dfrac{\in p_\rho \gamma}{(1-\in)\rho\tau_v} & \left[\eta(T-T_p) - \dfrac{(\gamma-1)(u-v)^2}{(1-\in)}\right] \\ \dfrac{(v-u)}{(1-\in)} \\ 0 \\ \Gamma(T_p - T) \end{bmatrix}$$

$\Gamma = \dfrac{\tau_v}{\tau_T}$ and $\eta = \Gamma \dfrac{C_m}{C_p}$.The relationship between 'τ_T' and 'τ_v' is given by

$\tau_T = \dfrac{3}{2} P_r \dfrac{C_m}{C_v} \tau_v$, where P_r is Pr adtl number.

The six families of characteristics of the equation (2.7) are given by:

$\dfrac{dx}{dt} = u$, representing the gas-particle trajectory, and

$\dfrac{dx}{dt} = v$ (repeated three times), represent the solid particle trajectory, and triple degeneracy corresponds to case for non-equilibrium flows and is a consequence of neglecting the partial pressure of the particle.

The remaining two characteristics representing waves propagating in $\pm x$ direction are

$$\frac{dx}{dt} = \underset{\sim}{u} \pm \underset{\sim}{a}. \tag{2.8}$$

where

$$\underset{\sim}{u} = \left[K + 1(1-\in)u\right]/2(1-\in).$$

$$\underset{\sim}{a} = \left[\left\{K + (1-\in)u\right\}^2 - 4(1-\in)(uK - T)\right]^{1/2} ./2(1-\in)$$

and $$K = \left[u(1-\in\gamma) + \in v(\gamma-1)\right].$$

In the particular case when $\in \to 0$, equation (2.8) reduces to

$$\frac{dx}{dt} = u \pm a, \text{ where } a = T^{1/2}. \tag{2.9}$$

3. Characteristic Front

In studying the wave phenomenon governed by hyperbolic equations, it is usually more natural and convenient to use the characteristics of governing system as the reference coordinate system. Let us introduce the characteristic variables 'α' and 'ψ' such that;

$$\alpha,_t + u\alpha,_x = 0, \tag{3.1}$$

$$\psi,_t + (\underset{\sim}{u} + \underset{\sim}{a})\psi,_x = 0, \tag{3.2}$$

The leading characteristic front can be represented by $\alpha = 0$ and if a gas particle crosses this front at time 't' it's path will be represented by $\psi = t$. Keeping in view of the properties of α and ψ, it is obvious that the function $x(\alpha,\psi)$ and $t(\alpha,\psi)$ satisfy the following partial differential equations;

$$x,_\alpha = ut,_{\alpha,}\ x,_\psi = (\underset{\sim}{u} + \underset{\sim}{a})t,_\psi$$

The transformation from space time (x,t) to the plane of characteristic parameters (α,ψ) will be one to one if and only if Jacobian

$$J = (x,_\alpha t,_\psi - x,_\psi t,_\alpha) = (u - \underset{\sim}{u} - \underset{\sim}{a})\mathrm{t},_\alpha t,_\psi$$

or

$$J = \left(\frac{\in(\gamma-1)(u-v)}{2(1-\in)} - \underset{\sim}{a}\right) t,_\alpha t,_\psi ,$$

does not vanish or does not become infinity anywhere.

Since $t,_\psi \neq 0$, from physical considerations a breakdown of solution in terms of characteristic parameters will arise if and only if $t,_a = 0$,

In terms of characteristic coordinates equations given by (2.7) reduces to the following form.

