Accessing the Performance Measure in an Organization through Additive Weibull Distribution

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Abstract

Stochastic model is developed to attain the expected time to reach the performance measure level, in the background with assumptions that the times between judgments epochs are independent and identically distributed (i.i.d) random variable. Attempt made using Additive Weibull Distribution to check if this particular distribution fits the shock model approach.

Keywords: Expected Time, Variance, Recruitment, Additive Weibull distribution, Manpower Planning.

1. Introduction

Manpower planning has attracted the attention of management scientists in the past few years as a fruitful area both for theoretical analysis and practical application. Manpower planning consists of placing right number of people, right kind of people at the right place, right time, doing the right things for which they are suited for the achievement of goals of the organization. In recent times managers and administrators have realized that the effective function of their manpower has a great impact on the overall performance of the firm. Poor manpower planning can result in "incessant employment and terminations. Apart from this, more and more firms are developing strategic manpower plans and are integrating those plans with other strategic planning activities.

Manpower planning is useful in business and industrial sectors. Since the human behavior is random in nature, stochastic model provide the basic frame work for efficient analysis and design of a manpower systems. In [[3], [4], [6] and [7] the authors have obtained the mean time to recruitment in a threshold organization using shock model approach. The Weibull distribution has been used in many different fields with many applications, see for example Lai, Xie [8] and Murthy [5]. The hazard function of the Weibull distribution can only be increasing, decreasing or constant. Thus it cannot be used to model lifetime data with a bathtub shaped hazard function, such as human mortality and machine life cycles. For many years, researchers have been developing various extensions and modified forms of the Weibull distribution, with different number of parameters.

Mathematical model is obtained for the expected time of breakdown point to reach the threshold level through Additive Weibull distribution. One can see for more detail in Esary *et al.,* (1973), Sathiyamoorthi (1980), Pandiyan *et al.,* (2010) Kannadasan *et al.,* (2013) and Kannadasan *et al.,* (2015) about the expected time to cross the threshold level of the organization.

2. Assumptions of the Models

In order to develop the mathematical model, four basic assumptions are made concerning the distributions of the involved random variables.

- The organization takes decisions on revising policies at random times, where the inter-decision times, which are called epochs, are i.i.d. random variables.
- At every epoch a random number of persons quit the organization.
- The organization is exposed to a break down situation when the number of exits of personnel exceeds a "threshold level".
- The threshold level is represented by a random variable following an Additive Weibull distribution.

3. Notations

X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i = 1,2,3, \dots k$ and $X_i's$ are i.i.d and $X_i = X$ for all i.

Y : A continuous random variable denoting the threshold level having the Additive Weibull Distribution.

$g(.)$: The probability density functions (p.d.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ i.e., p.d.f. of $\sum_{i=1}^{k} X_i$

$g*(.)$: Laplace transform of $g(.)$; $g_k^*(.)$: Laplace transform of $g_k(.)$

$h(.)$: The probability density functions of random threshold level which has the Additive Weibull Distribution and $H(.)$is the corresponding probability generating functions.

U : A continuous random variable denoting the inter-arrival times between decision epochs.

$f(.)$: p.d.f. of random variable U with corresponding Probability generating function.

$V_k(t)$: $F_k(t) - F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in (0, t]

$S(.)$: The survivor function i.e., $P[T > t]$;$1 - S(t) = L(t)$

4. Model Description

A random variable X is said to have an Additive Weibull distribution if its cumulative distribution function (cdf) is

$$F(x) = 1 - e^{-\alpha x^{\theta} - \gamma x^{\beta}} \qquad x \geq 0$$

θ and β are the shape parameters and α and γ are scale parameters.