$$\left[\frac{\in(\gamma-1)(u-v)}{2(1-\in)}-\underset{\sim}{a}\right]u,_{\alpha}t,_{\psi}+\frac{1}{\gamma(1-\in)}\left(T,_{\alpha}t,_{\psi}-T,_{\psi}t,_{\alpha}\right)$$
$$+\frac{T}{\rho\gamma(1-\in)}\left(\rho,_{\alpha}t,_{\psi}-p,_{\psi}t,_{\alpha}\right)$$
$$+\left[\frac{\in(\gamma-1)(u-v)}{2(1-\in)}-a\right]\frac{\in p_{p}}{(1-\in)^{2}p}(u-v)t,_{\alpha}t,_{\psi}=0, \qquad (3.5)$$

$$\left[\frac{\in(\gamma-1)(u-v)}{2(1-\in)}-\underset{\sim}{a}\right]p,_{\alpha}t,_{\psi}+p\left[u,_{\alpha}t,_{\psi}-u,_{\psi}t,_{\alpha}\right]+\frac{\rho(v-u)}{(1-\in)}\left[\in,_{\alpha}t,_{\psi}-\in,_{\alpha}t,_{\alpha}\right]$$
$$+\frac{\in p}{(1-\in)}\left[v,_{\alpha}t,_{\psi}-v,_{\psi}t,_{\alpha}\right]=0, \qquad (3.6)$$

$$\frac{\in(\gamma-1)(u-v)}{(1-\in)}T,_{\psi}t,_{\alpha}+\left[\frac{\in(\gamma-1)(u-v)}{2(1-\in)}-\underset{\sim}{a}\right]T,_{\alpha}t,_{\psi}$$
$$+\frac{T(\gamma-1)(u-v)}{(1-\in)}\left[\in,_{a}t,_{\psi}-\in,_{\psi}t,_{\alpha}\right]$$
$$+T(\gamma-1)\left[u,_{\alpha}t,_{\psi}-u,_{\psi}t,_{\alpha}\right]+\frac{\in T(v-u)(\gamma-1)}{(1-\in)\rho}\left[\rho,_{\rho}t,_{\psi}-\rho,_{\psi}t,_{\alpha}\right]$$
$$+\frac{\in T(\gamma-1)}{(1-\in)}\left[v,_{\alpha}t,_{\psi}-v,_{\psi}t,_{\alpha}\right]+\frac{\in\rho_{\rho}\gamma}{(1-\in)\rho}\left[\frac{\in(\gamma-1)(u-v)}{(1-\in)}-\underset{\sim}{a}\right]$$
$$\left[\eta\left(T-T_{p}\right)-\frac{(\gamma-1)(u-v)^{2}}{(1-\in)}\right]t,_{\alpha}t,_{\psi}=0, \qquad (3.7)$$

$$(u-v)v,_{\psi}t,_{\alpha}+\left[\frac{(u-v)\{\in(\gamma+1)-2\}}{2(1-\in)}-\underset{\sim}{a}\right]v,_{\alpha}t,_{\psi}$$
$$+\left[\frac{\in(\gamma-1)(u-v)}{2(1-\in)}-\underset{\sim}{a}\right]\frac{(u-v)}{(1-\in)}t,_{\alpha}t,_{\psi}=0, \qquad (3.8)$$

$$(u-v)v,_{\psi} t,_{\alpha} + \left[\frac{(u-v)\{\in(\gamma+1)-2\}}{2(1-\in)} - \underset{\sim}{a} \right] \in,_{\alpha} t,_{\psi}$$
$$+ \in \left(v,_{\alpha} t,_{\psi} - v,_{\psi} t,_{\alpha} \right) = 0, \tag{3.9}$$

$$(u-v)T_{p,\psi} t,_{\alpha} + \left[\frac{(u-v)\{\in(\gamma+1)-2\}}{2(1-\in)} - \underset{\sim}{a} \right] T_{p,\alpha} t,_{\psi}$$
$$+\Gamma \left[\frac{\in(u-v)(\gamma-1)}{2(1-\in)} - \underset{\sim}{a} \right] (T_p - T) t,_{\alpha} t,_{\psi} = 0. \tag{3.10}$$

If we consider the case in which the wave front is an outgoing characteristic propagating into a uniform region, boundary conditions are given by