The probability density function (pdf) of the Additive Weibull distribution is

$$f(x) = \left(\alpha\theta x^{\theta-1} + \gamma\beta x^{\beta-1}\right) e^{-\alpha x^{\theta} - \gamma x^{\beta}}$$

The hazard rate function is given by

$$\bar{H}(x) = 1 - F(x)$$

$$= e^{-\alpha x^{\theta} - \gamma x^{\beta}}$$

Now, assuming that the threshold Y follows Additive Weibull distribution with parameter λ, it can be proved that

$$P(X_i < Y) = \int_0^{\infty} g_k(x)\bar{H}(x)\, dx$$

$$= \int_0^\infty g_k(x)\ e^{-\alpha x^\theta - \gamma x^\beta}$$

On simplification it can be shown that

$$= \left[g^* e^{-x(\alpha+\gamma)}\right]^k \tag{1}$$

The survival function $S(t)$ which is the probability that an individual's survives for a time t

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t)\, P(X_i < Y)$$

It is also known from renewal theory that

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ [g^*(\alpha + \gamma)]^k \tag{2}$$

Using convolution theorem for Laplace transforms, $F_k(t) = 1$ and on simplification, it can be shown that $L(T) = 1 - S(t)$

Taking Laplace Transform of $L(T)$, We get

$$= 1 - \left\{\sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\ [g^*(\alpha + \gamma)]^k\right\}$$

On simplification it can be shown that

$$= \left\{[1 - g^*(\alpha + \gamma)] \sum_{k=1}^{\infty} F_k(t)\ [1 - g^*(\alpha + \gamma)]\right\}^{k-1} \tag{3}$$

By taking Laplace-Stieltjes transform, it can be shown that

$$l^*(s) = \frac{[1 - g^*(\alpha + \gamma)]\, f^*(s)}{[1 - g^*(\alpha + \gamma) f^*(s)]} \tag{4}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter.

Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the equation (4) we get,

$$= \frac{[1 - g^*(\alpha + \gamma)] \left(\frac{c}{c+s}\right)}{\left[1 - g^*(\alpha + \gamma)\left(\frac{c}{c+s}\right)\right]}$$

On simplification it can be shown that

$$= \frac{[1 - g^*(\alpha + \gamma)]\, c}{[c + s - g^*(\alpha + \gamma)c]} \qquad (5)$$

We know that

$$E(T) = \frac{-d}{ds}\, l^*(s) \;\; given\; s = 0$$

$$= \frac{(-1)[1 - g^*(\alpha + \gamma)]c}{[c + s - g^*(\alpha + \gamma)c]^2}$$

$$= \frac{1}{c[1 - g^*(\alpha + \gamma)]} \qquad (6)$$

$$g^*(.) \sim exp(\mu)\;, \qquad g^*(\lambda) \sim exp\left(\frac{\mu}{\mu + \lambda}\right),$$

$$E(T) = \frac{1}{c\left[1 - \frac{\mu}{\mu + \alpha} - \frac{\mu}{\mu + \gamma}\right]}$$

$$= \frac{(\mu + \gamma)(\mu + \alpha)}{c\,[\alpha\gamma - \mu^2]} \qquad (7)$$

We know that

$$E(T^2) = \frac{d^2}{ds^2}\;\; l^*(s)\;\; given\;\; s = 0$$

$$E(T^2) = \frac{2}{c^2\left[1 - \frac{\mu}{\mu + \alpha} - \frac{\mu}{\mu + \gamma}\right]^2}$$

$$= \frac{2(\mu+\gamma)^2(\mu+\alpha)^2}{c^2\,[\alpha\gamma-\mu]^2} \qquad (8)$$

$$V(T) = E(T^2) - [E(T)]^2$$

$$= \frac{2(\mu+\gamma)^2(\mu+\alpha)^2}{c^2\,[\alpha\gamma-\mu]^2} - \frac{(\mu+\gamma)^2(\mu+\alpha)^2}{c^2\,[\alpha\gamma-\mu]^2}$$

$$= \frac{(\mu+\gamma)^2(\mu+\alpha)^2}{c^2\,[\alpha\gamma-\mu]^2}$$

(7) and (8) give the mean and variance of the time to recruitment for the present model.