$u = v = 0, \rho = 1, T = 1 = T_p$ and $\in = \in_{0'}$ at $\alpha = 0$

and $u = F'(\alpha), x = F(\alpha), t = \alpha,$ at $\psi = 0.$

In order to solve the problem we consider $F(t) = \beta f(t),$ where $\beta(\ll 1),$ Characterizes the amplitude of disturbance and $f(t) \sim 0(1).$ We can now assume a solution of system (3.5) to (3.10) of the form

$$Q(\alpha,\psi) = Q^{(0)}(\alpha,\psi) + \beta Q^{(1)}(\alpha,\psi) + 0(\beta^2) \tag{3.11}$$

where 'Q' is any one of the independent variables u, ρ, T, x etc. If $\beta = 0$ the piston is at rest and hence,

$$u^{(0)} = v^{(0)} = 0, \quad \rho^{(0)} = 1, \quad T^{(0)} = T_p^{(0)} = 1,$$
$$\in^{(0)} = \in_0, \quad \underset{\sim}{a}^{(0)} = \frac{1}{\sqrt{(1-\in_0)}}, \quad \underset{\sim}{u}^{(0)} = 0. \tag{3.12}$$

Substituting the expression (3.11) in equations (3.5 to 3.10) and collecting the terms of order $\beta^{(0)}$. we have:

$$x_{,\alpha}^{(0)} = 0, \quad x_{,\psi}^{(0)} = t_{,\psi}^{(0)}. \tag{3.13}$$

Thus boundary conditions are:

$$t^{(0)} = \alpha, \quad x^{(0)} = 0, \quad \text{at } \psi = 0,$$
$$t^{(0)} = \psi \quad \text{at } \alpha = 0. \tag{3.14}$$

Equation (3.13) on integration subject to boundary conditions (3.14) yield,

$$x^{(0)} = \psi, \quad t^{(0)} = \alpha + \psi. \tag{3.15}$$

The complete solution to $0\left(\beta^{(0)}\right)$ be given by (3.12) and (3.15) and to this order the characteristics in the physical plane are

$$x=\psi,$$
$$t-x=\alpha. \qquad (3.16)$$

When the expansion (3.11) is inserted in the governing equations (3.5), to (3.10) and (3.3), the collection of terms of order β gives the following set of equations for the first order solutions,

$$\left[\rho^{(1)}+T^{(1)}-\gamma u\sqrt{(1-\epsilon_o)}\right]_{,\alpha}-\left[\rho^{(1)}+T^{(1)}\right]_{,\psi}-\frac{\gamma\epsilon_0\rho_p}{(1-\epsilon_0)^{3/2}}\left(u^{(1)}-v^{(1)}\right)=0, \qquad (3.17)$$

$$\left[u^{(1)}-\rho^{(1)}\right]_{,\alpha}-u,_{\psi}^{(1)}+\frac{\epsilon_0}{\sqrt{(1-\epsilon_0)}}\left(v,_{\alpha}^{(1)}-v,_{\psi}^{(1)}\right)=0 \qquad (3.18)$$

$$\left[\frac{(\gamma-1)u^{(1)}}{\sqrt{(1-\epsilon_0)}}-T^{(1)}\right]_{,\alpha}-\frac{(\gamma-1)u_{,\psi}^{(1)}}{\sqrt{(1-\epsilon_0}}+\frac{\epsilon_0(\gamma-1)}{\sqrt{(1-\epsilon_0)}}\left(v,_{\alpha}^{(1)}-v,_{\psi}^{(1)}\right)$$
$$-\frac{\eta\epsilon_0\rho_p\gamma\left[T^{(1)}-T,_{p}^{(1)}\right]}{(1-\epsilon_0)}=0, \qquad (3.19)$$

$$v,_{\alpha}^{(1)}+\frac{(u^{(1)}-v^{(1)})}{(1-\epsilon_0)}=0, \qquad (3.20)$$

$$\epsilon,_{\alpha}^{(1)}-\epsilon_0\sqrt{(1-\epsilon_0)}\left(v,_{\alpha}^{(1)}-v,_{\psi}^{(1)}\right)=0, \qquad (3.21)$$

$$T_{p,\alpha}^{(1)}+\Gamma\left(T_p^{(1)}-T^{(1)}\right)=0, \qquad (3.22)$$

and

$$x_{,\alpha}^{(1)}=u^{(1)},\ x_{,\psi}^{(1)}=\underset{\sim}{u}^{(1)}+\underset{\sim}{a}^{(1)}+t_{,\psi}^{(1)}. \qquad (3.23)$$