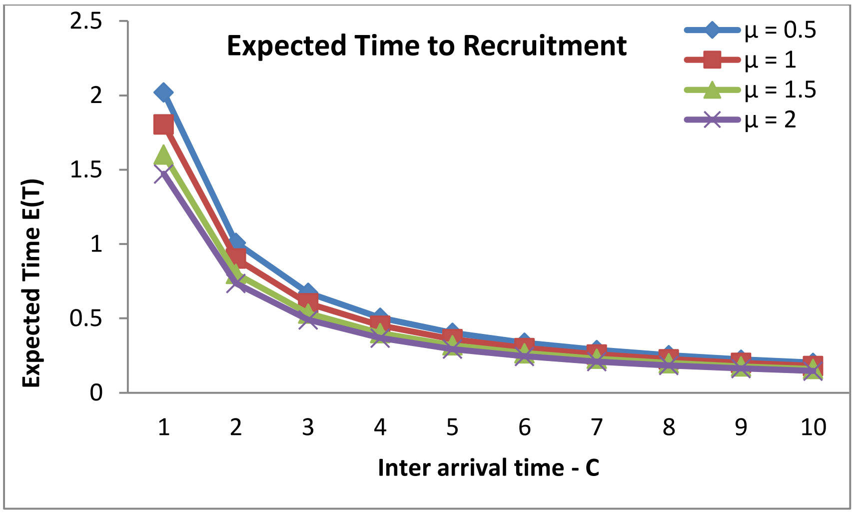

Figure: 1a

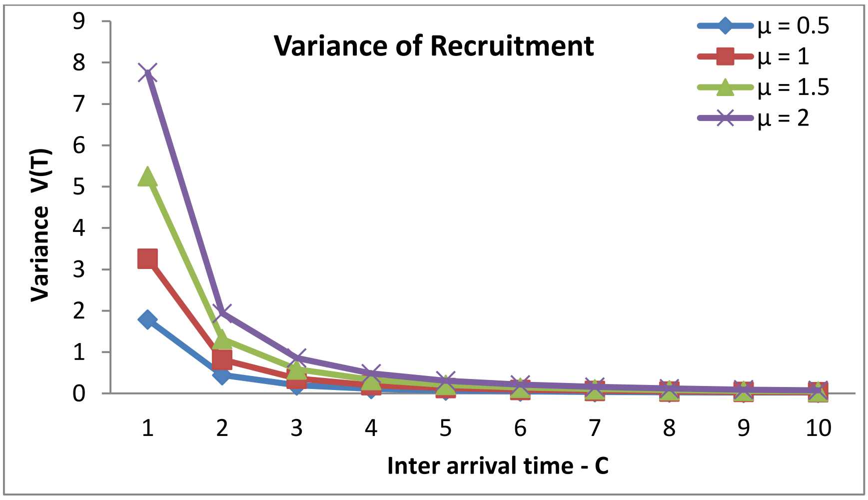

Figure: 1b

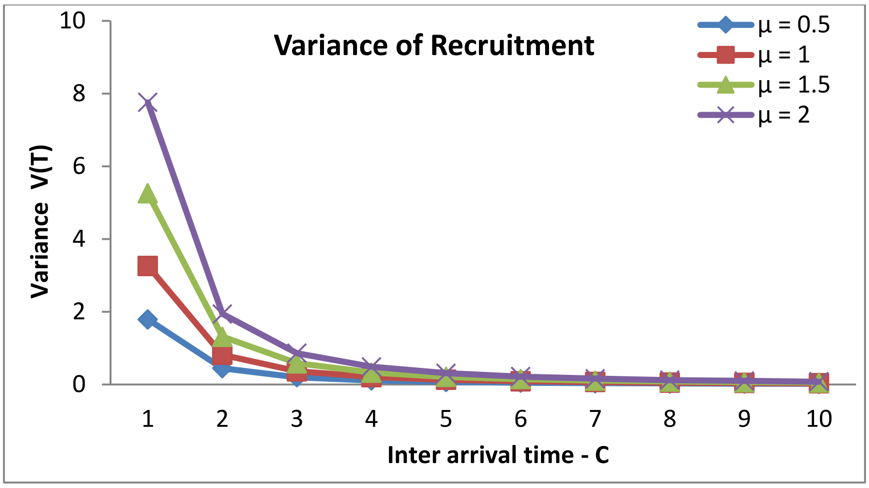

Figure: 2a

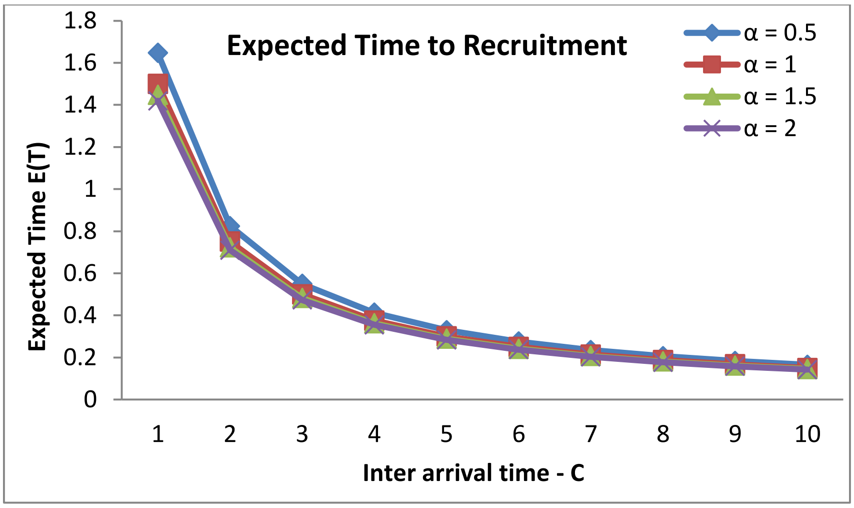

Figure: 2b

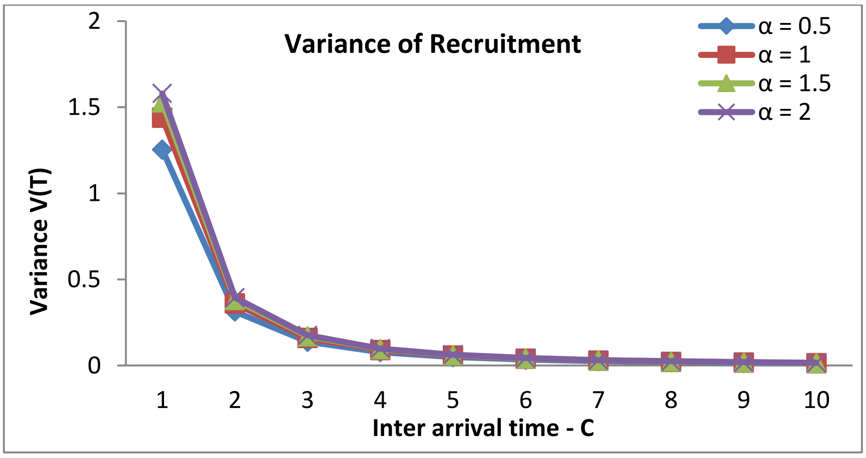

Figure: 3a

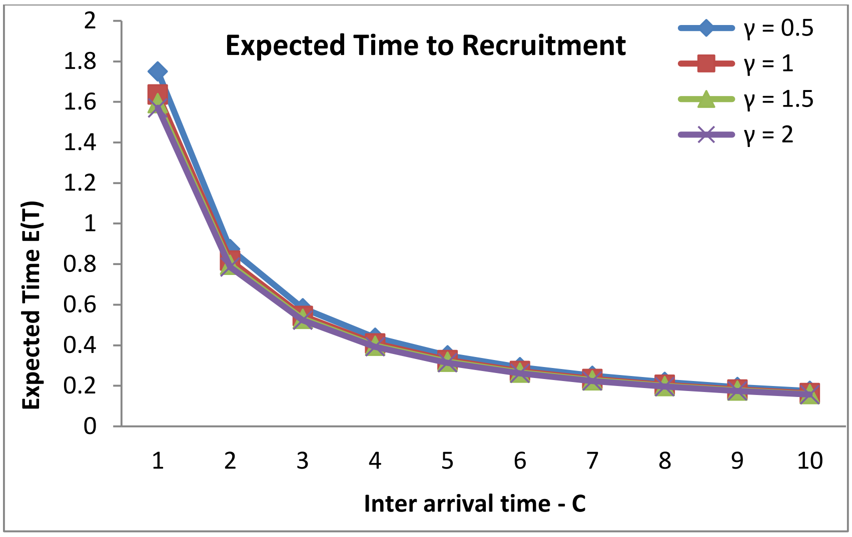

Figure: 3b

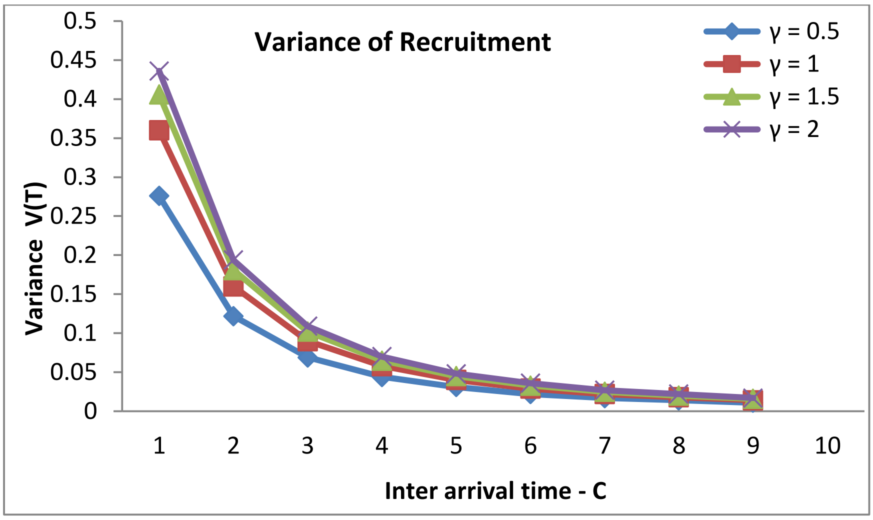

Figure: 3c

5. Conclusion

From the results for the performance measures, we note that the mean and variance of the time to recruitment increase with α and γ, μ parameters kept fixed. i.e., as the inter-arrival time increases the time to recruitment in an organization decreases in all the cases found. Hence, the results obtained are better from the organization's point of view when the inter-decision times are correlated. In the context of attrition, the model developed in this paper can be utilized to plan for adequate