The boundary conditions for the first order solutions can be obtained as

$$t^{(1)}=0,\ x^{(1)}=f(\alpha),\ u^{(1)}=f'(\alpha)\ \text{at}\ \psi=0, \qquad (3.24)$$

$$u^{(1)} = v^{(1)} = 0.\ T^{(1)} = T_p^{(1)} = 0,\ \ \in^{(1)} = 0,\ t^{(1)} = 0 \text{ at } \alpha = 0. \tag{3.25}$$

Since the Parandtl number of many gases is close to 2/3 and C_m / C_p, is of order unity for many gas-particle combinations hence for such cases temperature and velocity relaxation times are approximately equal, i.e., $\tau_v = \tau_T$. In such approximate case $\eta = 1$ and $\Gamma = 1$. The Laplace transform of $Q(\alpha, \psi)$ with respect to α is dented by $\hat{Q}(\xi, \psi)$, where

$$\hat{Q}(\xi, \psi) = \int_0^\infty Q(\alpha, \psi) e^{-\alpha\xi} d\alpha,$$

equation (3.17) to (3.23) assume the following form

$$\frac{d}{d\psi}\left[\hat{\rho}^{(1)} + \hat{T}^{(1)}\right] - \gamma\xi\sqrt{(1-\in_o)}[-\hat{u}^{(1)} + \frac{1}{\gamma\sqrt{(1-\in_0)}}][\hat{\rho}^{(1)} + \hat{T}^{(1)}] + \frac{\gamma \in_0 \rho_p}{\sqrt{(1-\in_0)}}\left[\hat{u}^{(1)} - \hat{v}^{(1)}\right] = 0, \tag{3.26}$$

$$\frac{d\hat{u}^{(1)}}{d\psi} + \frac{\in_0}{\sqrt{(1-\in_0)}}\frac{d\hat{v}^{(1)}}{d\psi} + \xi[\hat{\rho}^{(1)} - \hat{u}^{(1)} - \frac{\in_0}{\sqrt{(1-\in_0)}}\hat{v}^{(1)}] = 0, \tag{3.27}$$

$$\frac{d\hat{u}^{(1)}}{d\psi} + \in_0 \frac{d\hat{v}^{(1)}}{d\psi}\xi[\hat{u}^{(1)} - \frac{\sqrt{(1-\in_0)}}{\gamma-1}\hat{T}^{(1)}] - \in_0 \xi\hat{v}^{(1)}, + \frac{\gamma}{\gamma-1}\frac{\in_0 \rho_p}{\sqrt{(1-\in_0)}}(\hat{T}^{(1)} - \hat{T}_p^{(1)}) = 0, \tag{3.28}$$

$$(1-\in_0)\xi\hat{v}^{(1)} - [\hat{u}^{(1)} - \hat{v}^{(1)}] = 0, \tag{3.29}$$

$$\frac{d\hat{v}^{(1)}}{d\psi} + \frac{\xi}{\in_0\sqrt{(1-\in_0)}}[\hat{\in}^{(1)} - \in_0\sqrt{(1-\in_0)}\ \hat{v}^{(1)}] = 0, \tag{3.30}$$

$$\xi\hat{T}_p^{(1)} + \left(\hat{T}_p^{(1)} - \hat{T}^{(1)}\right) = 0, \tag{3.31}$$

and

$$\hat{x}^{(1)} = \frac{\hat{u}^{(1)}}{\xi} \ , \ \frac{d}{d\psi}[\hat{x}^{(1)}] = \underset{\sim}{\hat{u}}^{(1)} + \underset{\sim}{\hat{a}}^{(1)} + \frac{d}{d\psi}\hat{t}^{(1)} \tag{3.32}$$

respectively, similarly the transformed boundary conditions are:

$$\hat{t}^{(1)} = 0, \ \hat{x}^{(1)} = \hat{f}(\xi), \ \hat{u}^{(1)} = \xi \hat{f}(\xi), \text{ at } \psi = 0, \tag{3.33}$$

$$\hat{u}^{(1)} = 0, \ \hat{v}^{(1)} = \hat{\in}^{(1)} = \hat{T}^{(1)} = \hat{T}_p^{(1)} = \hat{t}^{(1)} = 0, \text{ at } \alpha = 0. \tag{3.34}$$

In view of equations (3.27) to (3.31) equation (3.26) yields following second order linear differential equation;

$$\frac{d^2\hat{u}^{(1)}}{d\psi^2} - 2\xi \frac{d\hat{u}^{(1)}}{d\psi} - \xi^2 \frac{\gamma A \hat{u}^{(1)}}{(1+\xi)} = 0, \tag{3.35}$$

where 'A' is constant given by

$$A = -\frac{(1+\xi)}{B + (C/D)} \left[\frac{(C/D) + B - \gamma}{\gamma} - \frac{\in_0 \rho_p}{\sqrt{(1-\in_0)\{1+\xi(1-\in_0)\}}} \right]$$

and

$$B = \left[1 + \frac{\in_0}{\sqrt{(1-\in_0\{1+\xi(1-\in_0)\}}} \right],$$

$$C = \frac{(\gamma-1)}{\sqrt{(1-\in_0)}} \left[1 + \frac{\in_0}{\{1+\xi(1-\in_0)\}} \right],$$

$$D = \left[1 + \frac{\gamma \in_0 \rho_p}{(1-\in_0)(1+\xi)} \right].$$

Solution of equation (3.35) satisfying the condition that $\hat{u}^{(1)}$ is bounded as $\psi \to \infty$ is

$$\hat{u}^{(1)} = K_c \exp(-\Lambda\psi) \tag{3.36}$$

where $\Lambda = -\xi + \xi\left(1 + \frac{A\gamma}{1+\xi}\right)^{1/2}$ and constant K_c is determined by the boundary conditions;

$$\hat{u}^{(1)} = \xi \hat{f}(\xi), \text{ at } \psi = 0,$$

which gives $\hat{u}^{(1)} = \xi \hat{f}(\xi) \exp(-\Lambda\psi)$. (3.37)

Thus equations (3.26) to (3.32), give

$$\hat{u}^{(1)}(\xi,\psi) = \xi f(\xi) \exp(-\Lambda\psi), \tag{3.38}$$

$$\hat{v}^{(1)}(\xi,\psi) = \xi \hat{f} \exp(-\Lambda\psi)/\left[1+\xi(1-\in_0)\right], \tag{3.39}$$

$$\hat{\rho}^{(1)}(\xi,\psi) = \xi\hat{f}(\xi) \exp(-\Lambda\psi)[1+\frac{\in_0}{\sqrt{(1-\in)}}\left\{1+\xi(1-\in_0)\right\} + \frac{\Lambda}{\xi}(1+\frac{\in_0}{\sqrt{(1-\in_0)}})], \tag{3.40}$$

$$\overset{\Lambda}{T}^{(1)}(\xi,\psi) = \frac{(\gamma-1)(1+\xi)\sqrt{(1-\in_0)}\left[1+\xi(1-\in_0)+\in_0\right]}{\left[1+\xi(1-\in_0)\right]\left[(1-\in_0)(1+\xi)+\gamma\in_0\rho_p\right]}(\Lambda+\xi)\hat{f}(\xi) \exp(-\Lambda\psi), \tag{3.41}$$