provision of manpower in the organization. The goodness of fit for the distribution assumed in this paper can be tested by collecting relevant data. Further, the observations on the performance measures given in this paper will be useful to enhance the simplification of the calculation of the manpower profile in future manpower development expectation, not only on industry but also in a wider field.

When α is kept fixed with other parameters γ, μ the inter-arrival time $'c'$, which follows exponential distribution, is an increasing parameter. Therefore, the value of the expected time $E(T)$ to cross the threshold is decreasing, for all cases of the parameter value $\alpha = 0.5,1,1.5,2$. When the value of the parameter α increases, the expected time is found decreasing, this is observed in Figure 1a. The same case is found in variance $V(T)$ which is observed in Figure 1b.

When γ is kept fixed with other parameters α, μ the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the threshold is found to be decreasing, in all the cases of the parameter value $\gamma = 0.5,1,1.5,2$. When the value of the parameter γ increases, the expected time is found decreasing. This is indicated in Figure 2a. The same case is observed in the variance $V(T)$ which is observed in Figure 2b.

When μ is kept fixed with other parameters α, γ the inter-arrival time $'c'$ increases, the value of the expected time $E(T)$ to cross the threshold is found to be decreasing, in all the cases of the parameter value $\mu = 0.5,1,1.5,2$. When the value of the parameter μ increases, the expected time is found decreasing. This is indicated in Figure 3a. The same case is observed in the variance $V(T)$ which is observed in Figure 3b.

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