$$\hat{T}_p^{(1)}(\xi,\psi) = \frac{(\gamma-1)\sqrt{(1-\in_0)}[1+\xi(1-\in_0)+\in_0](\Lambda+\xi)}{\left[1+\xi(1-\in_0)\right]\left[(1-\in_0)(1+\xi)+\gamma\in_0\rho_p\right]}\hat{f}(\xi) \exp(-\Lambda\psi), \tag{3.42}$$

$$\hat{\in}^{(1)}(\xi,\psi) = \frac{\in_0\sqrt{(1+\in_0)}}{\left[1+\xi(1-\in_0)\right]}(\Lambda+\xi)\hat{f}(\xi) \exp(-\Lambda\psi), \tag{3.43}$$

$$\hat{x}^{(1)} = \hat{f}(\xi) \exp(-\Lambda\psi), \tag{3.44}$$

and

$$\hat{t}^{(1)}(\alpha,\psi)=\frac{\hat{f}(\xi)\{\exp(-\Lambda\psi)-1\}}{2(1-\epsilon_0)\{1+\xi(1-\epsilon_0)\}}[\xi\{2(1+\xi(1-\epsilon_0))-\epsilon_0(2-(\gamma+1)(1-\epsilon_0))\}$$

$$+2(1-\epsilon_0)\Lambda\{1+\xi(1-\epsilon_0)\}+\frac{\Lambda+\xi}{(1-\epsilon_0)}\{\frac{(1-\epsilon_0)^{3/2}(\gamma-1)(1+\xi)\{1+\xi(1-\epsilon_0)+\epsilon_0\}}{(1-\epsilon_0)(1+\xi)+\gamma\epsilon_0\rho_p} \qquad (3.45)$$

$$+\frac{(1+\epsilon_0)\epsilon_0}{\sqrt{(1-\epsilon_0)}}\}]$$

respectively.

On inversion, it follows that

$$u(a,\psi)=\frac{\beta}{2\pi i}\int_{S-i\infty}^{S+i\infty}\xi f(\xi)\exp(\alpha\xi\text{-}\Lambda\psi)d\xi+0(\beta^2), \qquad (3.46)$$

$$x(\alpha,\psi)=\psi+\frac{\beta}{2\pi i}\int_{S-i\infty}^{S+i\infty}\hat{f}(\xi)\exp(\alpha\xi\text{-}\Lambda\psi)d\xi+0(\beta^2), \qquad (3.47)$$

$$t(\alpha,\psi)=\psi+\alpha+\frac{\beta}{4\pi i}\int_{S-i\infty}^{S+i\infty}\frac{\hat{f}(\xi)\exp(\alpha\xi)[\exp(-\Lambda\psi)-1]}{(1-\epsilon_0)[1-\xi(1-\epsilon_0)]}$$

$$[\xi\{2(1+\xi(1-\epsilon_0))-\epsilon_0(2-(\gamma+1)(1-\epsilon_0))\}$$

$$+2\Lambda(1-\epsilon_0)\{1+\xi(1-\epsilon_0)\}+\frac{\Lambda+\xi}{(1-\epsilon_0)}\{\frac{(1-\epsilon_0)^{3/2}(\gamma-1)(1+\xi)(1-\epsilon_0)+\epsilon}{(1-\epsilon_0)\{(1+\xi)+\gamma\epsilon_0\rho_p}$$

$$+\frac{\epsilon_0(1+\epsilon_0)}{\sqrt{(1-\epsilon_0)}}\}]d\xi+0(\beta^2),$$

(3.48)

where S is a positive number satisfying the condition that all the singularities of these integral are to the left of line $\xi=S$ in the complex plane.

4. Conclusions

Equations (3.46) to (3.48) are solutions up to the first order of non-linear equations (3.3) and (3.5) to (3.10) along with prescribed boundary conditions (2.10). The trajectory of the outgoing waves and particle paths in the $(x\text{-}t)$ plane is described by equations (3.46) and (3.48), which evidently show the convergence of characteristics. The formation of a shock wave is characterized by $t,_\alpha=0$.

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